John H. Van Vleck's Legacy to Radio Astronomy

by
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CfA

Innovation and Discovery in Radio Astronomy:
A Celebration of the Career of Ron Ekers
Queenstown, New Zealand
13–17 September 2016

John H. Van Vleck
(1899–1980)

Professor of physics, Harvard, 1936–1980

PhD in physics, Harvard (under E.C. Kemble), 1922
  First American thesis on quantum theory

Grandson of John Monroe Van Vleck
  Professor of astronomy and mathematics, Wesleyan University, 1853–1912

Main interests
  Quantum theory of magnetism (Nobel Prize* 1977)
  Quantum theory of molecules (Van Vleck line shape)

*with Nevill Mott and Philip Anderson

Nobel Prize Lecture
"The Key to Understanding Magnetism"

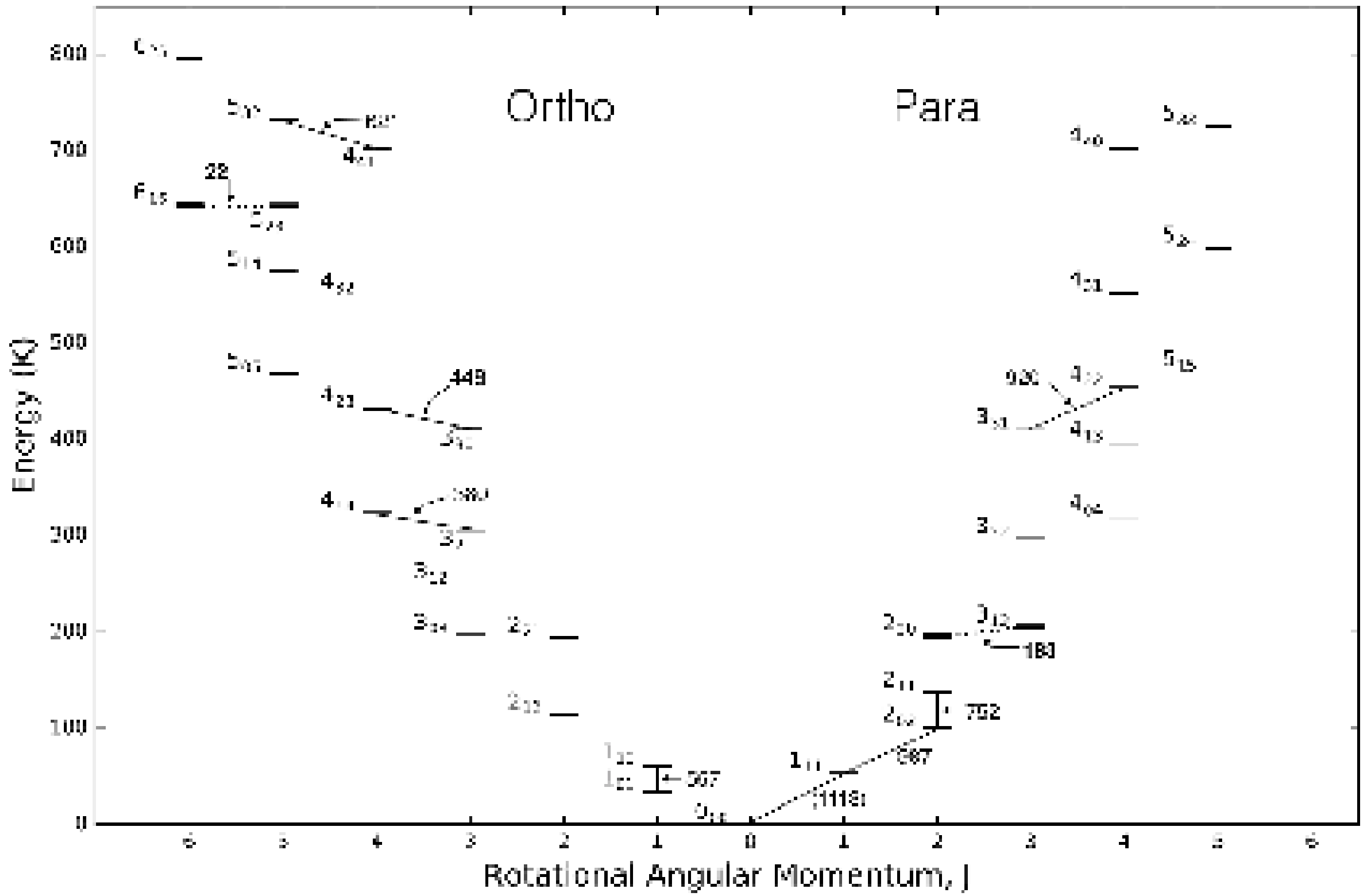
"This brings me up to the years of World War II, during which very little was done in the way of pure research."

But Van Vleck wrote two classified reports that had great impact on the later development of radio astronomy.

"The Atmospheric Absorption of Microwaves"
Report 43-2, Radiation Laboratory, MIT
April 27, 1942 (declassified August 1960)

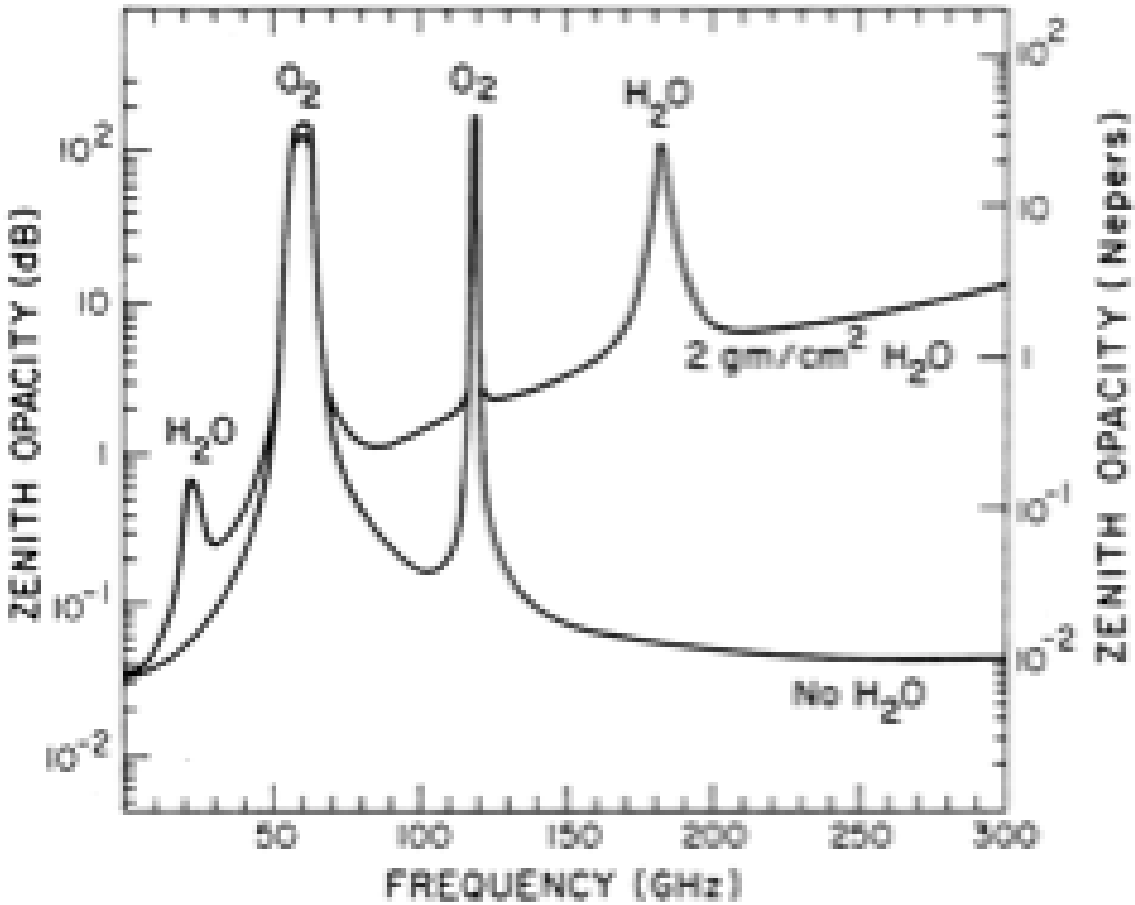
"The Spectrum of Clipped Noise"
Report 51, Radio Research Laboratory, Harvard University
July 21, 1943 (declassified March 1946)

Rotational Spectrum of Water Vapor

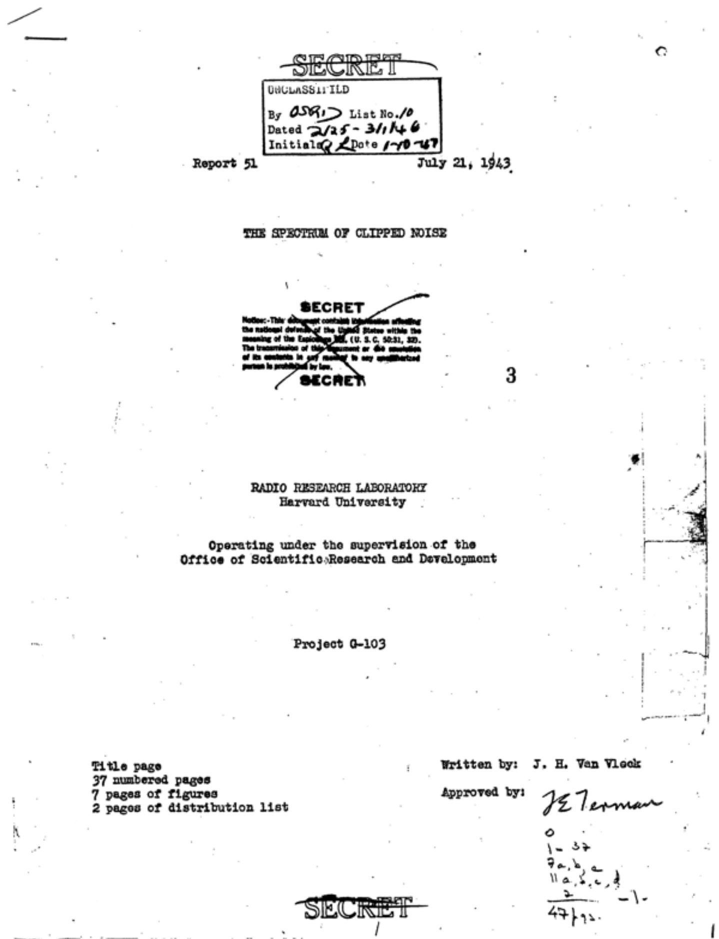


First accurate measurements with infrared spectroscopy by Randall et al., 1937

Atmospheric Absorption Due to Water Vapor and Oxygen



Cover of Van Vleck's Classified Memo on Clipped Noise



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Dated 2/25 - 3/1/46
Initials RL Date 1-10-67

Report 51 July 21, 1943

THE SPECTRUM OF CLIPPED NOISE

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RADIO RESEARCH LABORATORY
Harvard University

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Project G-103

Title page
37 numbered pages
7 pages of figures
2 pages of distribution list

Written by: J. H. Van Vleck

Approved by: F E Terman

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1 - 37
7 a, b, c
11 a, b, c, d
2
47 pp.

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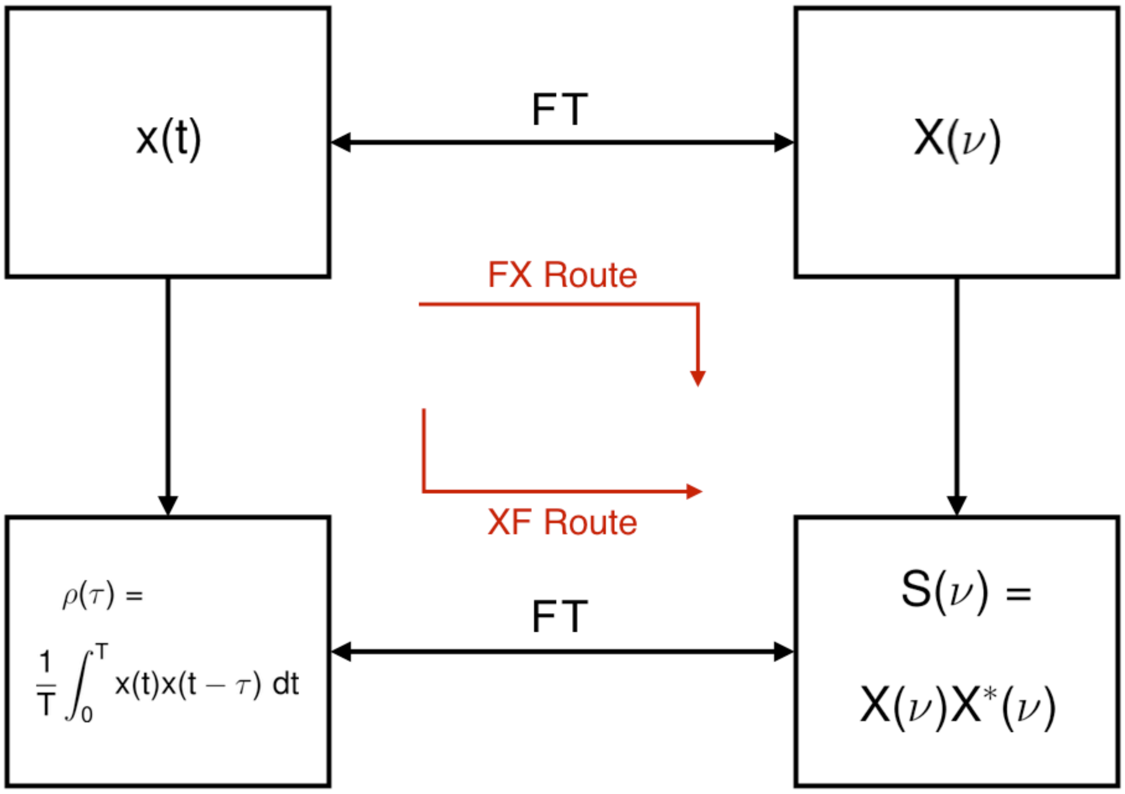
Fourier Transform Paradigm

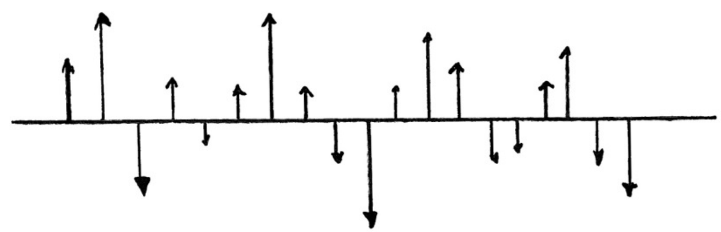


Van Vleck Clipping Correction

Random voltage

(Van Vleck, 1943, fig. 1)

Digital representation



$\longrightarrow \sigma^2\rho(\tau)$

Clipped representation

1101011100111001100

$\longrightarrow \rho_c(\tau)$

$$\rho(\tau) = \sin\left[\frac{\pi}{2}\rho_c(\tau)\right]$$

Van Vleck's Derivation of the "Arcsine" Relation

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uncl

III EXTREME CLIPPING

With extreme clipping the function f(X) involved in (10) has the form

$$f(X) = +1, \quad (X>0), \qquad f(X) = -1, \quad (X<0). \qquad (13)$$

We have here assumed a normalization such that after clipping the mean square amplitude is unity, or in other words that the ordinates of the horizontal straight lines in Figure 2 are ±1. The expression (10) becomes

$$R(t) = \frac{1}{2\pi\sqrt{1-r^2}}\left[\int_0^\infty\int_0^\infty e^{-\alpha}\,dX\,dY + \int_{-\infty}^0\int_{-\infty}^0 e^{-\alpha}\,dX\,dY - \int_0^\infty\int_{-\infty}^0 e^{-\alpha}\,dX\,dY - \int_{-\infty}^0\int_0^\infty e^{-\alpha}\,dX\,dY\right] \qquad (14)$$

where

$$\alpha = (X^2+Y^2-2rXY)/2(1-r^2).$$

We can simplify (14) by using the relation

$$\frac{1}{2\pi\sqrt{1-r^2}}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\alpha}\,dX\,dY = 1, \qquad (15)$$

which is readily verified mathematically and which is also obvious from the fact that the correlation would be unity if instead of (13) we had f(X) = 1 for all values of the argument. Also, it is convenient to introduce polar coordinates

$$X = \rho\cos\varphi, \quad Y = \rho\sin\varphi$$

It is thus found that (14) can be written

$$R(t) = 4\int_0^{\pi/2} d\varphi \int_0^\infty \frac{1}{2\pi\sqrt{1-r^2}}\, e^{-\frac{\rho^2(1-r\sin 2\varphi)}{2(1-r^2)}}\,\rho\, d\rho - 1. \qquad (16)$$

Integration gives

$$R(t) = \frac{2\sqrt{1-r^2}}{\pi}\int_0^{\pi/2}\frac{d\varphi}{1-r\sin 2\varphi} - 1 = \frac{2}{\pi}\sin^{-1}(r). \qquad (17)$$

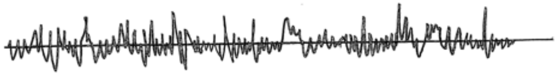
We thus have the rather simple and elegant result that the effect of extreme clipping is to make the correlation function $2/\pi$ times the arc sine of the original correlation function before clipping. In case there

Digital Spectrometers for Radio Astronomy

1943	Van Vleck	$\rho(\tau) = \sin\left[\frac{\pi}{2}\rho_c(\tau)\right]$
1950	Lawson & Uhlenbeck	Summary of VV
1966	Van Vleck & Middleton	Publication of VV
1963	Weinreb	MIT/DI line
1963	Goldstein	JPL/Venus radar
1965	Cooley-Tukey	FFT algorithm, $t = n \log n$
1974	Yen	Urged use of FFT
1984	Chikada	First FX correlator
1997	Escoffier (ALMA)	Last XF correlator (?)
2005	Bunton et al.	FX (& PFBs) ⟶

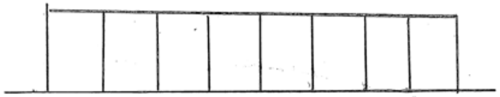
Windows for the FX and XF Schemes

Gaussian noise segment



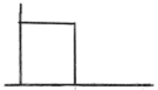
time

FX: segment blocks



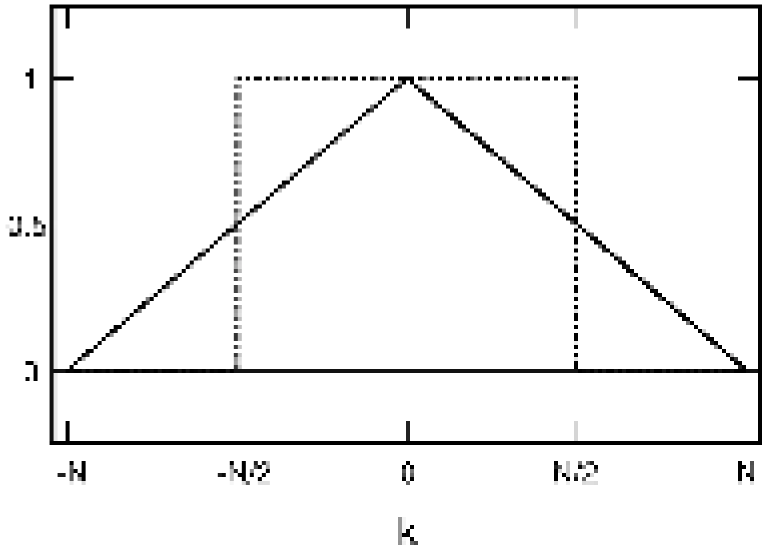
time

XF: delay window

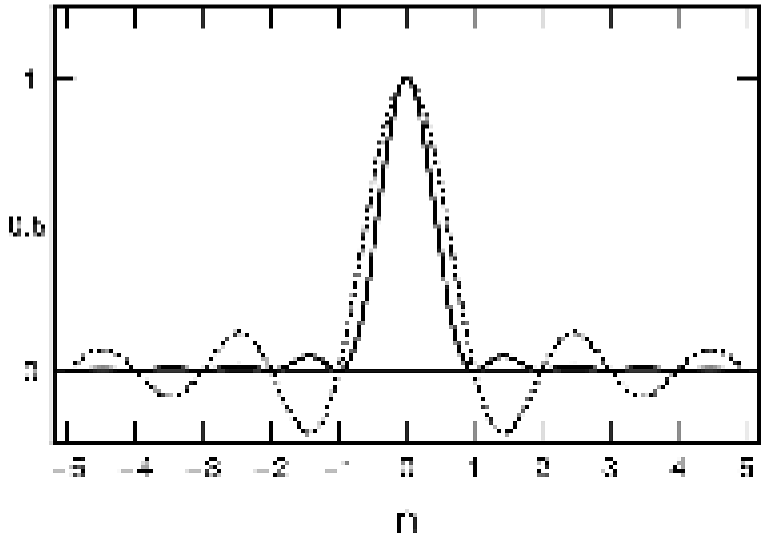


delay (lag)

Comparison of Window and Spectral Responses



Lag Density (number of multiplications)



Spectral response functions

Solid line: FX, dotted line: XF

SNR Ratio (FX/XF) vs. Linewidth Simulation