

The Effect of Noise on the Dust Temperature - Spectral Index Correlation

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ABSTRACT

We investigate how uncertainties in flux measurements affect the results from modified blackbody SED fits. We show that an inverse correlation between the dust temperature T and spectral index β naturally arises from least squares fits due to the uncertainties, even for sources with a single T and β . Fitting SEDs to noisy fluxes solely in the Rayleigh-Jeans regime produces unreliable T and β estimates. Thus, for long wavelength observations ($\lambda \gtrsim 200 \mu\text{m}$), or for warm sources ($T \gtrsim 60 \text{ K}$), it becomes difficult to distinguish sources with different temperatures. We assess the role of noise in recent observational results that indicate an inverse and continuously varying $T - \beta$ relation. Though an inverse and continuous $T - \beta$ correlation may be a physical property of dust in the ISM, we find that the observed inverse correlation may be primarily due to noise.

Subject headings: dust - infrared:ISM - methods:miscellaneous

1. Introduction

Recent advances in infrared and sub-millimeter observations have enabled detailed investigation of the properties of dust in a wide range of environments. Both ground and space based observatories, such as *SCUBA*, *Bolocam*, *MAMBO*, *Spitzer*, and *ISO*, as well as the balloon borne experiment *PRONAOS*, have revealed much about the nature of dust emission. The near-future observatories *Herschel* and *Planck* will also be capable of detecting dust in a variety of environments. One conclusion drawn from spectral energy distribution

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(SED) fits to flux measurements by *PRONAOS* is that the dust emissivity spectral index β has an inverse correlation with the dust temperature T , and that this correlation is well described as a hyperbola (Dupac et al. 2003). Désert et al. (2008), using longer wavelengths fluxes of galactic sources from the Archeops experiment, and Yang & Phillips (2007), through observations of luminous infrared galaxies (LIRGS), also find a similar anti-correlation.

In this short paper, we present an investigation of the effect of noise on the T and β estimates from SED fits to flux measurements. In recent work, we focused on starless cores, with $T \lesssim 20$ K, and showed that SED fits may result in inversely correlated and erroneous T and β estimates, due to line of sight temperature variations, noise, or both (Shetty et al., ApJ submitted, hereafter Paper I). Here, we consider warmer sources. We investigate how uncertainties in the fluxes, coupled with the $T - \beta$ degeneracy that arises from the functional form of the modified blackbody spectrum describing dust emission (Blain et al. 2003; Sajina et al. 2006), lead to a spurious inverse $T - \beta$ correlation from SED fits.

2. Modified Blackbody Assumption

The common assumption about the SED due to dust is that it is a blackbody modified by a power law in the frequency (Hildebrand 1983). For optically thin dust emission, the flux density S_ν takes the form

$$S_\nu = \Omega B_\nu(T) \kappa_0 \left(\frac{\nu}{\nu_0} \right)^\beta N. \quad (1)$$

Here, Ω is the solid angle of the observing beam, $B_\nu(T)$ is the Planck function, and N is the dust column density. The term $\kappa_0(\nu/\nu_0)^\beta$ is the frequency dependent opacity of the emitting dust. Dust in the interstellar medium is usually characterized by $1 \lesssim \beta \lesssim 2$ (e.g. Draine & Lee 1984; Mathis 1990, and references therein). Equation (1) only accurately characterizes sources with a single T (isothermal) and a single β .

With 4 or more fluxes, a direct fit of equation (1) to the fluxes can be employed to estimate three parameters: β , T , and $\kappa_0 N$ (Paper I, and references therein). As discussed in Paper I, the fitting results are sensitive to noise and temperature variations along the line of sight. A single temperature fit to fluxes near the peak of the SED may result in erroneous T and β estimates, due to line of sight temperature variations. For fluxes in the Rayleigh-Jeans (R-J) tail, the best fit T may provide an accurate estimate for the “column temperature,” or density weighted temperature. However, in the R-J tail, the fits are very sensitive to noise uncertainties. In this work, we consider isothermal sources, focusing on the effect of noise on T and β estimates from SED fits.

3. Effect of Noise on SED Fits

In order to assess the effect of noise on the SED fits, we employ simple Monte Carlo experiments. We compute the fluxes at various wavelengths from sources with given temperatures and spectral indices, using equation (1). To incorporate the effect of noise, we then multiply each flux by a factor $1+\epsilon$, where ϵ is a random value drawn from a Gaussian distribution with a chosen dispersion σ (and mean of 0). We then perform a minimized Chi-squared fit of equation (1) to the noisy fluxes, and compare the resulting β and T estimates to those of the sources.

Figure 1 shows the 75% and 50% probability contours of the best fit T and β for three isothermal sources, with $T=20, 60,$ and 100 K, and with $\beta=2$. To obtain good statistics, we perform fits to numerous sets of data. The fluxes are sampled at $10 \mu\text{m}$ intervals, in the range $\lambda \in 100\text{-}600 \mu\text{m}$. The noise levels of the fluxes are 5% and 10%. For a 5% error in the fluxes, the mean (or median) of the T and β fits provide a good estimate of the source parameters for the 20 K source. Though an inverse $T - \beta$ relationship is apparent, the spread about the source parameters is very small. However, for the warmer sources, the fits show a clear inverse $T - \beta$ relationship; the mean or median T and β also do not correspond to the source values. We note that weighting the fluxes by signal-to-noise, or fitting in $\log(\lambda)\text{-}\log(S_\nu)$ space, can improve the fits, but an inverse $T - \beta$ relationship still emerges. In this work all the fluxes are weighted equally, in order for direct comparison with recent investigations in §5.

Also shown in Figure 1(b) are fits to fluxes from a 20 K source with $\beta=1.5$. These fits illustrate that sources with different values of the spectral index, but with identical temperatures, would give vertically offset, but otherwise similar, points in a $T - \beta$ plane.

In practice, only a few fluxes are typically available from observations for fitting an SED. To simulate this more realistic scenario, we consider fits to sparsely sampled fluxes. Figure 2 shows the 75% and 50% likelihood contours of the best fit T and β from fits to five fluxes, with $\lambda=100, 200, 260, 360,$ and $580 \mu\text{m}$. In comparison with Figure 1, the spread in both β and T estimates is larger, especially for fits to fluxes from the 20 K source. The fits from the warmer 60 K and 100K sources are less distinguishable from each other, compared with the analogous fits incorporating many more fluxes shown in Figure 1. Further, fits to fluxes from the warmer sources are less likely to recover the source temperature: for the 60 K source, and with $\sigma=5\%$, only $\sim 20\%$ of the fits recover temperatures within 10% of the source temperature; for the 100 K source, only $\sim 10\%$ of the fits accurately recover the source temperature within 10%.

Also shown in Figure 2 (as a dashed line) is the best fit hyperbola to *PRONAOS* and

IRAS data found by Dupac et al. (2003). The wavelengths of the five fluxes considered in the fits in Figure 2 are the same as those considered by Dupac et al. (2003). After describing the degeneracy between β and T in the next section, we discuss implications of our analysis on recent observational interpretations in §5.

4. Degeneracy between the Spectral Index and the Temperature

We have shown that modest levels of noise can result in grossly misleading T and β estimates from a least squares fit of equation (1).¹ The best fit β is anti-correlated with the best fit T due the degeneracy between β and T (Dupac et al. 2001; Blain et al. 2003; Sajina et al. 2006; Désert et al. 2008).

In the R-J part of the SED, fits to fluxes from various warm sources may be difficult to distinguish (e.g. the 60 and 100 K fits in Figure 2). The reason for the confusion in the resulting fits is illustrated in Figure 3. Figure 3(a) compares two SEDs, with $T=60$ K and 29.3 K, and $\beta=2$ and 3.4, respectively. The latter is a fit to five noisy fluxes from the 60 K source. The two SEDs are rather similar at large wavelengths, especially in the range $\lambda \in 100\text{-}600 \mu\text{m}$, as shown in Figure 3(b). In this wavelength range, there are numerous combinations of T and β that are consistent with the observed SED, and therefore it is difficult to constrain them. Thus, fitting modified blackbody SEDs solely to fluxes in the R-J regime produces unreliable T and β estimates. Moreover, the degeneracy between T and β is such that an overestimate in β results in an underestimate in T .

For isothermal sources, fits to fluxes near the peak of the SED provide more accurate estimates of the source parameters (Paper I). Thus, the 100-600 μm wavelength range considered here can be shown to provide accurate estimates for source with $T \lesssim 20$ K. This is why the mean value of T and β recovers the source values for the cold 20 K source in Figures 1 and 2, but not so for the warmer sources. Shorter wavelength observations in the Wien regime may mitigate the effect of noise for warmer sources; but, at wavelengths $\lambda \lesssim 100 \mu\text{m}$, embedded sources as well as transiently heated very small grains (in cold regions) may contribute to the observed flux (Li & Draine 2001).

¹Other techniques, such as Bayesian or likelihood based methods, may provide better estimates of the source parameters.

5. Comparison with Recent Observations

Dupac et al. (2003) find an inverse correlation between the best fit T and β from *PRONAOS* data. In their analysis, they consider the degeneracy between T and β by performing statistical tests on noisy model fluxes. They find that noise is insufficient to account for the derived $T - \beta$ trend, and suggest that an inverse $T - \beta$ correlation is an intrinsic characteristic of dust in the ISM. In our extension of their work, we find a greater scatter in T and β (see Fig. 4 of Dupac et al. (2001)). That the best fits populate the T and β plane with a very similar shape to that produced simply by the presence of noise is strongly suggestive that noise is affecting these fitting results (see Figs 1 and 2).

The T and β fits from Dupac et al. (2003) show lower values of β at $T \gtrsim 20$ K, compared with the distribution in Figures 1 and 2. As already discussed, comparing isothermal sources with different values of β would simply result in vertically offset points on the $T - \beta$ plane. We thus consider whether sources with different temperatures, but with $\beta=1.5$, are consistent with PRONAOS data, focusing on M17 (Dupac et al. 2002) and Orion (Dupac et al. 2001), which are the two sources that span the full range $T \in 20$ -80 K in Figure 3 of Dupac et al. (2003).

Figure 4 shows numerous best fit T and β to five noisy fluxes from 30 K, 40 K, and 50 K isothermal sources, with $\beta=1.5$. For the warmer sources, the fits are similar to many of the points in Figure 4 of Dupac et al. (2002) at $T \gtrsim 20$ K. As Dupac et al. (2002) describe, at some positions the IRAS 100 μm fluxes were not included in the fit. Figure 4 also shows such fits. Though this simple model cannot account for the points at low T in Figure 4 of Dupac et al. (2002), many of the points at $T \gtrsim 20$ K are very similar. Thus, a model with constant $\beta \sim 1.5$, for dust at temperatures $\gtrsim 30$ K, may account for many of the PRONAOS derived T and β points (at $T \gtrsim 20$ K). Such temperatures may be realistic for much of the dust in M17 (e.g. Goldsmith et al. 1997).

To verify whether any substantial differences can be easily identified in cases where β is constant or variable, we perform the Monte-Carlo experiments described in §3 assuming models with different $\beta - T$ dependencies. The first model has $\beta=1.5$ for all T ; the second has $\beta=1/(0.4+0.008T)$, which is the fit found by Dupac et al. (2003). We construct a series of sources with $T \in 15$ -80 K, in increments of 5 K, and fit equation (1) to numerous sets of noisy fluxes at 100, 200, 260, 360, and 580 μm . Figures 5(a)-(b) show the results for fits to fluxes with noise levels of 5%, for both models, along with the best fit hyperbola from Dupac et al. (2003).

Figures 5(a)-(b) show that the fit parameters from the lowest temperature sources at 15, 20, and 25 K are distinct from the warmer sources. As already discussed, at these

temperatures, given the wavelengths of the fluxes considered, the peaks of the emergent SEDs are well sampled, so the fits are not very sensitive to noise.

The main difference between the constant- β and the inverse-hyperbolic $\beta(T)$ models is that there is less scatter in the fit β from the model where β depends on T . This difference occurs because the sources with low temperatures have higher spectral indices (and vice versa); thus, this inverse-hyperbolic $\beta(T)$ function reduces the scatter about β for a given T relative to the scatter from the constant β case.

The extent of the scatter in Figure 5 is dependent on the chosen level of noise, the form of $\beta(T)$, as well as the *temperatures of the sources considered in this test* (and the wavelengths of the sampled fluxes). For example, for the range $T \in 40\text{--}50$ K in the constant- β model (green points in Fig. 5(a)), the fits are well described by the best fit hyperbola from Dupac et al. (2003). Further, simply increasing the noise level in the model where β varies inversely with T (Fig 5(b)) could produce a $T - \beta$ scatter plot that is similar to the constant β model with the same temperature range (Fig. 5(a)). The primary effect of an intrinsic inverse $T - \beta$ correlation is to reduce the spread in the $T - \beta$ plane due to noise in the flux measurements.

Despite the differences in the scatter between the models in Figure 5(a)-(b), there is a striking similarity in the shape of the $T - \beta$ distribution: even though the models have different forms in $\beta(T)$, fits to the fluxes result in a similar inverse $T - \beta$ correlation. Due to this similarity, it is very difficult to distinguish an intrinsic $T - \beta$ anti-correlation from the artificial anti-correlation arising due to noise. Alternative estimates of the actual range in source temperatures, such as molecular line observations or radiative transfer modeling, would be necessary to accurately account for the spread in T and β due to the intrinsic degeneracy between the parameters in least squares SED fitting.

One method to quantify the contribution of measurement uncertainties to derived correlations is the use of correlation coefficients (e.g. Kelly 2007). Dupac et al. (2001) (in §4.2) compare the correlation coefficient of their best fits from Orion observations to a model with no correlation between T and β . The model has numerous uncorrelated T and β pairs, from which noisy fluxes are constructed; fits to those noisy fluxes result in a correlation coefficient between T and β of -0.4. The discrepancy between this value and the value from the observations, -0.92, indicates that a uniform and uncorrelated distribution in T and β can be ruled out (see also Yang & Phillips 2007). Such a test, however, has not ruled out scenarios where β is constant, the temperature range is limited, or both.

The correlation coefficients from $T - \beta$ fits to noisy fluxes from any isothermal model are $\lesssim -0.90$, which are remarkably similar to the value from the fits to Orion observations. In a case where there is a small range in T , such as the green points in Figure 5(a) ($T \in 40\text{--}50$

K), we compute a correlation coefficient of -0.8, also similar to the value found from the observations. These statistical tests suggest that a constant β , with a limited range in the source temperature, cannot be ruled out in interpreting the *PRONAOS* Orion observations.

If the *PRONAOS* observations detected sources with $T \lesssim 20$ K, and given the relative flux uncertainties of $\sim 5\%$ (Dupac et al. 2001; Pajot et al. 2006), then the lack of points with $T \lesssim 20$ and $\beta \lesssim 1.2$ is suggestive that β for such cold sources cannot be a constant value of 1.5. One possibility is that $\beta=2$ for the coldest sources, but $\beta = 1.5$ for sources with $T \gtrsim 30$ K. Similar to an inverse hyperbolic function in $\beta(T)$, such a step function would result in a $T - \beta$ distribution with less scatter compared to a scenario where β is constant. Figure 5(c) shows the fits from this step function in $\beta(T)$. This model illustrates that other functional forms of $\beta(T)$ may produce a $T - \beta$ distribution similar to that found by the *PRONAOS* observations.

The observationally inferred $T - \beta$ correlation is certainly consistent with, e.g. an inverse intrinsic hyperbolic $\beta(T)$ function. Yet, our analysis suggests that the intrinsic functional form of $\beta(T)$ cannot be reliably derived from a fit to the T and β values obtained from a least-squares SED fit to the observed fluxes. Other functional forms of $\beta(T)$, in combination with the noise levels, may also be consistent with the observations. The agreement between the $T - \beta$ correlation coefficients from fits to *PRONAOS* Orion observations and our isothermal, single β models suggest that T and β may not be correlated. Other statistical tests are needed to quantify the $T - \beta$ degeneracy.

A more complete understanding of the distribution in the source T and β is necessary to accurately infer any physical $T - \beta$ correlation. The scatter in the best fit parameters about the hyperbola in the Dupac et al. (2003) study is likely a consequence of various factors. Since the wavelengths of the observed fluxes are in the R-J part of the emergent SED from warm sources, noise does significantly bias the fitting. Additionally, the dataset presented in Dupac et al. (2003) contains numerous sources, with different spectral indices and temperatures, between sources and perhaps even within individual sources; these issues may all contribute to the scatter in the distribution of T and β .

6. Summary

We have shown that a natural consequence of fitting modified blackbody SEDs to flux measurements, with modest noise uncertainties, is an inverse correlation between the temperature T and spectral index β . Such an inverse correlation arises even for isothermal sources with a constant β . Least squares fits to fluxes from the Rayleigh-Jeans regime of the

emergent SED are very sensitive to noise uncertainties. Consequently, various isothermal sources with different T s but identical β s may be indistinguishable through simple SED fits.

We find that the spurious $T - \beta$ anti-correlation due to noise, from numerous isothermal sources with a limited range in T and β , is similar to the observationally inferred anti-correlation (§5). A hyperbolic $T - \beta$ relation may indeed be a physical characteristic of dust, but the observational results show similarities with other explanations, such as a single β for isothermal sources with $T \gtrsim 30$ K, or a step function where $\beta \approx 2$ for $T \lesssim 20$ K and $\beta \approx 1.5$ for $T \gtrsim 30$ K. Thus, least squares SED fits to measured fluxes may not be able to reveal any intrinsic correlation between β and T .

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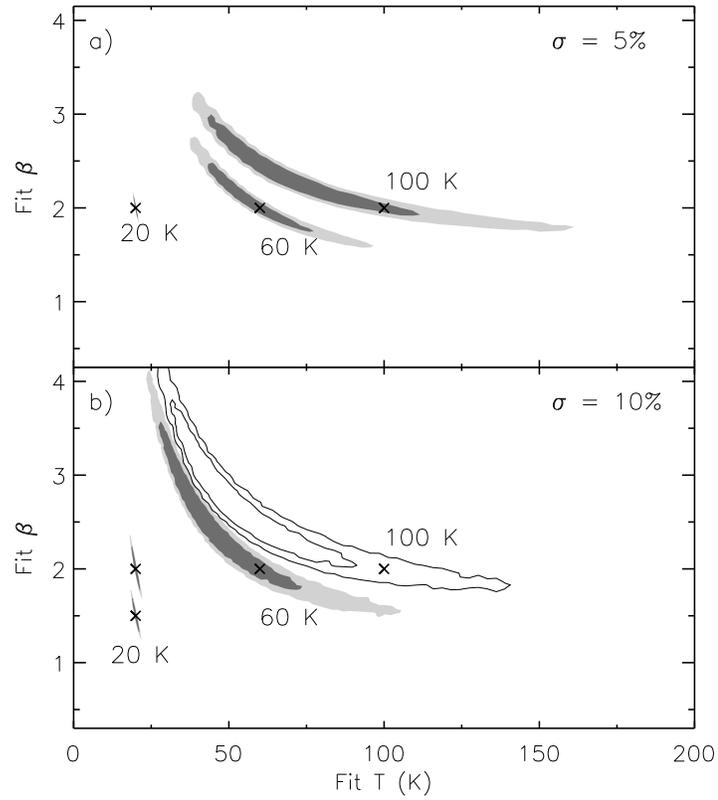


Fig. 1.— The 75% and 50% probability contours for the best fit T and β to noisy fluxes, in the wavelength range 100-600 μm (sampled at 10 μm increments), from 20 K, 60 K, and 100 K isothermal sources with $\beta = 2$ (marked by crosses). Gaussian distributed noise is included in each flux, with (a) $\sigma=5\%$ and (b) $\sigma=10\%$. In (b), best fit T and β from a 20 K source with $\beta=1.5$ is also shown.

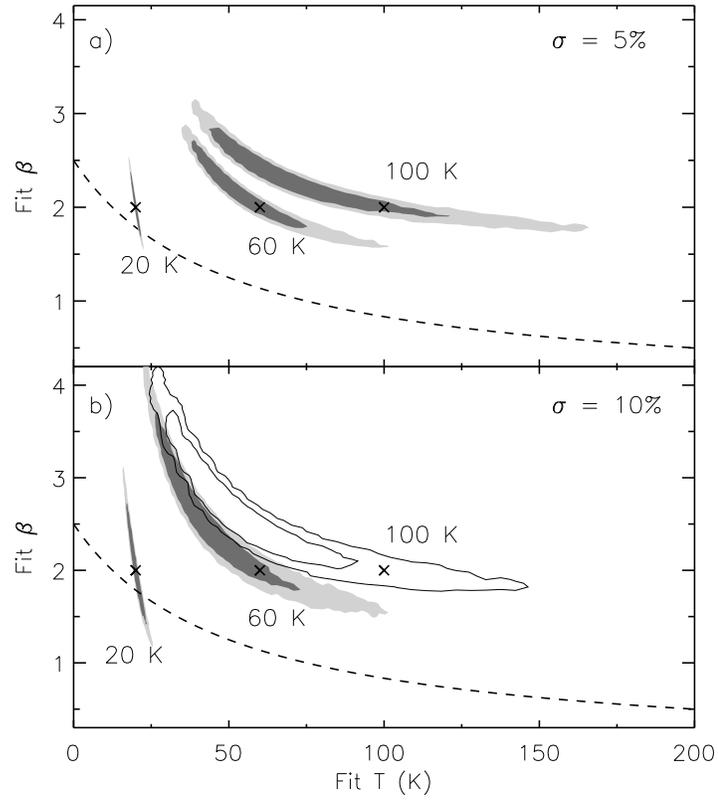


Fig. 2.— The 75% and 50% probability contours for the best fit T and β to noisy fluxes, with (a) $\sigma=5\%$ and (b) $\sigma=10\%$. The wavelengths of the sampled fluxes are $\lambda=100, 200, 260, 360,$ and $580 \mu\text{m}$, from three isothermal sources (as in Fig. 1). The dashed line is the best fit hyperbola from Dupac et al. (2003).

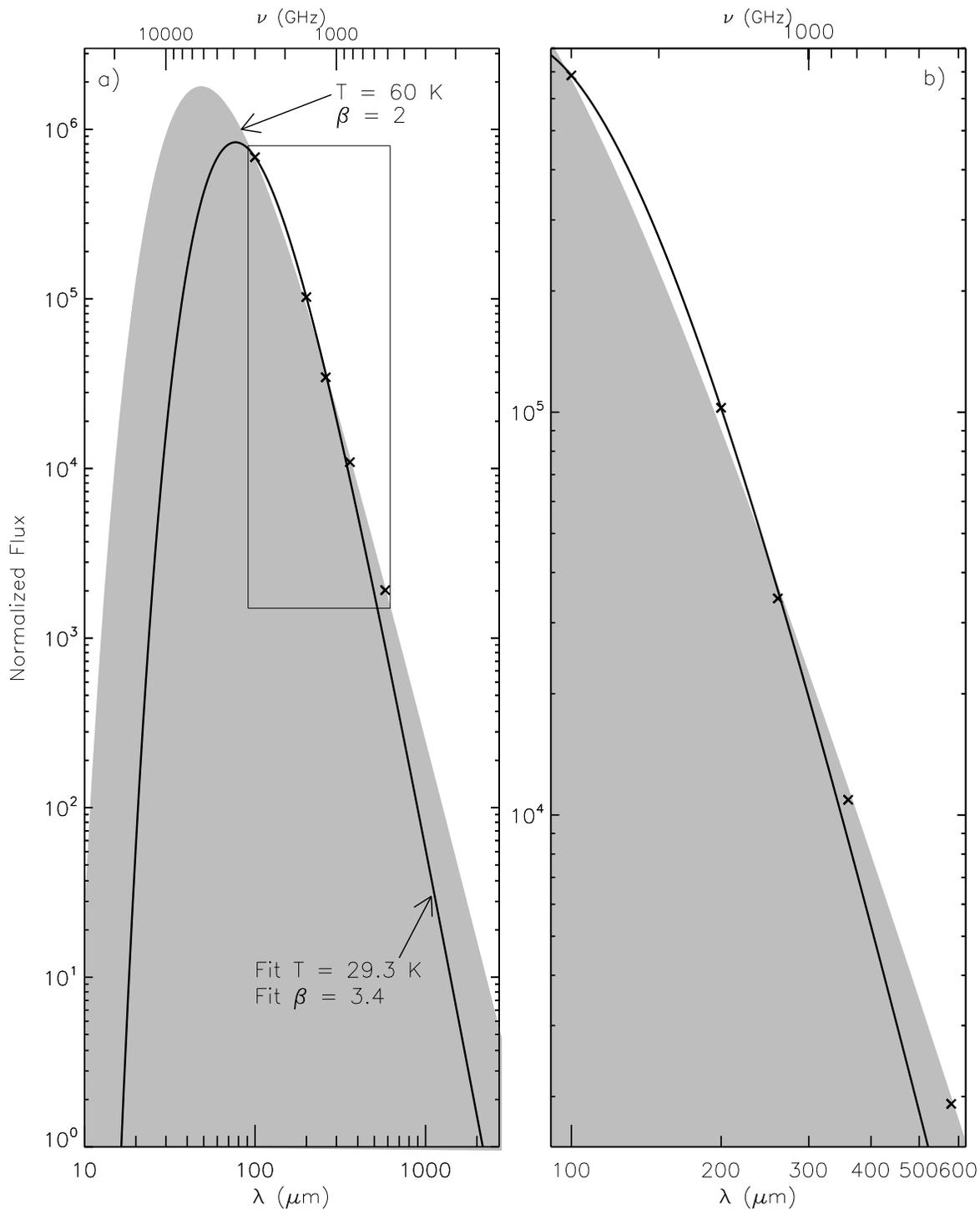


Fig. 3.— Actual and fit SEDs from a 60 K isothermal source. The boundary of the shaded region is the dust SED from a 60 K source. The solid curve shows a fit to the few noisy fluxes (crosses) at $\lambda=100, 200, 260, 360,$ and $580 \mu\text{m}$. The errors in those fluxes are 2.1%,

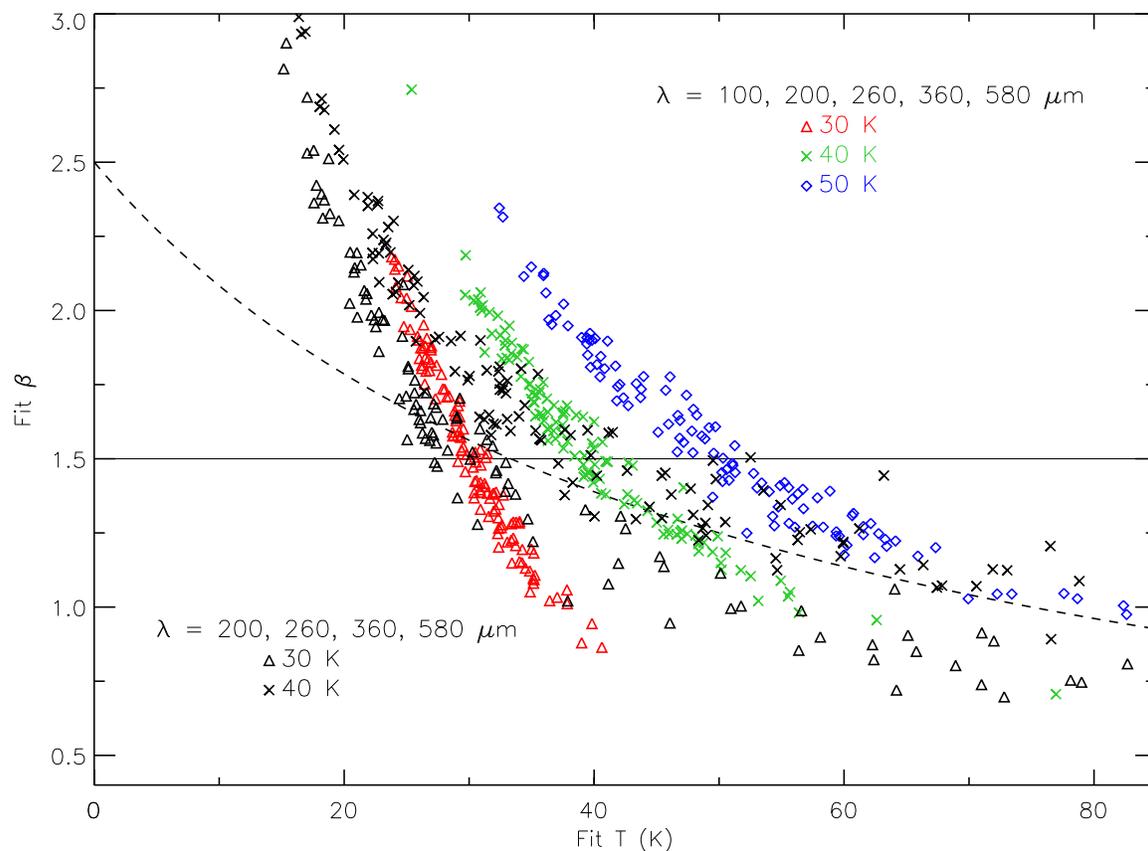


Fig. 4.— Best fit β and T to noisy fluxes (with $\sigma=5\%$) from isothermal sources with $T \in 30$ - 50 K. Colored points show fits to fluxes with $\lambda=100, 200, 260, 360,$ and $580 \mu\text{m}$. Black points show fits to fluxes from a 30 and a 40 K source excluding the $100 \mu\text{m}$ flux. The horizontal line indicates the spectral index of the source, $\beta=1.5$. The dashed line shows the best fit to data presented by Dupac et al. (2003).

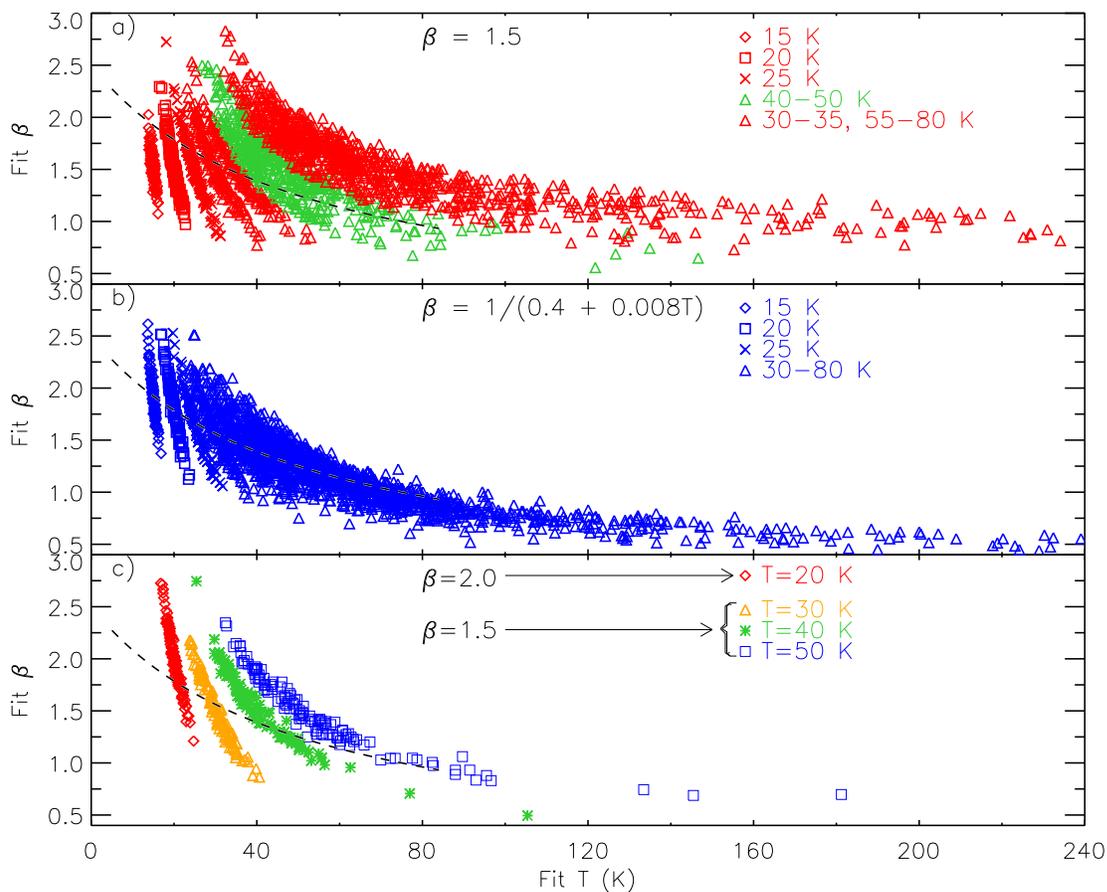


Fig. 5.— Best fit β and T to noisy fluxes (with $\sigma=5\%$) from isothermal sources with a range of temperatures, for three sets of models, along with the best fit from Dupac et al. (2003) (dashed line). (a) $\beta=1.5$; (b) $\beta=1/(0.4+0.008T)$; and (c) a step function in β , with $\beta=1.5$ for $T > 20$ K and $\beta=2.0$ for $T=20$ K. The fluxes have $\lambda=100, 200, 260, 360,$ and $580 \mu\text{m}$.