Astronomy 218
Homework 8

December 15, 2005
Due January 13, 2005 (Noon)
Deliver to M-337

1. A uniform spherical radio source, such as an H II region with constant temperature and density, has diameter $2a$ and is at a distance R from the earth, such that $a/R \ll 1$. If the opacity along the line of sight through the center of the sphere is τ,

   a) what is the opacity of rays not passing through the center of the sphere?

   b) Show that the received flux density will be

   $$S = B\Omega\left[1 + e^{-\tau}(1+\tau)\frac{2}{\tau^2} - \frac{2}{\tau^2}\right]$$

   where $\Omega = \pi a^2/R^2$ and $B =$ Planck function.

   c) what is S for $\tau \ll 1$ and $\tau \gg 1$?

   d) what is the apparent angular size of the source (full width at half intensity) for $\tau \ll 1$ and $\tau \gg 1$?

2. Consider an ionized wind from a star. As derived in class, the spectra has the form $S \propto \nu^{0.6}$, when the flow velocity is constant so that $n_e \propto r^{-2}$. Suppose the ionization has an inner (i.e. surface of star) and an outer cut off (ionizing photons exhausted). Hence, the electron density distribution is non zero only from $r_1 < r < r_2$, where r_1 and r_2 are the inner and outer radii, respectively.

   What is the spectrum of such a source? (Ignore emission from the star itself.) The spectrum will have several distinct regimes. Don't worry about details of the transition regions.

3. Show that the excitation temperature in a molecular cloud in which only two energy levels are considered and where transitions take place due to collisions or radiation is given by

   $$T_S = T_K\,\frac{T_0 + T_R}{T_0 + T_K}$$

   where T_R is the temperature of the radiation field and T_K is the kinetic temperature, and

$$T_0 = \frac{g_1}{g_2}\frac{h\nu}{k}\frac{C_{12}}{A}$$

where C_{12} is the collision rate, A is the spontaneous emission coefficient, and g_1 and g_2 are statistical weights.

Explain why molecules with larger Einsetin A coefficients will be excited in the denser regions of a cloud.

4. Consider the water maser in NGC 4258 (see Moran, Greenhill and Herrnstein, *J. Astrophys. Astr.* (1999) **20**,165-185, if necessary). These masers define an accretion disk with inner and outer radii of 0.14 and 0.28 pc, respectively. The disk is unresovled in the vertical direction and less than 0.0003 pc thick.

   a) Show that a disk in hyrdrostatic equilibrium by the relation

$$\frac{H}{R} \cong \frac{c_s}{v}$$

   where H is the thickness, R is the radius c_s is the sound speed and v is the Keplerian rotation speed. Proceed by considering a gas clump that is above the midplane of the disk. In hydrostatic equilibrium the vertical component of gravity due to the central mass (aka BH) is equal to the vertical pressure gradient ($p = \rho\, {c_s}^2$, where ρ is the density).

   b) Since $c_s \sim \sqrt{T}$, what is the maximum temperature of the accretion disk allowed by the measurements?

   c) For an isothermal accretion disk, how would the disk thickness vary with radius?