

# Efimov states close to a Feshbach resonance with large background scattering length

[Pietro Massignan](#)

October 30, 2009



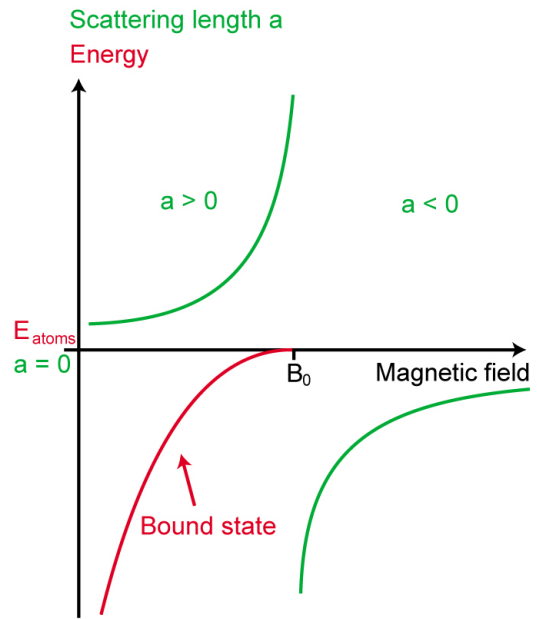
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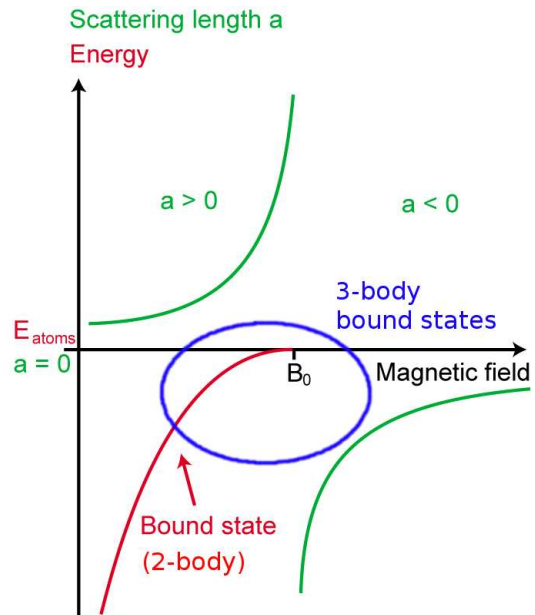
(work done in collaboration with Henk Stoof, ITF Utrecht)

Conference on “Efimov states in molecules and nuclei”, Rome

# Feshbach resonances in ultracold gases

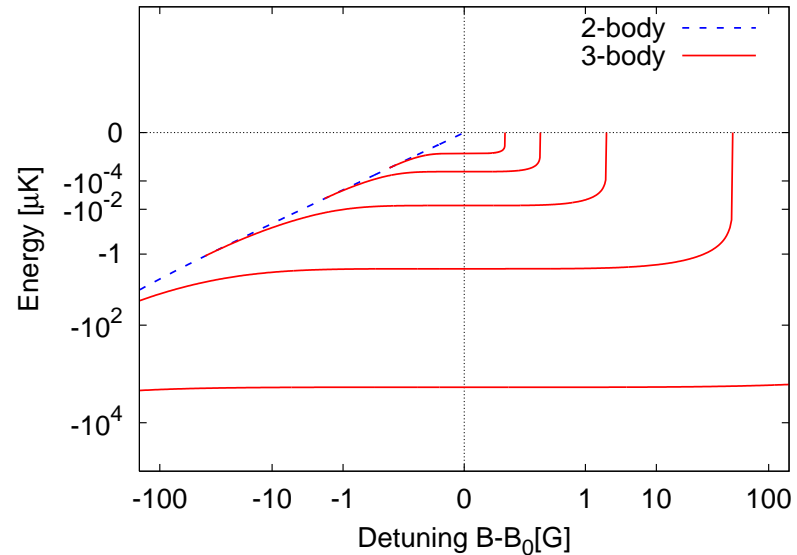
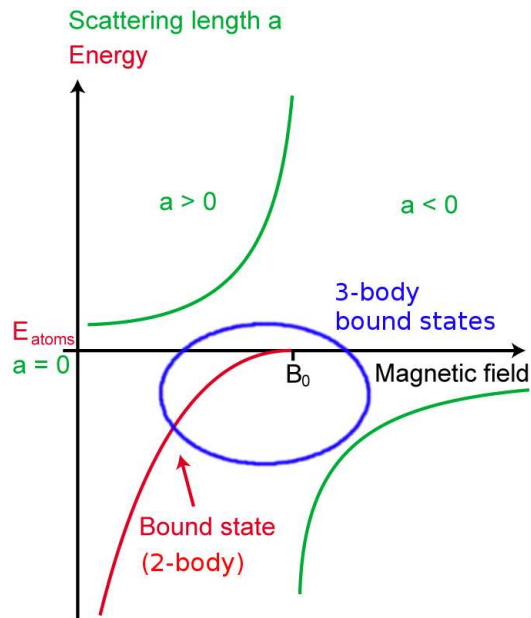


as  $|a| \gg R$  there's more...(Efimov, 1970s)



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rescaled units:  $E^{1/10}$  vs  $(B - B_0)^{1/5}$



Discrete scaling:  $\frac{a_{n+1}}{a_n} = 22.7, \quad \frac{E_{n+1}}{E_n} = 22.7^{-2}$

Review: Braaten and Hammer (2006).

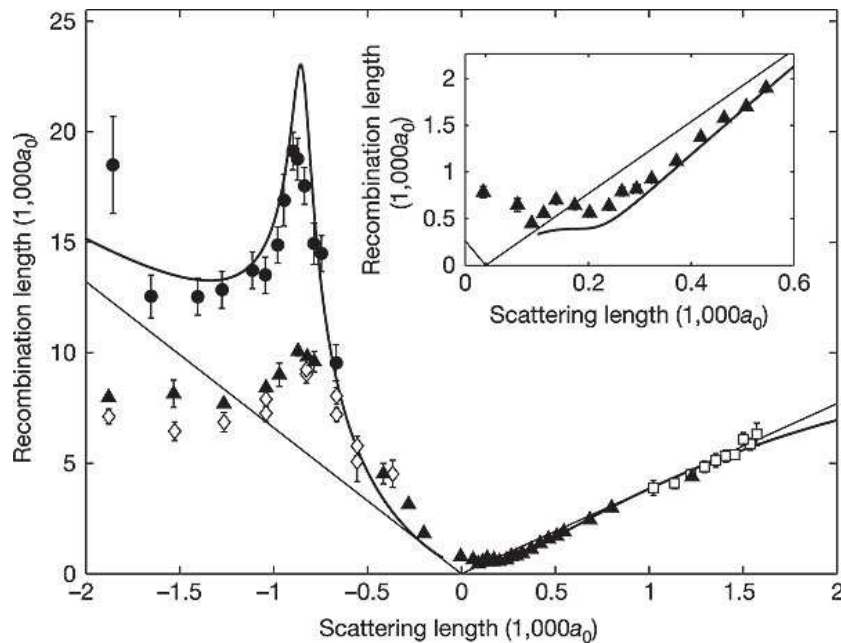
Universality: physics depends on  $a$  and an additional (3-body or low-energy) parameter.

## Innsbruck experiment (Nature 2006)

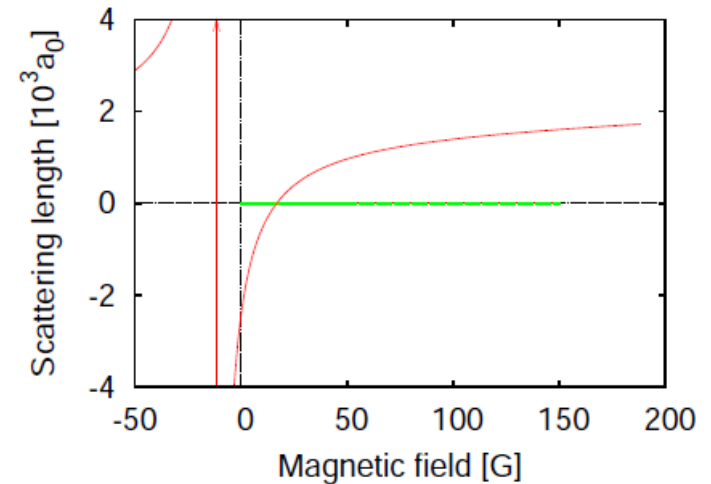
First experimental evidence for Efimov physics in cold gases.

Measurement of the recombination rate for  $A+A+A \rightleftharpoons A+D$  (A:atom, D:dimer) in a thermal gas of bosonic caesium.

$$dn/dt \propto -n^3 \rho_{\text{rec}}^4$$



$$\rho_{\text{rec}} \propto a \cdot C(a), \text{ with } C(a) \text{ log-periodic in } a$$

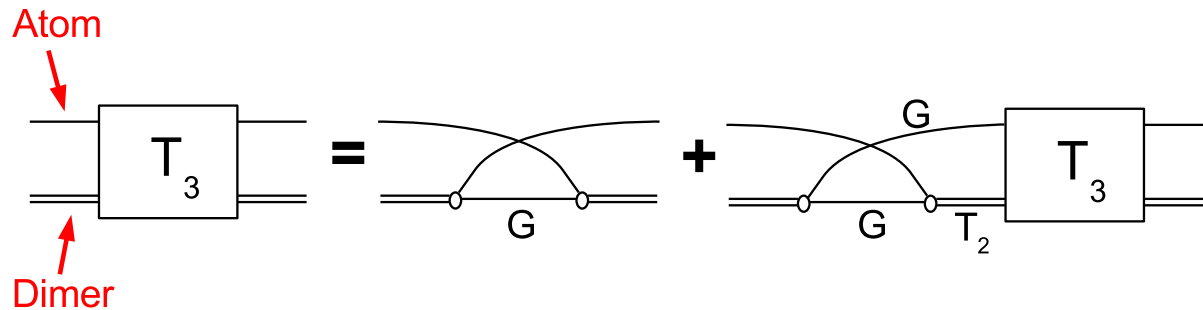


Universality? 1)  $B_0 < 0$ ,

2)  $|a| \sim R$ ,

3)  $a_{\text{bg}} \sim 1800a_0 \gg R$

## $T_3$ : T-matrix for atom-dimer scattering



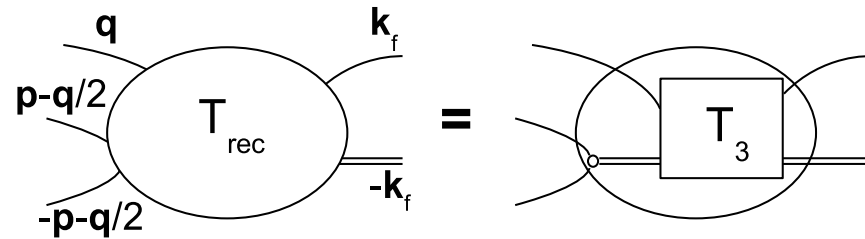
2-body s-wave interaction  $\rightarrow$  **STM** eq. (Skorniakov&Ter-Martirosian, 1956):

$$T_3(k, k'; E) = G(\dots) + \int_0^\infty dk'' G(\dots)G(\dots)T_2(\dots)T_3(\dots).$$

$G$ : atom propagator,  $T_2$  : atom-atom scattering matrix (or dimer propagator), more to come. . .

Scattering length:  $a_{AD} \propto T_3(0, 0; E_b)$ .

### 3-body recombination rate: $AAA \Rightarrow AD$

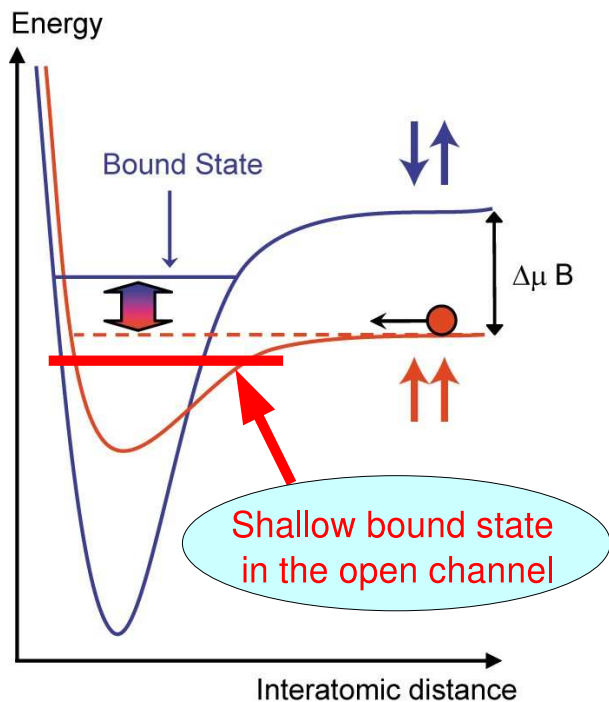


$$T_{\text{rec}}(p, q; E_b) \propto T_2(\dots) T_3(\dots)$$

$$\frac{dn}{dt} = -\frac{n^3}{3!} \int_{\Omega} \frac{d\Omega}{6\pi^2} k_f |T_{\text{rec}}|^2 = -\left(\frac{\sqrt{3}\hbar}{2m}\right) n^3 \rho_{\text{rec}}^4$$

( $n^3/3!$  is the density of triples in the gas)

## Dimer propagator for Feshbach resonances with large $a_{bg}$



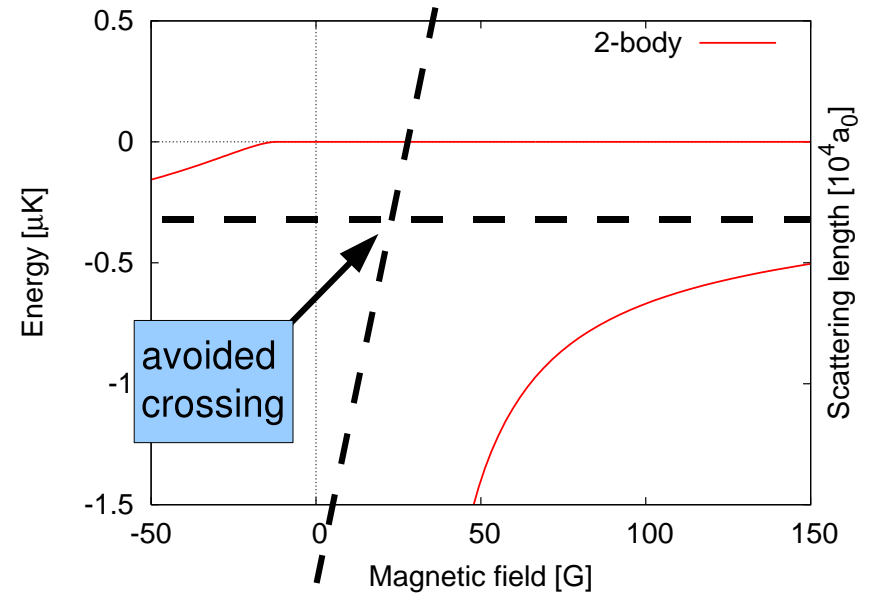
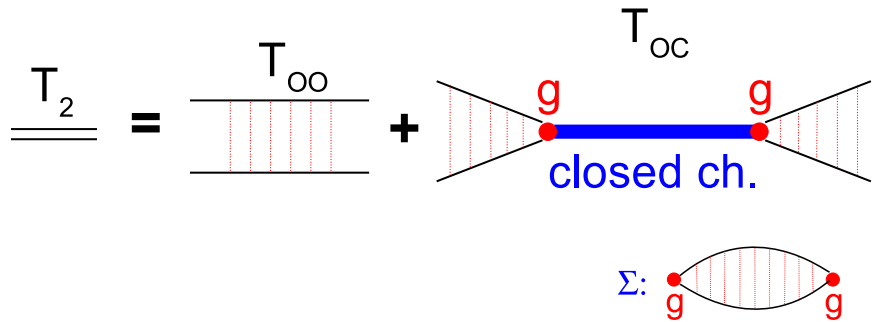
$$\text{Scattering length: } a = a_{bg} \left( 1 - \frac{\Delta B}{B - B_0} \right)$$

avoided crossing between the two bound states

$$T_2 = T_{oo} + T_{oc}$$

Bruun, Jackson, and Kolomeitsev (PRA 2005), Duine and Stoof (Phys. Rep. 2004).

## Pole structure of $T_2$



$$T_2(E) = \frac{8\pi\hbar^2}{m} \frac{1}{\left[ a_{\text{bg}} \left( 1 - \frac{\Delta B}{B - B_0 - E/\Delta\mu} \right) \right]^{-1} + ik} \quad k = \frac{\sqrt{mE}}{\hbar}$$

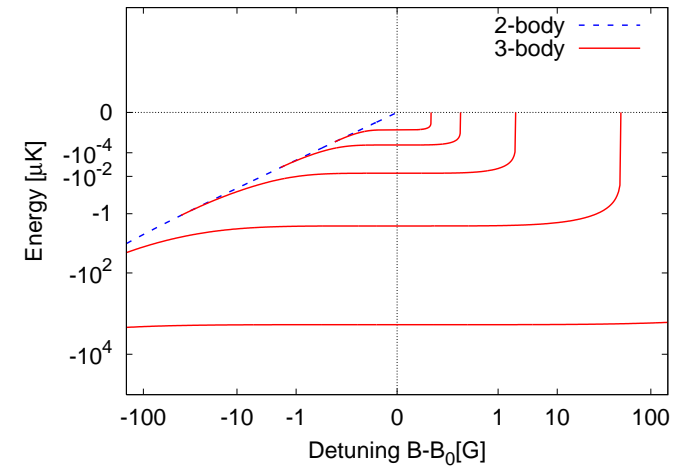
depending on the sign of  $a_{\text{bg}}$ ,  $T_2$  has 1 or 2 bound states: correct low-energy spectrum

## Cut-off dependence

The STM eq. with simple  $T_2 \sim \frac{1}{a^{-1} + ik}$

has a well-known UV cut-off dependence:

$$T_3(\dots) = \dots + \int_0^\Lambda dq \dots T_2(\dots) T_3(\dots).$$

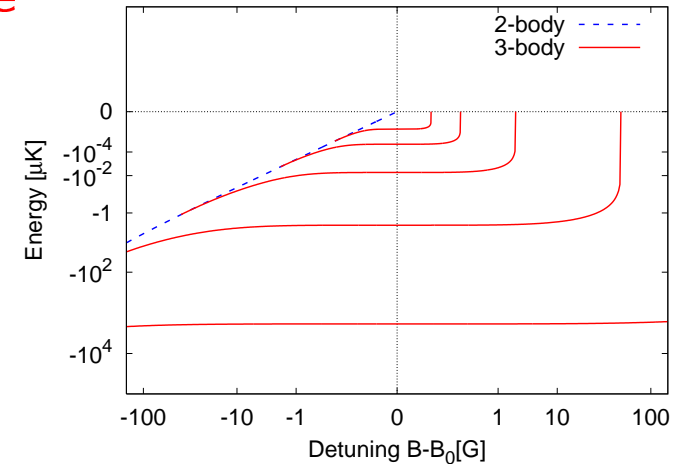


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### Possible solutions

- finite momentum cut-off  $\Lambda$  or, equivalently, high energy correction to  $T_2(E)$  fixed by an observable of the system
- for narrow resonances ( $R^* \gg R$ ), a natural cut-off is present:

$$f(E) \approx -\frac{1}{\frac{1}{a} + ik + R^* k^2} \quad \text{with } R^* = \frac{\hbar^2}{ma_{\text{bg}} \Delta B \Delta \mu}$$

Refs.: Danilov 1961, Fedorov and Jensen 2001, Petrov 2004

Same problem with our more refined  $T_2(E)$

$$T_2(E) = \frac{8\pi\hbar^2}{m} \frac{1}{\left[ a_{\text{bg}} \left( 1 - \frac{\Delta B}{B - B_0 - E/\Delta\mu} \right) \right]^{-1} + ik} \quad k = \frac{\sqrt{mE}}{\hbar}$$

Ok, let's include a  $k^2$  correction for  $T_2$  (which introduces an effective cut-off at  $k_{\text{max}} \sim 1/\tilde{R}$ ):

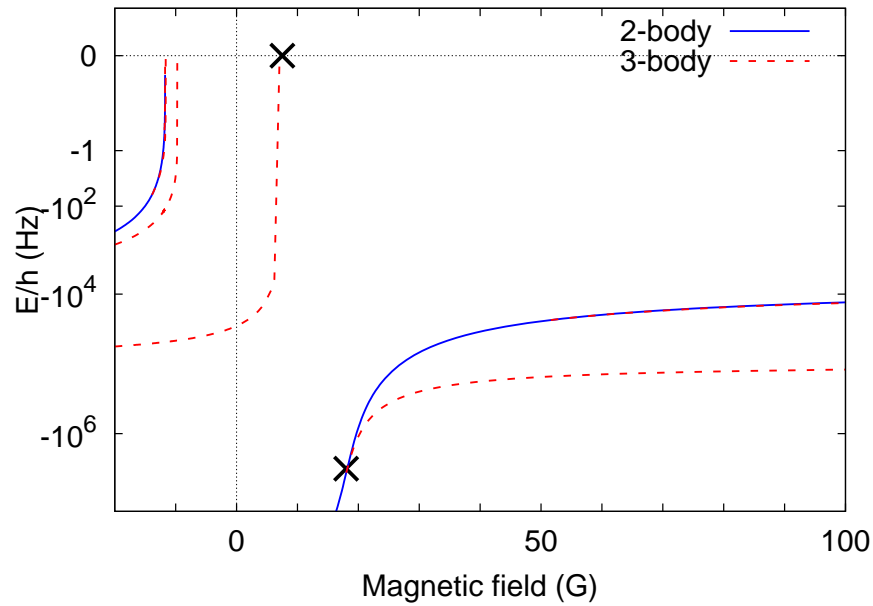
$$T_2(E) = \frac{8\pi\hbar^2}{m} \frac{1}{\left[ a_{\text{bg}} \left( 1 - \frac{\Delta B}{B - B_0 - E/\Delta\mu} \right) \right]^{-1} + ik + \tilde{R}k^2}$$

$$\text{with } \begin{cases} \tilde{R} = \tilde{R}_{\text{fit}} > 0 & \text{if } R^* \lesssim R \\ \tilde{R} = R^* \equiv \frac{\hbar^2}{ma_{\text{bg}}\Delta B\Delta\mu} & \text{if } R^* \gtrsim R \end{cases}$$

(if  $\tilde{R} < 0$  the additional deep pole of  $T_2$  has negative-norm)

# $^{133}\text{Cs}$ energy levels: comparison with Innsbruck experiments

$(R^* = 0.15a_0, \quad \tilde{R}_{\text{fit}} = 0.14a_0)$

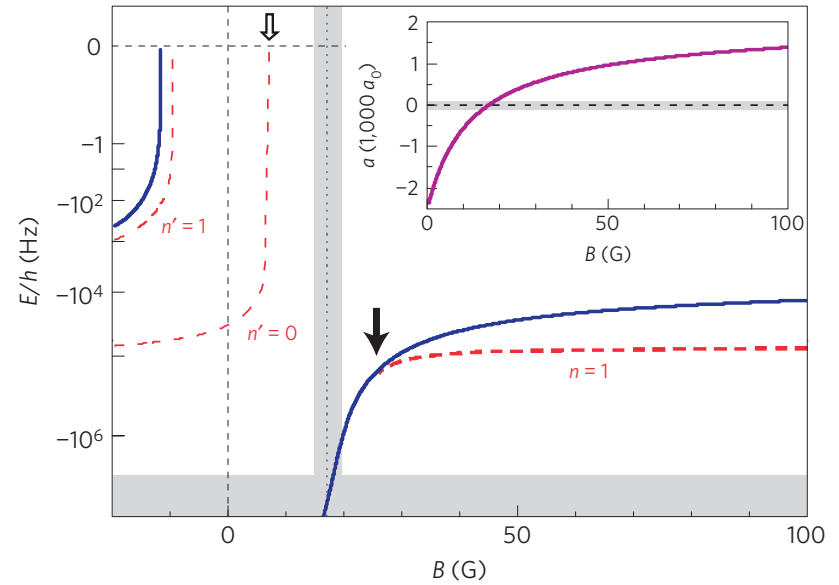
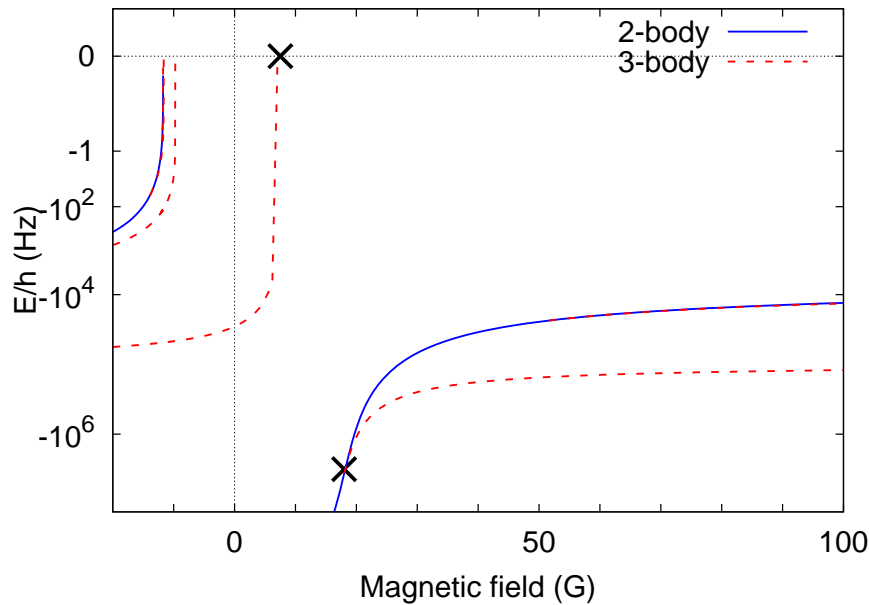


rescaled y-axis:  $E^{1/10}$  vs  $B$

Theory (left): P. Massignan and H. Stoof, PRA **78**, 030701 (2008)

# $^{133}\text{Cs}$ energy levels: comparison with Innsbruck experiments

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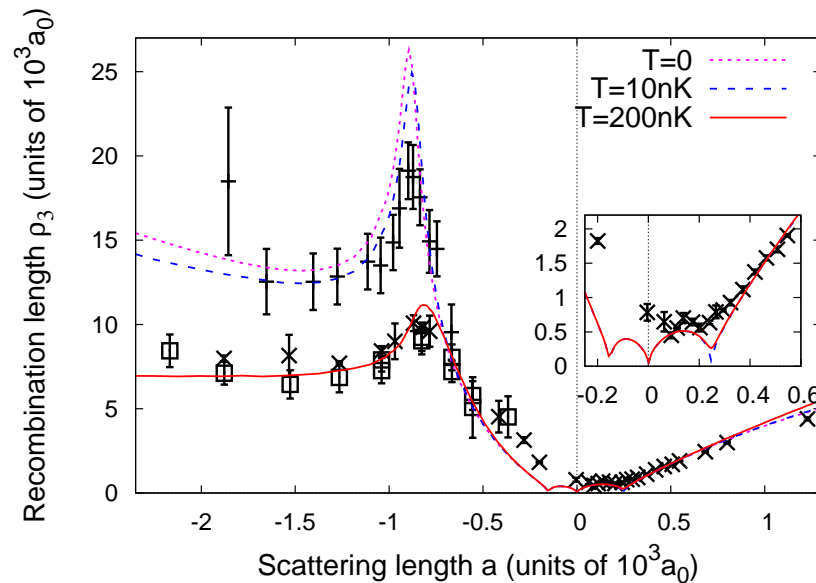


rescaled y-axis:  $E^{1/10}$  vs  $B$

Theory (left): P. Massignan and H. Stoof, PRA **78**, 030701 (2008)

Experiment (right): S. Knoop, F. Ferlaino, M. Mark et al., Nature Phys. **5**, 227 (2009)

## 3-body recombination: theory vs. experiment



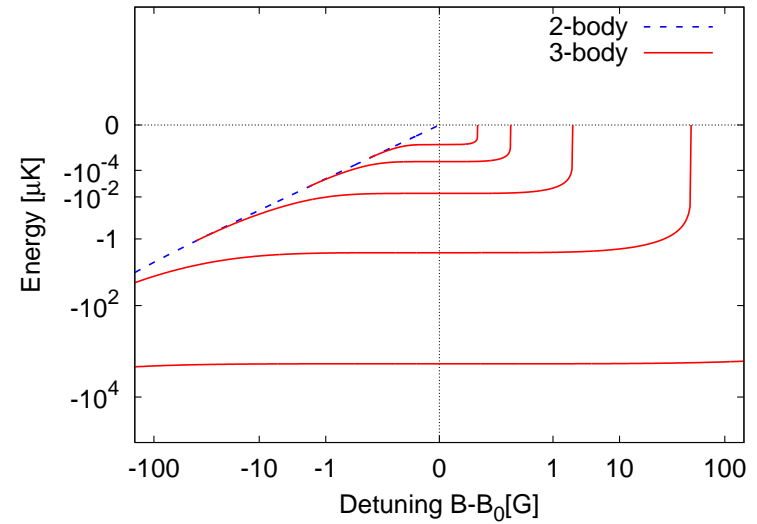
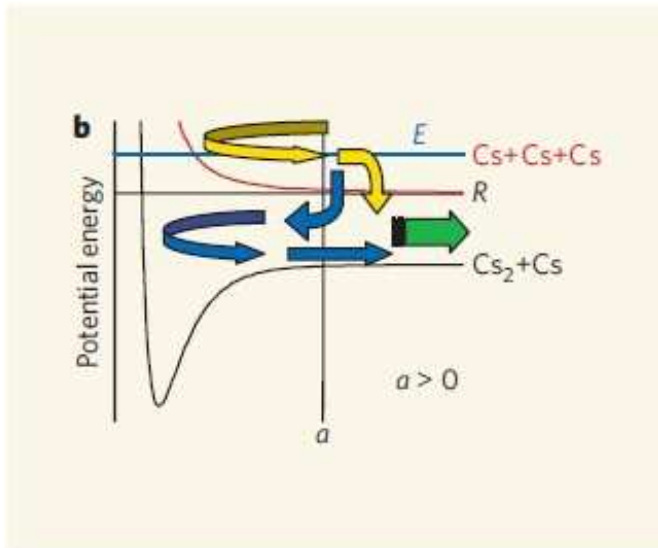
- ✓ positions of min/max with a single free parameter (instead of the 4 needed in the original paper)
- ✓ good accord with the temperature dependence

(see also: Jonsell; Yamashita, Frederico and Tomio; Braaten and Hammer; Lee, Köhler and Julienne)

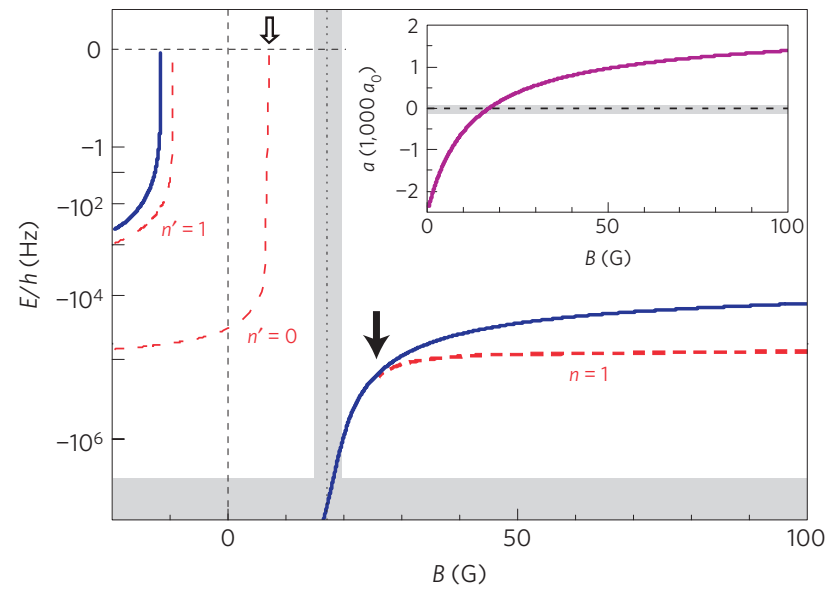
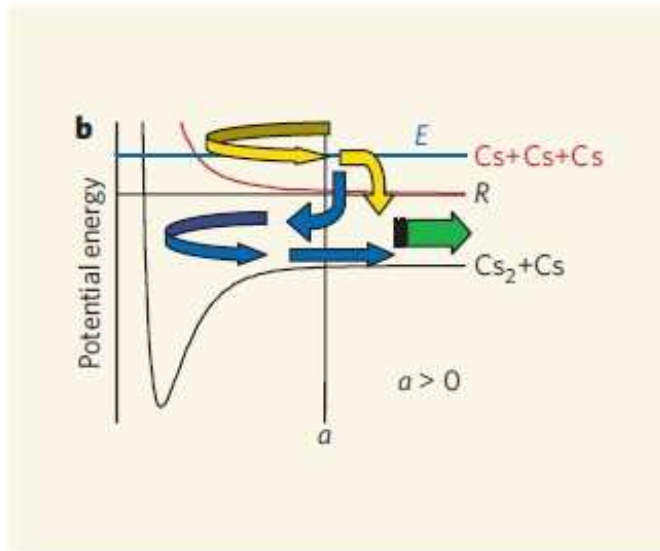
- **Additional prediction:** an interference Stückelberg minimum at negative  $a$  ( $a \simeq -130a_0$ )

P. Massignan and H. Stoof, PRA **78**, 030701 (2008)

# Stückelberg minima: destructive interference between two possible paths in the recombination process



## Stückelberg minima: destructive interference between two possible paths in the recombination process

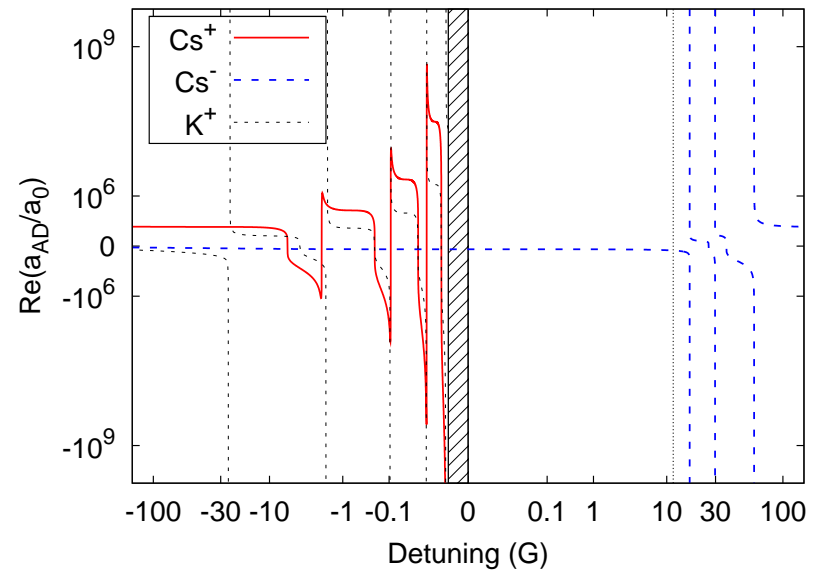
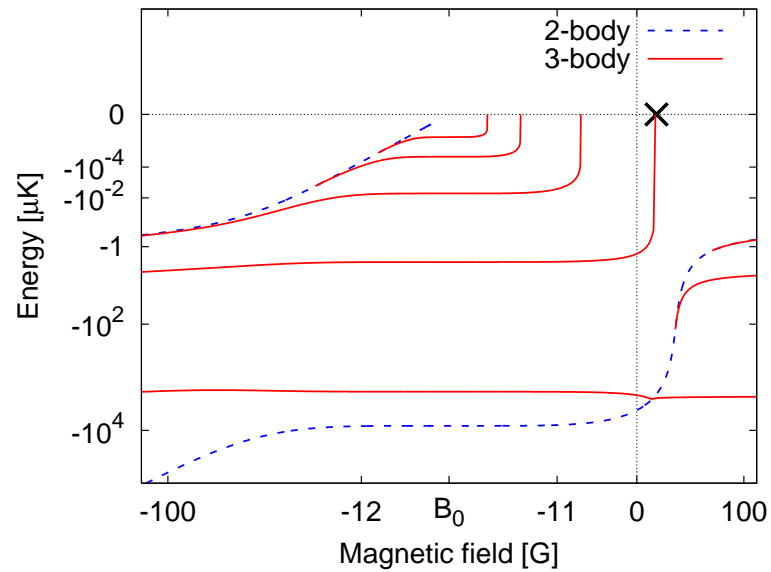


when  $a_{bg} \gg R$ , even for negative  $a$ !

(in Cs  $a_{bg} \sim 1800a_0$ )

## Atom-dimer scattering lengths

Scattering lengths:  $\frac{3\pi\hbar^2 a_{\text{AD}}^\pm}{m} = ZT_3(0, 0; E_b^\pm).$

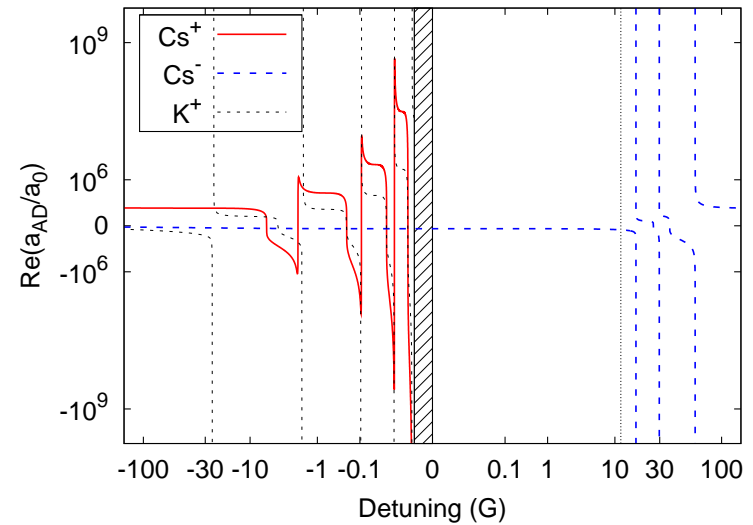
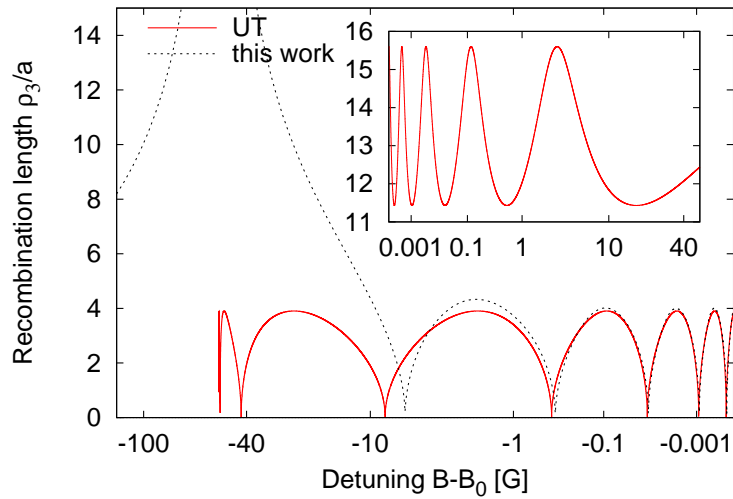


# Results for $^{39}\text{K}$ : recombination length and $a_{\text{AD}}$

Predictions for ongoing experiments @ LENS with potassium 39

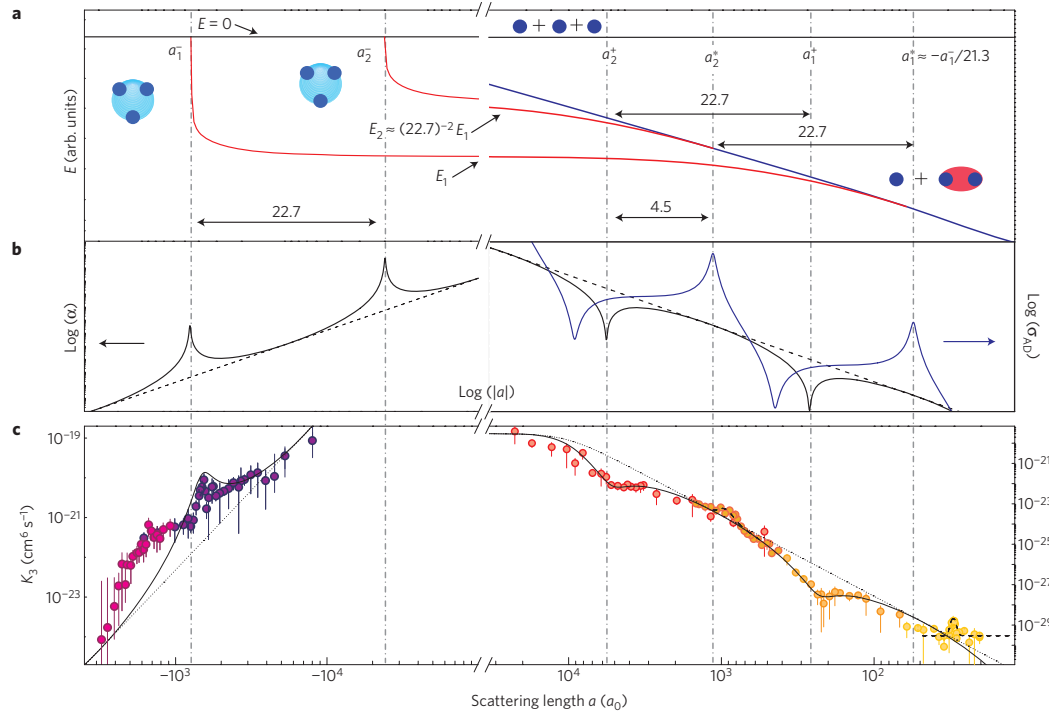
$(B_0 = 403\text{G}, a_{\text{bg}} = -29a_0, R^* = 29a_0 > R = 13a_0)$

$\eta = 0.06$



P. Massignan and H. Stoof, PRA **78**, 030701 (2008)

# $^{39}\text{K}$ recombination length measured at LENS



$[a/a_0]$	Theory	Exp.
$\rho_{\text{rec min.}}$	212	224
	3850	5650
AD res.	30.7	30.4
	810	930
$\rho_{\text{rec max.}}$	-550	-1500
		-650

Experiment: M. Zaccanti, B. Deissler, C. D'Errico et al., Nature Phys. 5, 586 (2009).

## Note added\*

The high-energy dependence discussed by Petrov is already in our T-matrix (without the  $k^2$  correction),

as may be seen in the limit  $|\frac{E}{\Delta\mu(B-B_0)}| \ll |\frac{B-B_0}{\Delta B}| \ll 1$ , where:

$$\frac{1}{T_2(E)} \propto \frac{1}{a_{\text{bg}} \left(1 - \frac{\Delta B}{B-B_0-E/\Delta\mu}\right)} + ik \sim \frac{1}{a} + ik + R^* k^2$$

Consequence:

- for narrow resonances, our model (without  $\tilde{R}$ ) should reduce to Petrov's
- it is not justified to assume  $\tilde{R} = R^*$ . The agreement between our a-priori predictions for  $^{39}\text{K}$  resonances and the measured values seems therefore *accidental*.

\*: thanks to Y. Castin, F. Werner, D. Petrov, E. Braaten, L. Pricoupenko

## Can the problem be fixed?

Yes!

In the broad resonance limit, keep as a fit parameter either of the following:

- the coefficient of the quadratic correction in the momentum  $\tilde{R}$
- a hard-core cut-off  $\Lambda$  in the STM integral eq.  $T_3(\dots) = \dots + \int_0^\Lambda dq \dots T_2(\dots)T_3(\dots)$
- a “soft” high-energy cut-off (finite range) in  $T_2(E)$  (see e.g. talk by M. Jona-Lasinio)

Is the model with  $\tilde{R} = \tilde{R}_{fit}$  useful?

Yes!

## In conclusion

We have:

- introduced a model that carefully describes Feshbach resonance with large  $a_{bg}$  and reproduces the energy dependence of the associated 3-body bound states
- recovered the temperature dependence of the 3-body recombination for  $^{133}\text{Cs}$
- predicted the existence of interference Stückelberg minima at negative  $a$
- calculated the atom-dimer scattering lengths for both  $^{133}\text{Cs}$  and  $^{39}\text{K}$
- predicted atom-dimer crossings for  $^{133}\text{Cs}$  at positive magnetic fields

Interesting developments:

- calculation of atom-dimer loss rates by inclusion of deeper bound states

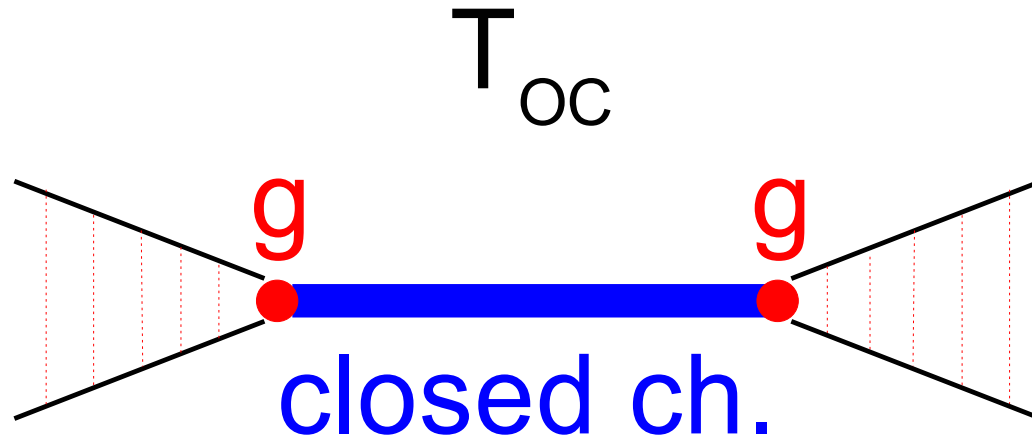
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## Open-channel scattering

$$T_{oo} = T_{bg} + T_{bg} \Pi T_{bg} + \dots$$

$$T_{oo} = \frac{T_{bg}}{1 - T_{bg}\Pi(E)} \quad \text{with } T_{bg} = \frac{4\pi\hbar^2 a_{bg}}{m}$$

pair propagator:  $\Pi(E) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left( \frac{1}{E + i0^+ - k^2/m} + \frac{1}{k^2/m} \right) = \frac{m^{3/2}\sqrt{-E}}{4\pi\hbar^3}$



$$T_{OC}(E) = \left( \frac{g}{1 - T_{bg}\Pi(E)} \right)^2 \frac{1}{E - \Delta\mu(B - B_0) - \hbar\Sigma(E)}$$

bare coupling between open and closed channels:  $g$

$$g^2 = \frac{4\pi\hbar^2 a_{bg}\Delta B\Delta\mu}{m}$$

self-energy of the closed ch. molecule:  $\hbar\Sigma(E) = g \frac{\Pi(E)}{1 - T_{bg}\Pi(E)} g$

