Realistic low energy description of the three boson problem

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Efimov States in Molecules and Nuclei
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Why yet another model?

- accurate description of the Feshbach mechanism
- inclusion of off-resonance effects
- quantification of non universal effects
- avoid complexities of collisional models

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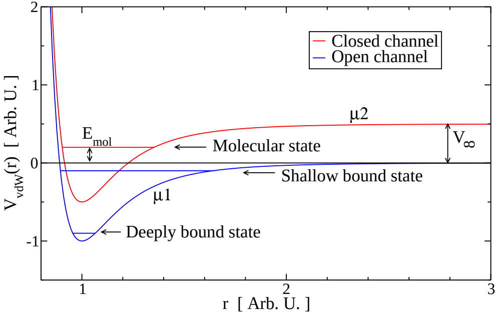
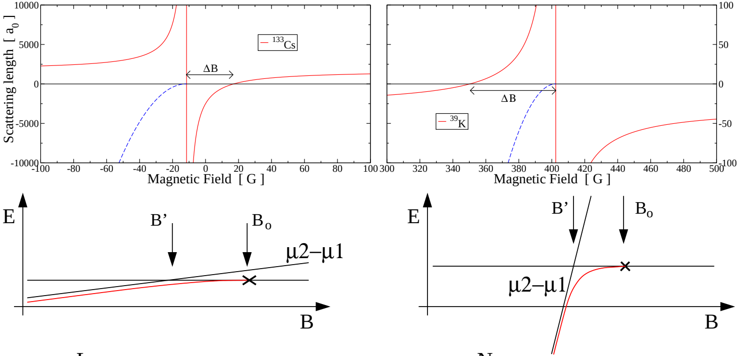
Van der Waals interatomic potential



- V_∞ is tuned with a magnetic field B
- $E_{\rm mol} \rightarrow$ internal energy of the molecular state
- $R_{vdW} \rightarrow$ range of the potential; $E_{vdW} = \frac{\hbar^2}{mR^2_{vdW}}$

Shape & Width of the Feshbach resonance



Large resonance

Narrow resonance

- $a = a_{\rm bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$ $\quad\Delta B \rightarrow$ width of the resonance
- $\delta\mu = \mu 2 - \mu 1$ $\quad E_{\rm mol} = \delta\mu(B - B')$

Basic facts about Efimov effect

- $|a| \gg R_{vdW} \rightarrow$ infinite series of three-body bound states.
- $E_n = e^{-2\pi/s_0} E_{n-1} \qquad s_0 = 1.00624.$
- The value of s_0 is universal in the resonant regime.
- The Efimov effect is due to an effective $\sim -1/\rho^2$ potential (ρ is the hyperradius cooordinate)

Modeling the resonance

- two-channel model to describe the Feshbach mechanism
- no zero range approximation but a simple finite range potential
- second quantized form:
  bosonic operators: atoms ($a_{\mathbf{k}}$), molecules ($b_{\mathbf{k}}$)
  commutation relation: $[a_{\mathbf{k}}, a_{\mathbf{k}'}] = [b_{\mathbf{k}}, b_{\mathbf{k}'}] = \delta(\mathbf{k} - \mathbf{k}')$
- coupling constants:
  open-open channel: g_0 (background scattering)
  open-closed channel: Λ (resonant scattering)

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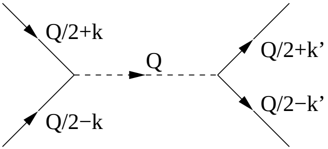
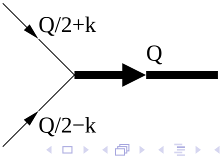
Model Hamiltonian

$$H = \int \frac{d\mathbf{k}}{(2\pi)^3} \left[\frac{\hbar^2 k^2}{2m} a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + \left(\frac{\hbar^2 k^2}{4m} + E_{\mathrm{mol}} \right) b^\dagger_{\mathbf{k}} b_{\mathbf{k}} \right]$$
$$+ \frac{g_0}{2} \int \frac{d\mathbf{k} d\mathbf{Q} d\mathbf{k}'}{(2\pi)^9} \chi^*_{\mathbf{k}} \chi_{\mathbf{k}'} \, a^\dagger_{\frac{\mathbf{Q}}{2}-\mathbf{k}'} a^\dagger_{\frac{\mathbf{Q}}{2}+\mathbf{k}'} a_{\frac{\mathbf{Q}}{2}+\mathbf{k}} a_{\frac{\mathbf{Q}}{2}-\mathbf{k}}$$
$$+ \Lambda \int \frac{d\mathbf{k} d\mathbf{Q}}{(2\pi)^6} \left(\chi^*_{\mathbf{k}} b^\dagger_{\mathbf{Q}} a_{\frac{\mathbf{Q}}{2}-\mathbf{k}} a_{\frac{\mathbf{Q}}{2}+\mathbf{k}} + \mathrm{h.c.} \right)$$

$\chi_{\mathbf{k}} = e^{-k^2 r_0^2/2}$ $\qquad$ $r_0 \rightarrow$ range of the model potential





Scattering amplitude & Scattering length

- Model parameters:
  r_0, g_0, Λ, $E_{\rm mol}$
- Physical parameters:
  R_{vdW}, a_{bg}, $\nu = \delta\mu(B - B_0)$, ΔB

$$a_{bg} = \frac{r_0 g_0 \sqrt{\pi}}{g_0 - g_0^c} \qquad \Delta B = \frac{8\pi\hbar^2\Lambda^2 a_{bg}}{m g_0^2 \delta\mu}$$

$$R^* = \frac{\hbar^2}{m a_{bg}\delta\mu\Delta B} = \frac{g_0^2}{8\pi a_{bg}^2 \Lambda^2}$$

- Scattering amplitude: $E = -\frac{\hbar^2 q^2}{m} < 0$

$$\frac{1}{f(E)} = q\ \mathrm{erfc}(q r_0) - \frac{e^{-q^2 r_0^2}}{a_{\rm bg}}\left(1 - \frac{\delta\mu\Delta B}{E - \delta\mu(B-B_0) + \delta\mu\Delta B}\right)$$

- Scattering length: $a = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$

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Features

1. We obtain the interesting property $\delta\mu\Delta B a_{bg} > 0$
2. The s-wave scat. len. a can be exactly identified with the standard expression $a_{\rm bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$
3. We obtain in a natural way $a_{bg} \sim r_0$ except in the neighborhood of a shape resonance
4. The model reduces exactly to the effective range approach[1] in the limit $r_0 \to 0$
5. The model can describe wide and narrow Feshbach resonances close or far from shape resonances

[1] D. Petrov, Phys. Rev. Lett. **93**, 143201 (2004)

What about r_0?

- In principle one should require $r_0 \sim R_{vdW}$
- We tune r_0 to reproduce the two-body spectrum.
  In the absence of the two-body spectrum: $r_0 = R_{vdW}$
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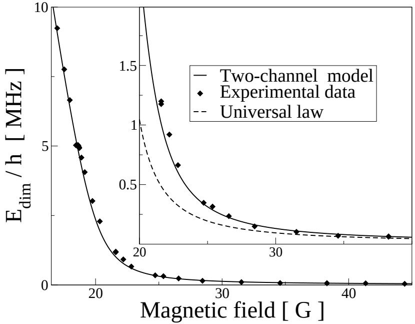
Three-body recombination

- $A + A + A \rightarrow A + D$
- $\dot{n} = -3\alpha_{rec} n^3$ $\quad n \rightarrow$ atomic density
- existence of a shallow dimer in the model
  α_{rec} can be computed exactly
- no shallow dimer in the model
  $\alpha_{rec} = \frac{\hbar L^6 P_<}{m R_{vdW}^2}$

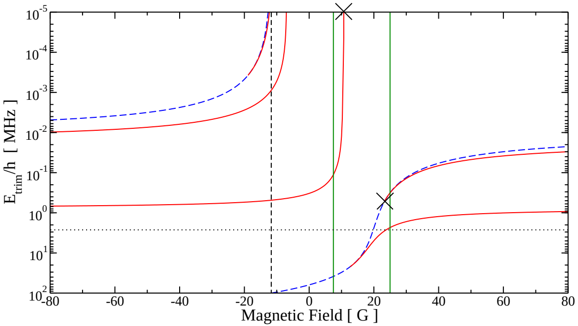
^{133}Cs: Dimer spectrum



- Fitting parameter to data[2]: $r_0 = 0.7 R_{vdW}$

[2] M. Mark *et al.*, Phys Rev. A **76**, 042514 (2007)

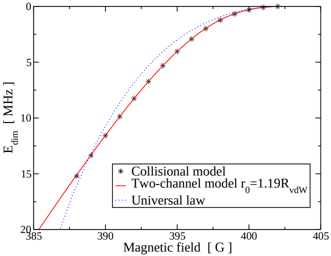
^{133}Cs: Trimer spectrum

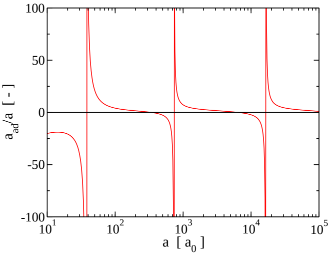


- Crosses: features observed at Innsbruck[3]
  predicted @ 10.3 G, 23.2 G; observed @ 7.5 G, ∼25 G

[3] T. Kraemer *et al*, Nature 440, 315 (2006)

^{39}K: Dimer spectrum & Efimov trimers

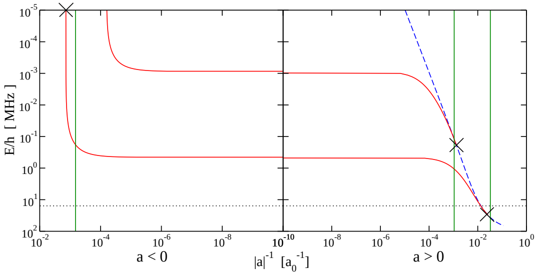




- Fitting parameter to data[4] : $r_0 = 1.19 R_{vdW}$

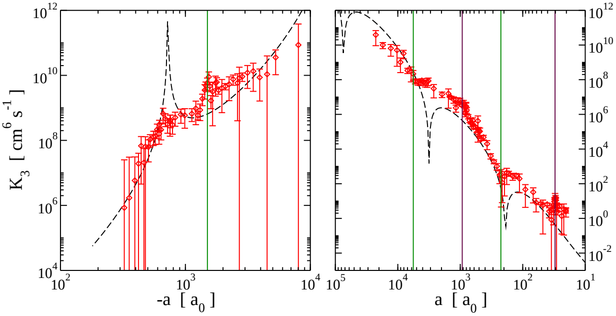
[4] Data from the collisional model by courtesy of A. Simoni

^{39}K: Trimer spectrum



- Trimers: predicted @ a = (-725.7, 38.0, 737.9, 16471.7) a_0; observed @ a = (-1500, 30.4, 930) a_0
- Deviation from universality in the model: 737.9/38.0 = 19.41, 16471.7/737.9 = 22.32

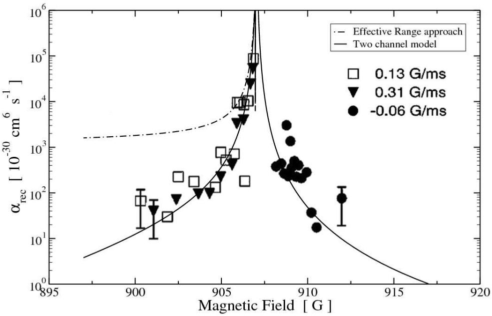
^{39}K: Three-body recombination



- Good agreement[5] though the model is unable to predict the correct three-body parameter.

[5] M. Zaccanti *et al.*, Nature Physics **5**, 586 (2009)

^{23}Na: Three-body recombination



- No available data[6] to fit: $r_0 = R_{vdW}$

[6] J. Stenger *et al.*, Phys. Rev. Lett. **82**, 2422 (1999)

- The presence of a real short length scale r_0 regularizes the theory with no cutoff needed.
- We quantitatively describe two-body physics, well beyond the universality region, by tuning the single parameter r_0.
- Even complicated spectra close to a shape resonance are correctly predicted.
- Three-body recombination rate is reproduced quantitatively though significant shifts show up in the position of the Efimov features.

- Work done in collaboration with Ludovic Pricoupenko, arxiv:0903.3808

Thank you!