Effective Field Theory for Few-Body Systems with Large Scattering Length

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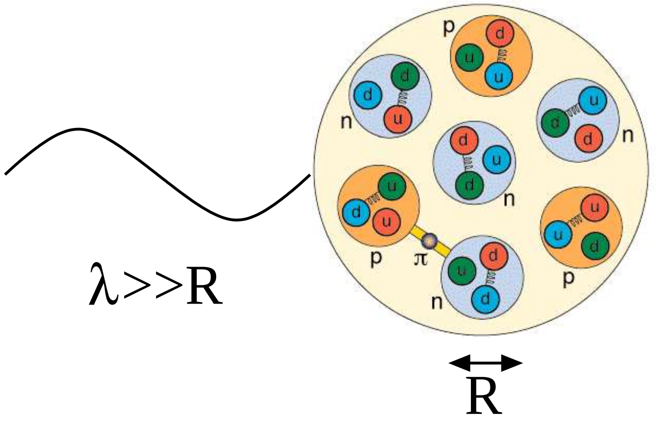
Agenda

- Introduction
- Resonant Interactions and Efimov Physics
- Effective Field Theory for Large Scattering Length
- Applications
  - Ultracold atoms
  - Halo nuclei
- Summary and Outlook

Effective Theory

- Separation of scales:
  $1/k = \lambda \gg R$
- Limited resolution at low energy:
  $\longrightarrow$ expand in powers of kR



Effective Theory

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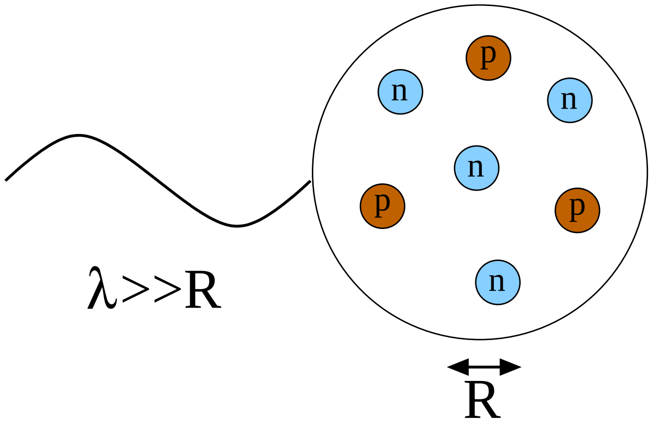


- Separation of scales:

  $1/k = \lambda \gg R$

- Limited resolution at low energy:

  $\longrightarrow$ expand in powers of kR

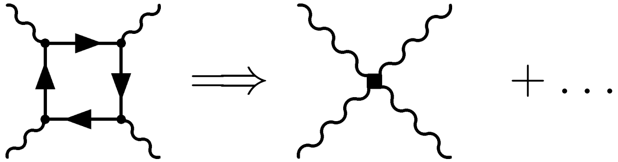
- Short-distance physics not resolved

  $\longrightarrow$ capture in low-energy constants using renormalization
  $\longrightarrow$ include long-range physics explicitly

- Systematic, model independent $\rightarrow$ universal properties
- Classic example: light-light-scattering (Euler, Heisenberg, 1936)

  Simpler theory for $\omega \ll m_e$:

  $\mathcal{L}_{QED}[\psi, \bar{\psi}, A_\mu] \rightarrow \mathcal{L}_{eff}[A_\mu]$



Resonant Interactions

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- Large scattering length: $|a| \gg \ell \sim r_e, l_{vdW}, \ldots$
- Natural expansion parameter: $\ell/|a|$, $k\ell$,...

$$a > 0 \quad \Longrightarrow \quad B_d = \frac{1}{2\mu a^2} + \mathcal{O}(\ell/a)$$

- Atomic physics:
  - ^{4}He: $a \approx 104$ Å $\gg r_e \approx 7$ Å $\sim l_{vdW} \longrightarrow B_d \approx 100$ neV
  - Feshbach resonances $\Longrightarrow$ variable scattering length
- Nuclear physics: S-wave NN-scattering, halo nuclei,...
  - $^1S_0$, $^3S_1$: $|a| \gg r_e \sim 1/m_\pi \longrightarrow B_d \approx 2.2$ MeV
  - ^{6}He: $2n$ separation energy $\approx$ 973 keV
- Particle physics:
  - $X(3872)$ as a $D^0\bar{D}^{0*}$ molecule? ($J^{PC} = 1^{++}$)

$$B_X = m_{D^0} + m_{D^{0*}} - m_X = (0.3 \pm 0.4)\ \text{MeV}$$

Efimov Physics

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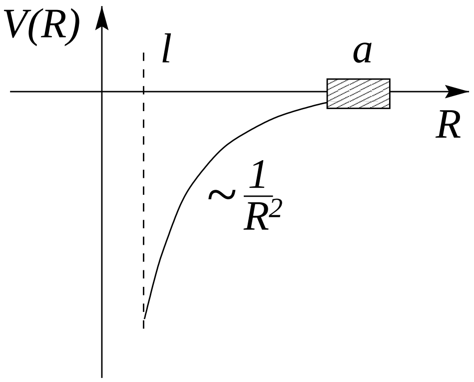
(V. Efimov, Phys. Lett. **33B** (1970) 563)

- Three-body system with large scattering length a
- Hyperspherical coordinates: $R^2 = (r_{12}^2 + r_{13}^2 + r_{23}^2)/3$
- Schrödinger equation simplifies for $|a| \gg R \gg l$:

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial R^2} + \frac{s_0^2 + 1/4}{R^2}\right] f(R) = \underbrace{-\frac{\hbar^2\kappa^2}{m}}_{E} f(R)$$



- Singular Potential: renormalization required
- Boundary condition at small R: breaks scale invariance

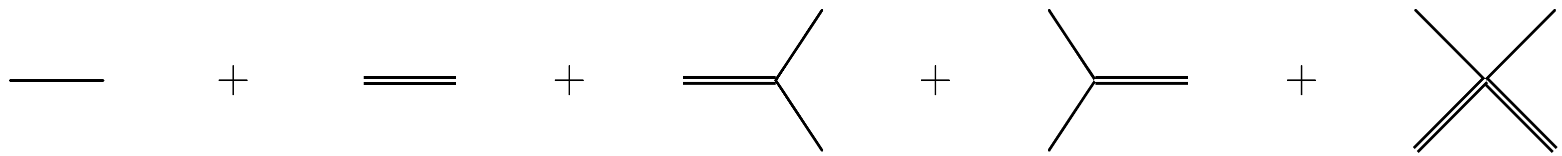
$\Longrightarrow$ dependence of observables on 3-body parameter (and a)

- EFT formulation: boundary condition $\Rightarrow$ 3-body interaction

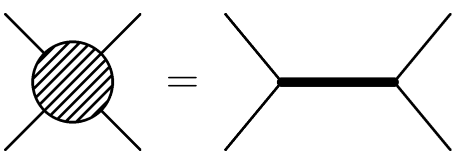
Two-Body System in EFT

- Effective Lagrangian (Kaplan, 1997; Bedaque, HWH, van Kolck, 1999)

$$\mathcal{L}_d = \psi^\dagger \left(i\partial_t + \frac{\vec{\nabla}^2}{2m} \right) \psi + \frac{g_2}{4} d^\dagger d - \frac{g_2}{4} (d^\dagger \psi^2 + (\psi^\dagger)^2 d) - \frac{g_3}{36} d^\dagger d \psi^\dagger \psi + ..$$



- Interacting dimeron propagator $\longrightarrow$ sum bubbles

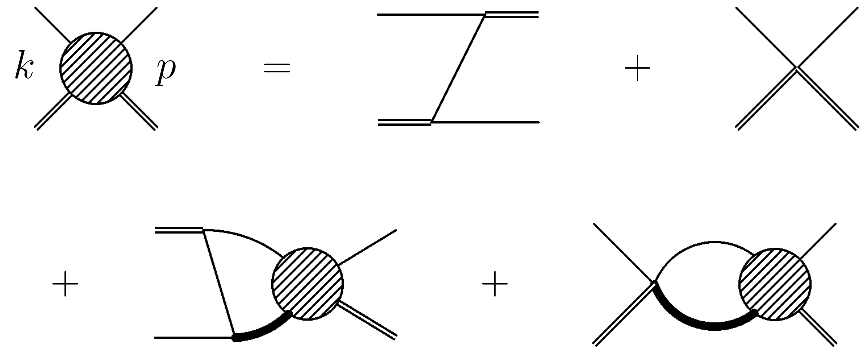
- Two-body amplitude $\mathcal{T}_2(k,k)$:  $\propto \dfrac{1}{1/a - ik} + \ldots$

- Matching: $g_2 \longleftarrow a, B_d$

- RG fixed points of g_2: $a = 0$ and $a = \infty$

- Higher order corrections: perturbation theory

- Three-body equation :

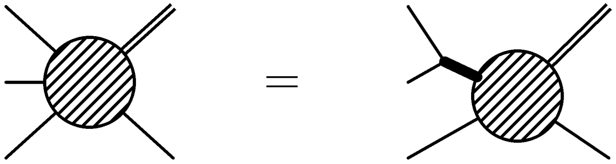


$$\mathcal{T}_3(k,p) = M(k,p) + \frac{4}{\pi}\int_0^{\Lambda} dq\, q^2\, M(q,p) D_d(q)\, \mathcal{T}_3(k,q)$$

with $$M(k,p) = \underbrace{F(k,p)}_{\text{1-atom exchange}} \underbrace{-\frac{g_3}{9g_2^2}}_{H(\Lambda)/\Lambda^2}$$

$(g_3 = 0,\ \Lambda \to \infty \longrightarrow$ Skorniakov, Ter-Martirosian '57)

- Three-body recombination:



Renormalization

- Observables are independent of regulator/cutoff Λ

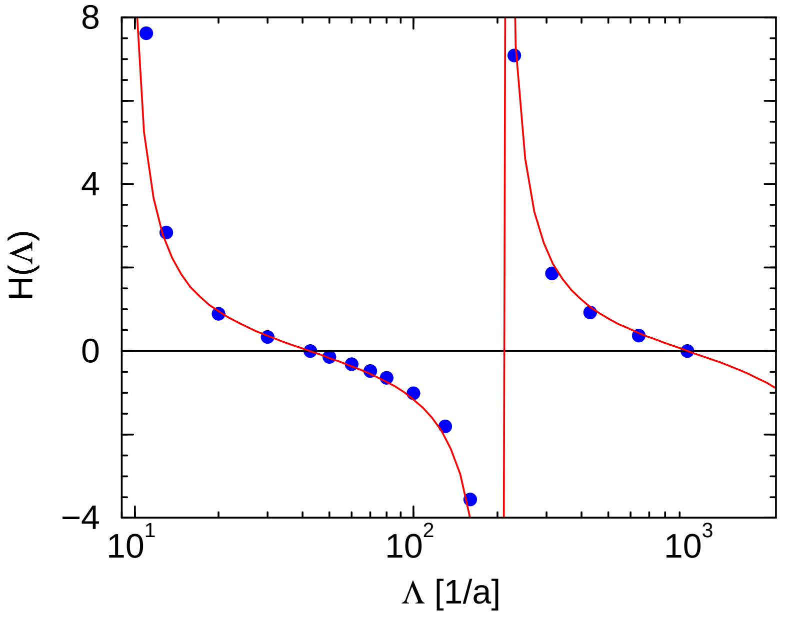
$\Rightarrow$ Running coupling $H(\Lambda)$

- $H(\Lambda)$ periodic: limit cycle

$$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

(cf. Wilson, 1971)

- Full scale invariance broken to discrete subgroup



$$H(\Lambda) = \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}\,, \qquad s_0 \approx 1.00624$$

- Limit cycle $\Longleftrightarrow$ Discrete scale invariance
- Matching: $\Lambda_* \longleftarrow B_t,\, K_3, \ldots \longrightarrow \kappa_*, a_*, a'_*$

Limit Cycle: Efimov Effect

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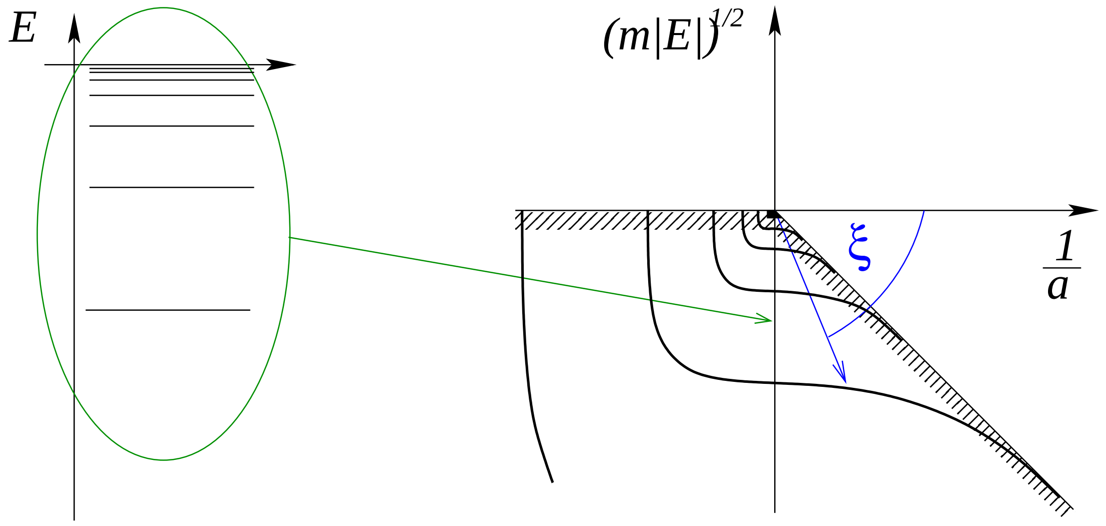
- Universal spectrum of three-body states

  (V. Efimov, Phys. Lett. **33B** (1970) 563)



  - Discrete scale invariance for fixed angle ξ
  - Geometrical spectrum für $1/a \to 0$

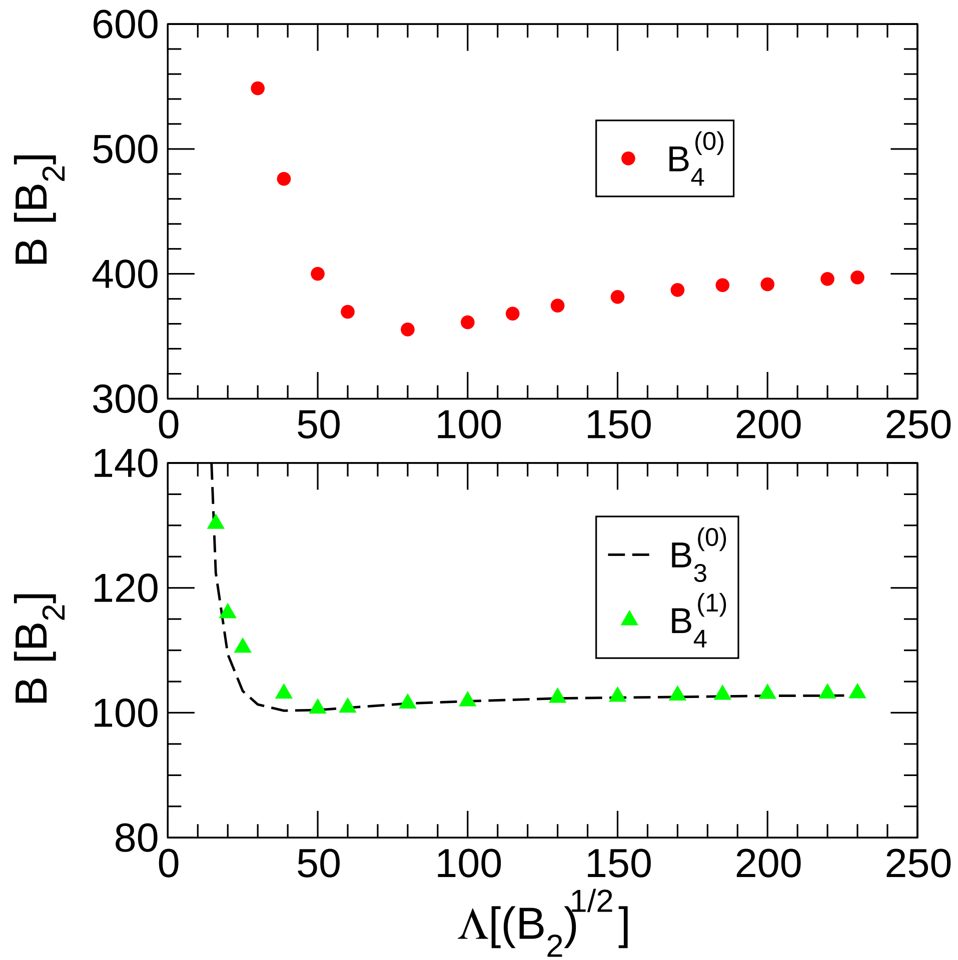
$$B_3^{(n)}/B_3^{(n+1)} \quad \xrightarrow{1/a\to 0} \quad 515.035\ldots$$

- Ultracold atoms $\Longrightarrow$ variable scattering length

Universal Correlations

- 2 Parameters at LO $\Rightarrow$ 3-body observables are correlated

  $\Longrightarrow$ Phillips line (Phillips, 1968)
- No four-body parameter at LO (Platter, HWH, Meißner, 2004)



Universal Correlations

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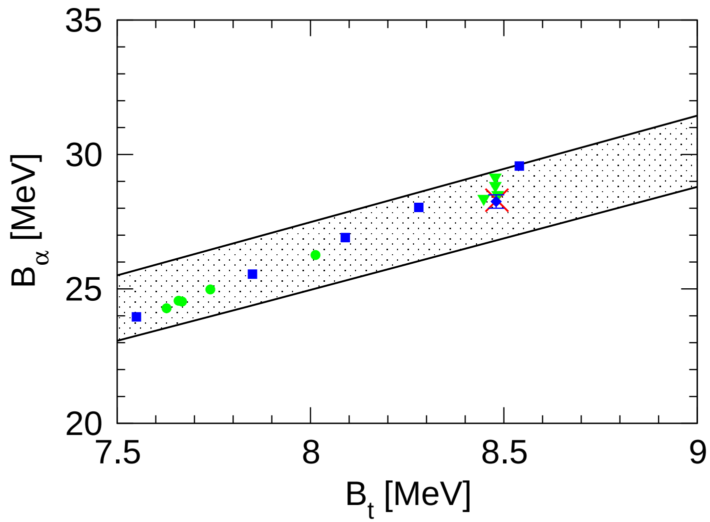
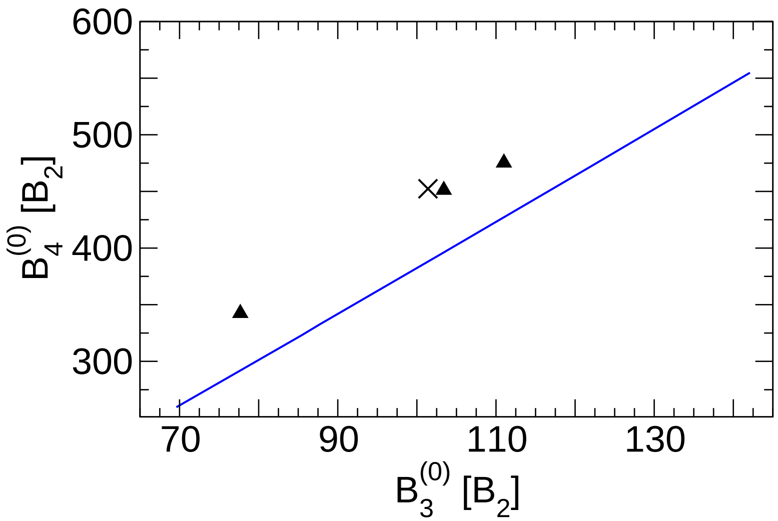
- No four-body parameter at LO (Platter, HWH, Meißner, 2004)

  $\Rightarrow$ 4-body observables are correlated $\Longrightarrow$ Tjon line





- Variation of 3-body parameter generates correlations
- Nuclear physics: Λ dependence of V_{low-k} (Bogner et al., 2004)
- Tjon line also at NLO (Kirscher et al., 2009)

More on the 4-Body System

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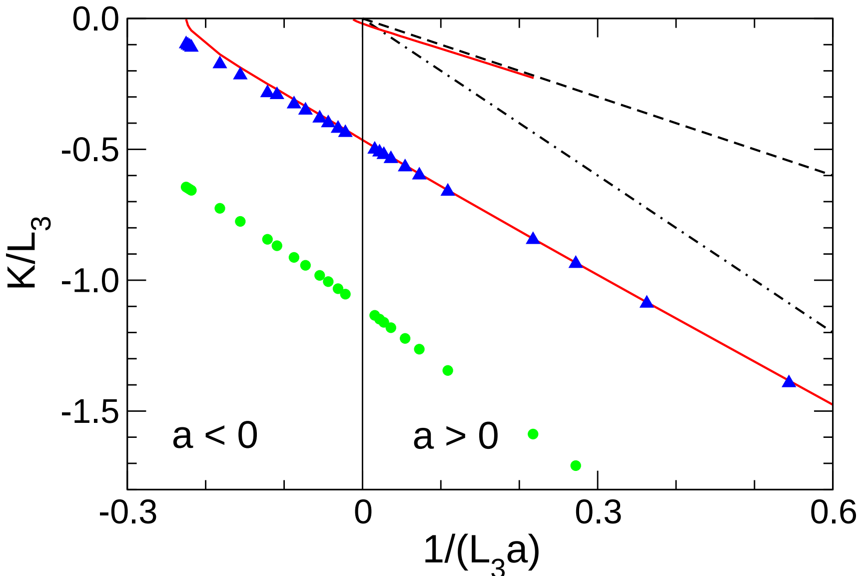
- Universal properties of 4-body system with large a
  - Bound state spectrum, scattering observables, ...
- "Efimov-plot": 4-body bound state spectrum as function of $1/a$

$$K = \text{sign}(E)\sqrt{m|E|}$$

$$B_4^{(0)} = 5B_3^{(0)} \qquad (1/a \equiv 0)$$

$$B_4^{(1)} = 1.01B_3^{(0)}$$

(Platter, HWH, EPJA **32** (2007) 113)



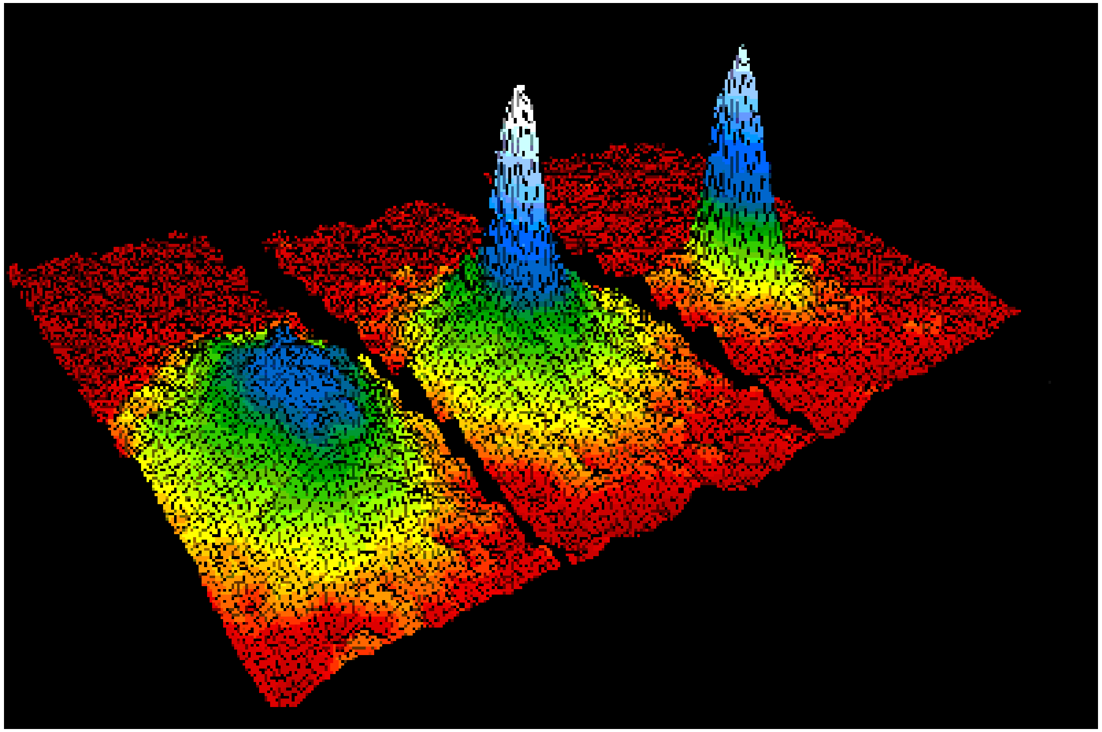
- Improved theoretical decription and signature in Cs loss data

  von Stecher, D'Incao, Greene, Nature Physics **5** (2009) 417

  Ferlaino, Knoop, Berninger, Harm, D'Incao, Nägerl, Grimm, PRL **102** (2009) 140401
- Four-body recombination (Wang, Esry; Mehta et al.)

Efimov Physics in Cold Atoms

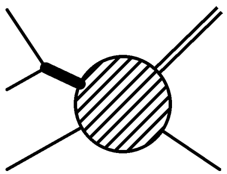
- Velocity distribution ($T =$ 400 nK, 200 nK, 50 nK)



(Source: http://jilawww.colorado.edu/bec/)

- Few-body loss rates provide window on Efimov physics
- Variable scattering length via Feshbach resonances

Three-Body Recombination



- Recombination into weakly-bound dimer:

  3 atoms $\rightarrow$ dimer + atom $\Rightarrow$ loss of atoms
- Recombination constant: $\dot{n}_A = -K_3\, n_A^3$
- Scattering length dependence for $a > 0$:

  (Nielsen, Macek, 1999; Esry, Greene, Burke, 1999; Bedaque, Braaten, HWH, 2000)

$$K_3 \approx 201.3 \sin^2\left[s_0 \ln(a\kappa_*) + 1.16\right] \frac{\hbar a^4}{m}, \qquad s_0 \approx 1.00624..$$

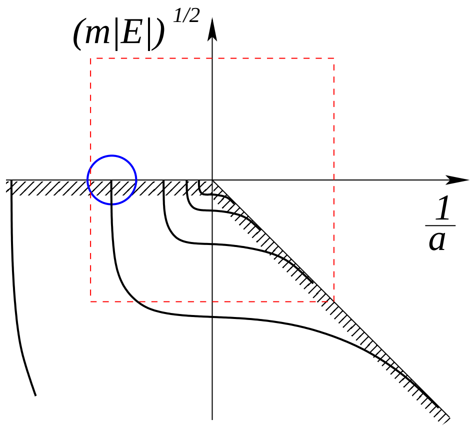
- Modification from deep dimers: Efimov states aquire width

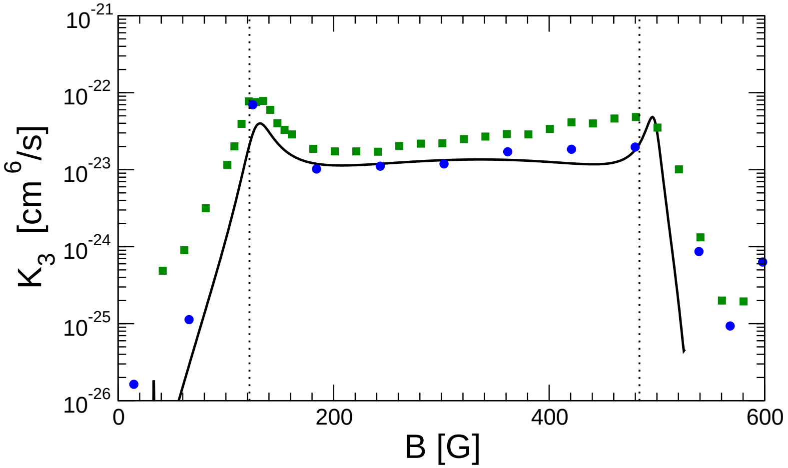
  $\Longrightarrow \quad \kappa_* \rightarrow \kappa_* \exp(i\eta_*/s_0)$
- Recombination into deep dimers $\Longrightarrow$ Efimov resonances
- Evidence for Efimov trimers in ^{133}Cs

  (Kraemer et al. (Innsbruck), Nature **440** (2006) 315)

Efimov Physics with Fermions

- Efimov effect for fermions $\Rightarrow \geq 3$ spin states ($|1\rangle, |2\rangle, |3\rangle, ..$)
- Experimental evidence for Efimov states in ^{6}Li
  - Ottenstein et al. (Heidelberg), Phys. Rev. Lett. **101** (2008) 203202
  - Huckans et al. (Penn State), Phys. Rev. Lett. **102** (2009) 165302





Braaten, HWH, Kang, Platter, Phys. Rev. Lett. **103** (2009) 073202

- Systematic normalization error: 70-90%
- Slow rise with B $\Leftrightarrow$ $\eta_* \propto 1/E_d$ (Wenz et al., arXiv:0906.4378)
- Related work: Naidon et al., arXiv:0811:4086; Schmidt et al., arXiv:0812.1191

Efimov Physics in ^{6}Li

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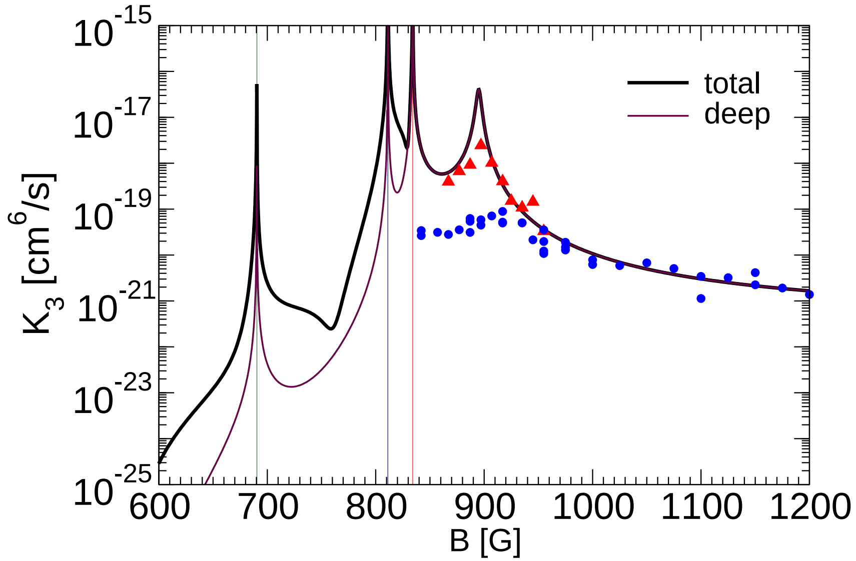
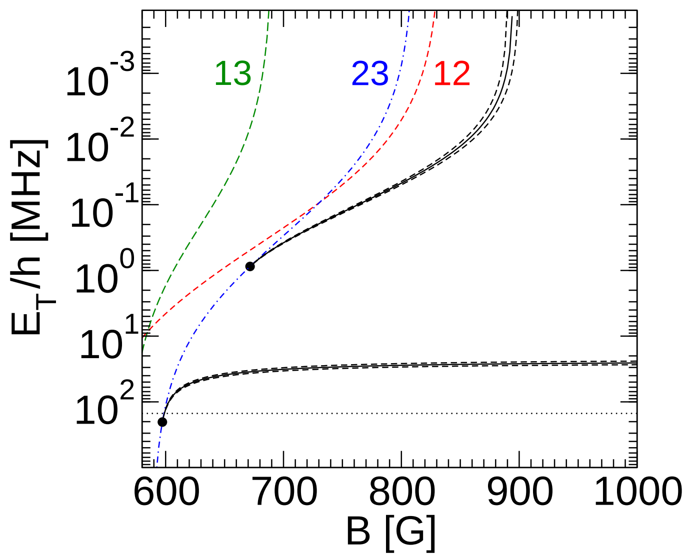
- Recombination resonances in high field region

  Williams et al. (Penn State), arXiv:0908.0789

- Recombination and bound state spectrum



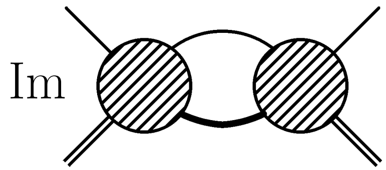


Braaten, HWH, Kang, Platter, arXiv:0908.4046

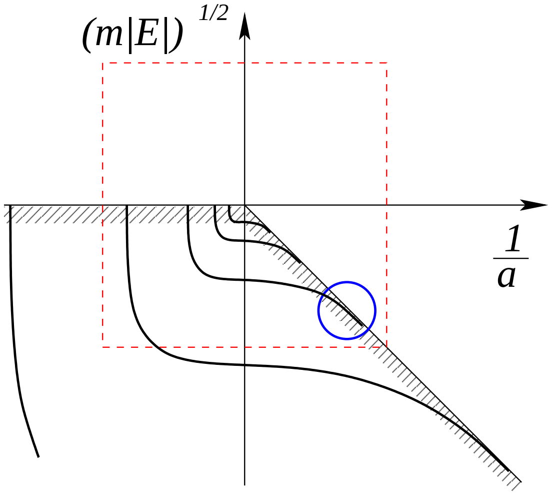
- Predictions for:
  - Two trimer states and widths
  - Atom-dimer relaxation resonance $(1-23)$

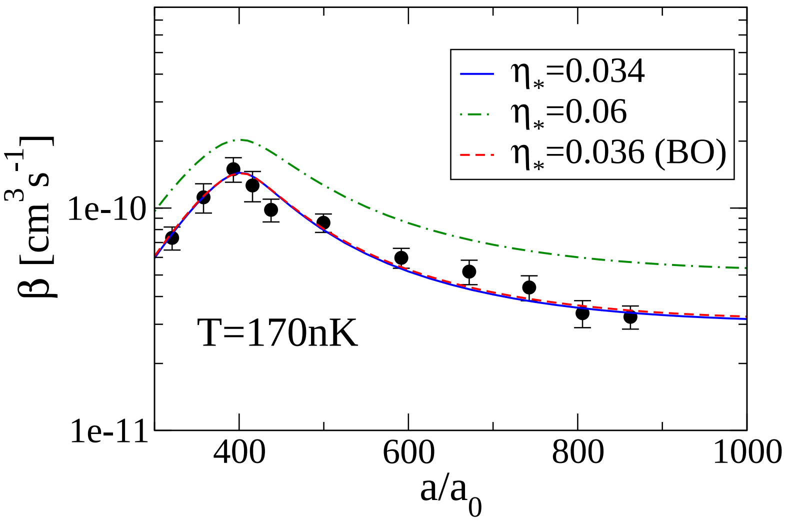
Dimer Relaxation in ^{133}Cs

- Dimer Relaxation: a + d $\to$ a + D (energetic)
- Relaxation constant: $\dot{n}_A = \dot{n}_D = -\beta\, n_A\, n_D$



- Recent experiment: Knoop et al. (Innsbruck), Nature Physics **5** (2009) 227
- Finite temperature $T \sim T_c$: Bose-Einstein average ?





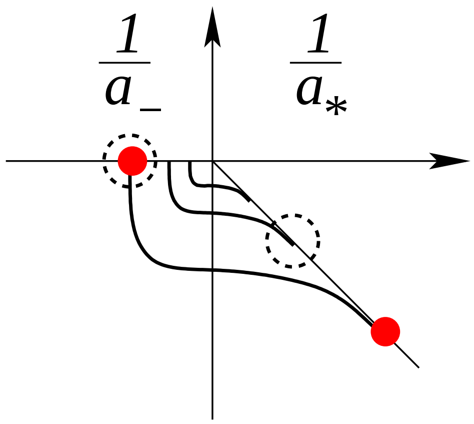
Helfrich, HWH, EPL **86** (2009) 53003

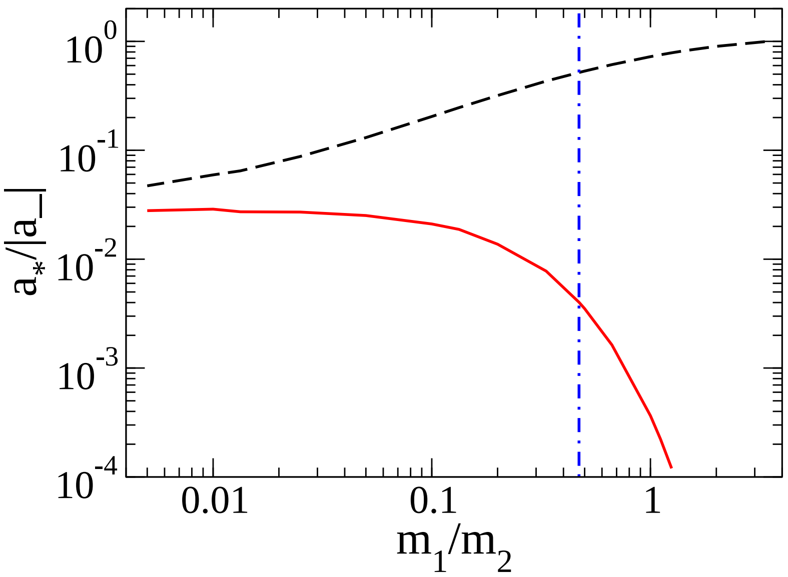
Heteronuclear Efimov Effect

- Recent experiment with heteronuclear mixture of ^{41}K and ^{87}Rb atoms (Barontini et al. (Florence), Phys. Rev. Lett. **103** (2009) 043201)

  $\Rightarrow$ Connected K-Rb-Rb resonances for $a > 0$ and $a < 0$

- Ratio of resonance positions: $a_* / |a_-|$ (Helfrich, HWH, in progress)





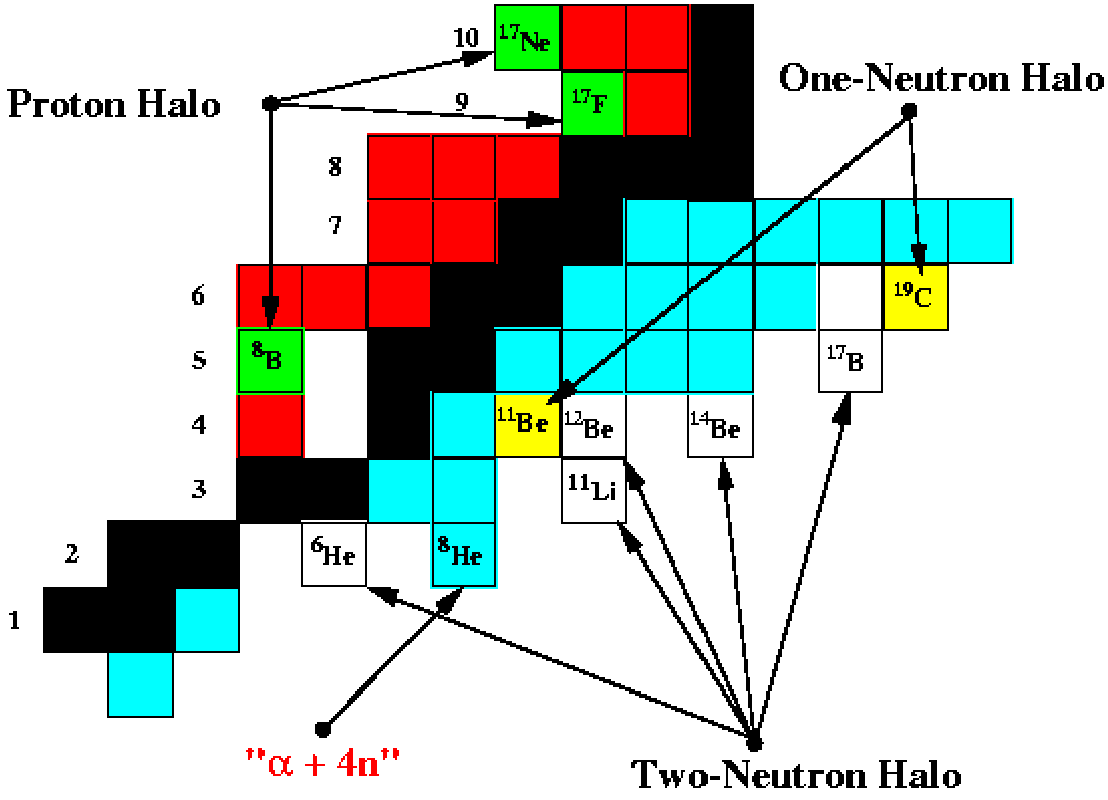
- K-Rb-Rb: $m_1/m_2 = 41/87 \approx 0.47 \quad \Rightarrow \quad \exp(\pi/s_0) \approx 128$

  $a_*/|a_-|$: 2.7 (Exp) $\Leftrightarrow$ 0.52 (Th)

Halo Nuclei

- Low separation energy of valence nucleons: $B_{valence} \ll B_{core}, E_{ex}$

  $\longrightarrow$ close to “nucleon drip line” $\longrightarrow$ scale separation $\longrightarrow$ EFT

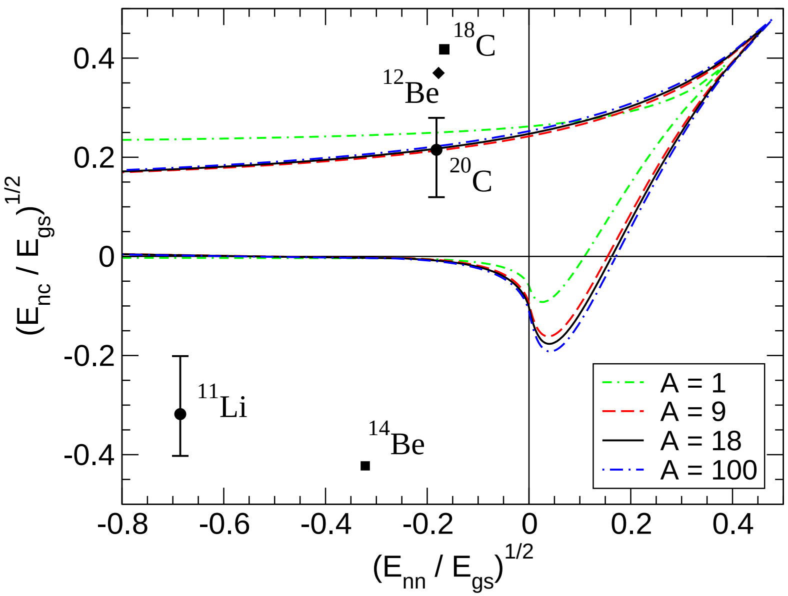


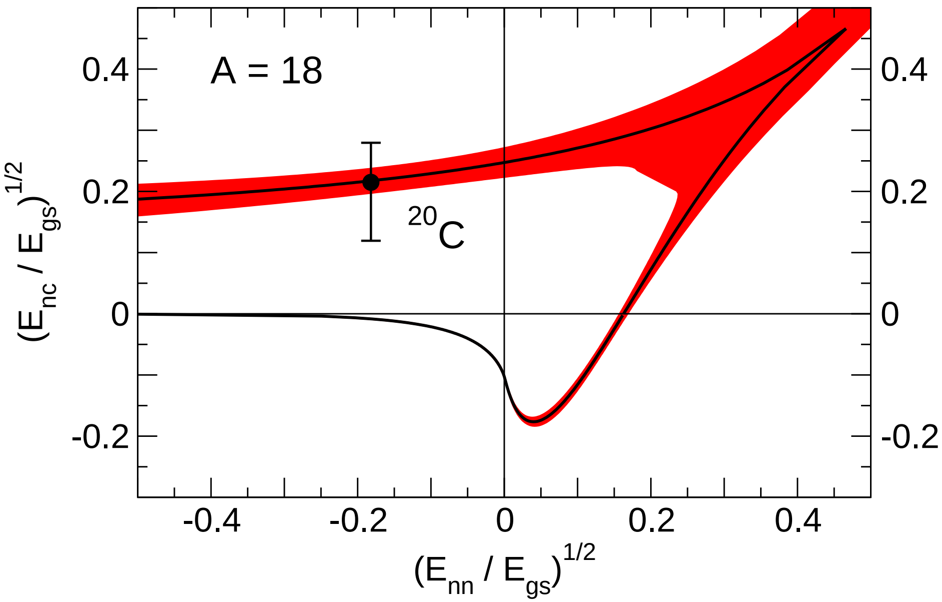
http://www.nupecc.org

- EFT for halo nuclei $\Longleftrightarrow$ cluster models

3-Body Halos

- Examples: $^{14}\mathrm{Be} \longleftrightarrow {}^{12}\mathrm{Be} + n + n, \quad {}^{20}\mathrm{C} \longleftrightarrow {}^{18}\mathrm{C} + n + n$
- “Effective" 3-body system: separation energy of valence nucleons small compared to binding energy of “core”
- Efimov effect in halo nuclei? $\Rightarrow$ excited states





Canham, HWH, Eur. Phys. J. A **37** (2008) 367 (cf. Amorim, Frederico, Tomio, 1997)

- Unchanged by NLO range corrections (Canham, HWH, to be published)

Form Factors and Radii

- Structure of halo nuclei $\to$ matter form factors, radii

nucleus	B_{nnc} [keV]	B_{nc} [keV]	$\sqrt{\langle r_{nn}^2 \rangle}$ [fm]	$\sqrt{\langle r_{nc}^2 \rangle}$ [fm]
^{14}Be	1120	-200.0	4.1 ± 0.5	3.5 ± 0.5
^{20}C	3506	161	2.8 ± 0.3	2.4 ± 0.3
	3506	60	2.8 ± 0.2	2.3 ± 0.2
^{20}C*	65 ± 6.8	60	42 ± 3	38 ± 3

Canham, HWH, Eur. Phys. J. A **37** (2008) 367

(cf. Yamashita, Tomio, Frederico, 2004)

- Input: TUNL Nuclear data evaluation project, ...
- Experiment: ^{14}Be $\to \sqrt{\langle r_{nn}^2 \rangle} = (5.4 \pm 1.0)$ fm

  (Marques et al., Phys. Rev. C **64** (2001) 061301)

Form Factors and Radii (NLO)

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- Range corrections: $r_e \approx 1/m_\pi = 1.4$ fm
- Structure of halo nuclei $\rightarrow$ matter form factors, radii

nucleus	B_{nnc} [keV]	B_{nc} [keV]	$\sqrt{\langle r^2_{nn}\rangle}$ [fm]	$\sqrt{\langle r^2_{nc}\rangle}$ [fm]
^{14}Be	1120	-200.0	3.9 ± 0.1	3.3 ± 0.1
^{20}C	3506	161	3.0 ± 0.1	2.5 ± 0.1
	3506	60	2.8 ± 0.1	2.4 ± 0.1
^{20}C*	65 ± 1.0	60	43.2 ± 0.5	38.7 ± 0.4

Canham, HWH, to be published

- Input: TUNL Nuclear data evaluation project, ...
- Experiment: ^{14}Be $\rightarrow \sqrt{\langle r^2_{nn}\rangle} = (5.4 \pm 1.0)$ fm
  (Marques et al., Phys. Rev. C **64** (2001) 061301)

Summary and Outlook

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- Effective field theory for large scattering length
  - Limit cycle in three-body system $\Leftrightarrow$ Efimov physics
  - Universal correlations (Phillips, Tjon line,...)
- Applications in atomic, nuclear, and particle physics
  - Cold atoms close to Feshbach resonance
  - Halo nuclei
  - Scattering properties of the $X(3872)$
- Future directions:
  - Cold atoms: heteronuclear systems, $N \geq 4$, 2d-systems, ...
  - Halo nuclei: reactions, external currents, ...
  - Hadronic molecules: universal properties, three-body molecules?
  - Three-nucleon system on the lattice: finite volume corrections, limit cycle in “deformed” QCD?

Additional Slides

N-Boson Droplets in 2D

- Study universal properties of N-boson droplets in 2 spatial dimensions

$$H = \int d^2x \left(\frac{\hbar^2}{2m} |\nabla \psi|^2 - \frac{g}{2} (\psi^\dagger \psi)^2 + \dots \right)$$

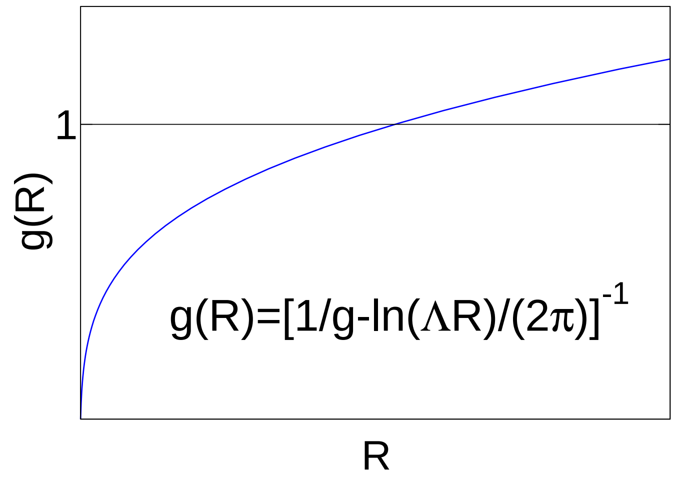
- Weakly attractive interaction: $g > 0$ (cf. Landau-Lifshitz)

  $\longrightarrow$ exponentially shallow dimer: $B_2 \sim \Lambda^2 \exp(-4\pi/g)$

- Define running coupling $g(R)$

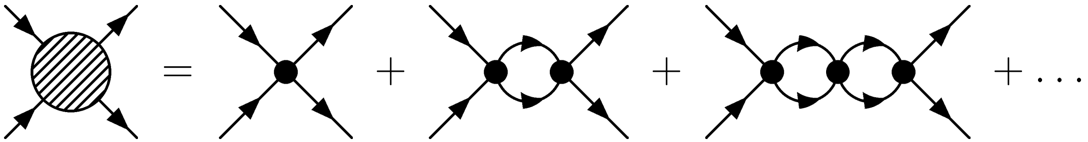
  $\rightarrow$ asymptotic freedom

  $\rightarrow$ calculate shallow N-body states



N-Boson Droplets in 2D

- All N-body energies are proportional to B_2
- Use classical field theory with running coupling $g(R)$ to calculate $B_N \Rightarrow$ RG improved variational calculation



- Ansatz: $$\psi(\mathbf{r}) = \frac{\sqrt{N}}{R\sqrt{2\pi C}}\, f\left(\frac{r}{R}\right) \longrightarrow N = \int d^2x\, \psi^\dagger \psi$$

  $\rightarrow$ minimize H with respect to size R and shape $f(r/R)$
- Minimization with respect to size R: $g(R) \sim 1/N + \mathcal{O}(1/N^2)$
- Both T and V are of $\mathcal{O}(N) \rightarrow g(R)$ stabilizes droplet
- Minimization with respect to shape $f \rightarrow$ bell shape

N-Boson Droplets in 2D: Results

- N-body bound states show exponential behavior

$$B_N = \left(c_0 + c_{-1}/N + c_{-2}/N^2 + \ldots\right)\, 8.567^N$$

$$\Rightarrow \boxed{B_N/B_{N-1} \approx 8.567, \qquad R_N/R_{N-1} \approx 0.3417, \qquad N \gg 1}$$

(HWH, D.T. Son, Phys. Rev. Lett. **93** (2004) 250408)

- Finite range interactions: $1 \ll N \ll N_c \approx 0.931 \ln(R_2/r_0)$
- How large is large?
- Few-body calculations for $N = 3, 4$ are available

Bruch, Tjon, Phys. Rev. A **19** (1979) 425

Nielsen, Jensen, Fedorov, Few-Body Syst. **27** (1999) 15

Platter, Hammer, Meißner, Few-Body Syst. **35** (2004) 169

N-Boson Droplets in 2D: Results

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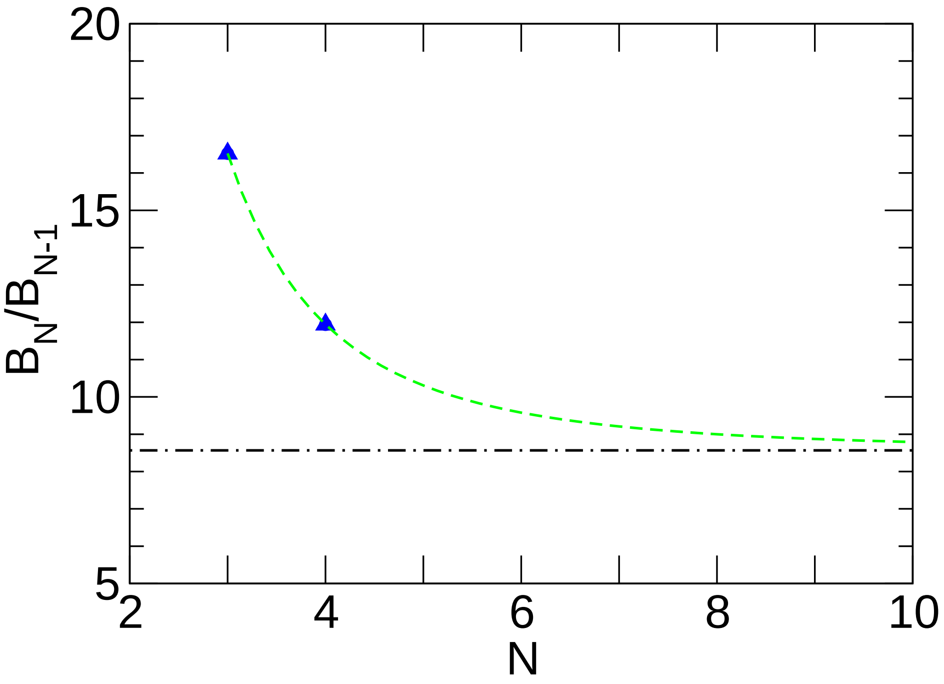
- How is the large-N limit approached?

$$N \gtrsim 6 \Leftrightarrow \frac{a}{r_e} \gg 600$$



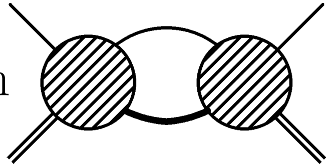
- Calculate explicitly for larger N

(cf. Blume, Phys. Rev. B **72** (2005) 094510; Lee, Phys. Rev. A **73** (2006) 063204)

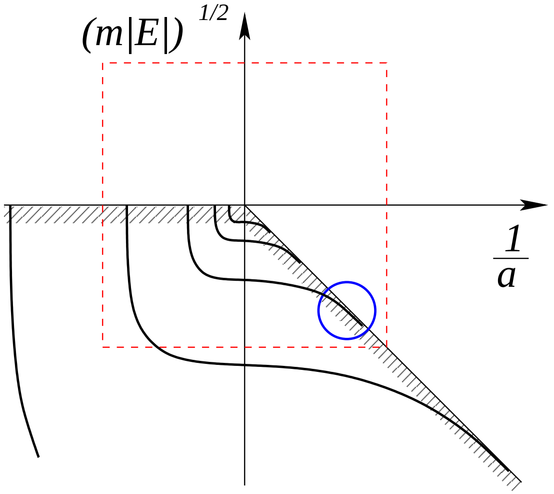
- Realization in experiment?

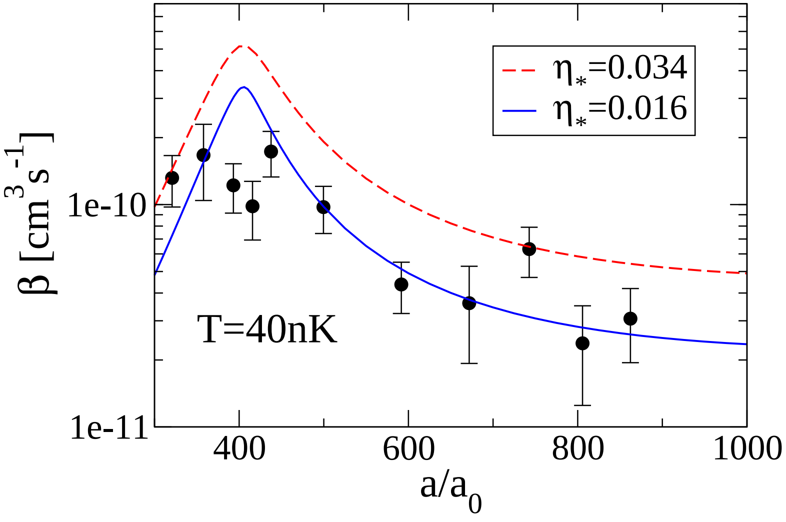
Dimer Relaxation in ^{133}Cs

- Dimer Relaxation: a + d $\rightarrow$ a + D (energetic)
- Relaxation constant: $\dot{n}_A = \dot{n}_D = -\beta\, n_A\, n_D$



- Recent experiment: Knoop et al. (Innsbruck), Nature Physics **5** (2009) 227
- Finite temperature $T \sim T_c$: Bose-Einstein average ?





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