Unnatural-parity excitations of three-charge systems

Efimov meeting

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Outline

1. Introduction
2. Three-charge systems
   - Natural parity
   - Unnatural parity excitation
3. Systems of four unit charges
   - Ground state of $(m_1^+, m_2^+, m_3^-, m_4^-)$
   - Borromean binding
   - Metastability
4. Conclusions

History

Very important activity on physics of two-electron systems, already in the late 20s

1. Journal Article — Add to marked items
Die Ionisierungsspannungen von Atomkonfigurationen mit zwei Elektronen
Egil A. Hylleraas
Naturwissenschaften, Volume 17, Number 50 / December, 1929
PDF (297.0 KB)

2. Journal Article — Add to marked items
Neue Berechnung der Energie des Heliums im Grundzustande, sowie des tiefsten Terms von Ortho-Helium
Egil A. Hylleraas
Zeitschrift für Physik A Hadrons and Nuclei, Volume 54, Numbers 5-6 / May, 1929
PDF (863.8 KB)

Berechnung der Elektronenaffinität des Wasserstoffs
H. Bethe
Zeitschrift für Physik A Hadrons and Nuclei, Volume 57, Numbers 11-12 / November, 1929
PDF (297.5 KB)

Journal Article — Add to marked item
Vergleich der Elektronenverteilung im Heliumgrundzustand nach verschiedenen Methoden
H. Bethe
Zeitschrift für Physik A Hadrons and Nuclei, Volume 55, Numbers 7-8 / July, 1929
PDF (256.1 KB)

1929

With of course a dramatic difference between the case of $[+2, -1, -1]$, such a He(α, e^-, e^-), and the case of $(+1, -1, -1)$ such as H$^-$. Problem raised later by Chandrasekhar

SOME REMARKS ON THE NEGATIVE HYDROGEN ION AND ITS ABSORPTION COEFFICIENT

S. CHANDRASEKHAR
Yerkes Observatory
Received June 28, 1944

ABSTRACT

Some remarks on the quantum theory of the negative hydrogen ion are made, and attention is drawn to certain facts which make the evaluation of its continuous absorption coefficient a problem of extreme difficulty.

and further generalised by Thirring (A course in Mathematical Physics)

Some Difficult Problems

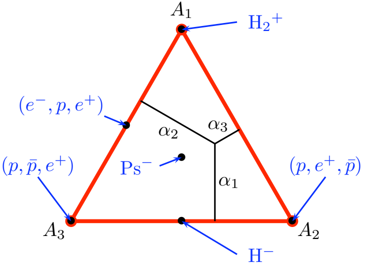
1. Investigate the three-body Coulomb system with charges $+, -, -$, and masses m_1, m_2, 1. For what region of the m_1, m_2-plane does there exist a point spectrum (cf. (4.3.27))? In particular, is there a bound state of e$^+$ H?

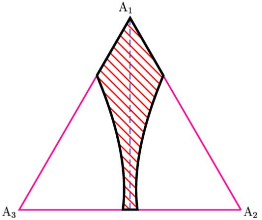
Halos. What is remarkable with H^-?

- H^- is the first halo in few-body quantum physics, followed by several others, such as $^6\mathrm{He}$ in nuclear physics, He clusters in molecular physics, and, perhaps, X(3872) in hadron physics.
- With $\mathrm{He} = (\alpha^{++}, e^-, e^-)$, once an electron is bound, it remains some attraction to trap the second one. Binding is obvious. Any other mass combination (m_1^{++}, m_2^-, m_3^-) also gives a stable atom.
- With H^-, the first electron neutralises the proton, hence it is not obvious what happens with the second electron.
- With H^-, a Hartree–Fock wave function $f(r_2)f(r_3)$ fails! You need at least $f(r_2)g(r_3) + g(r_2)f(r_3)$ with $f \neq g$, as done by Chandrasekhar, and named "unrestricted Hartee-Fock" by Goddard. Recent review Høgassen et al. (to appear in Am. J. Physics)
- The extension to other mass configurations gives either stability or instability.

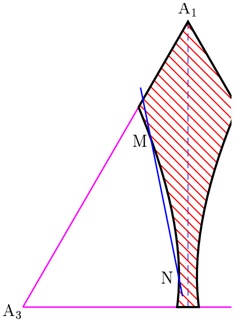
Survey of $(\pm,\mp,\mp)$ (Martin, R., Wu)

- $$H=\frac{\alpha_1}{2}\boldsymbol{p}_1^2+\frac{\alpha_2}{2}\boldsymbol{p}_2^2+\frac{\alpha_3}{2}\boldsymbol{p}_3^2-\frac{1}{r_{12}}-\frac{1}{r_{13}}+\frac{1}{r_{23}}\ ,$$
- Scaling: normalised inverse masses $\alpha_i = 1/m_i$, with $\sum_i \alpha_i = 1$,
- "Dalitz plot"
- With nice properties (convexity, star shape, stability along the axis)

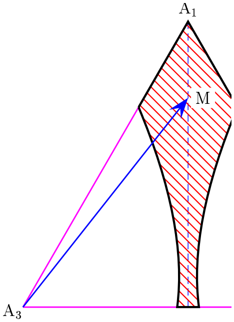




Rigorous properties



Convexity



Star shape

For the ground-state: any symmetric configuration (M^+, m^-, m^-) is stable (Hill, starting from the $M \to \infty$ case, and using frame attached to M, with a Hughes–Eckart term.

Mass dependence along the axis

At first sight: regular evolution of $(M^{\pm}, m^{\mp}, m^{\mp})$ along the symmetry axis,

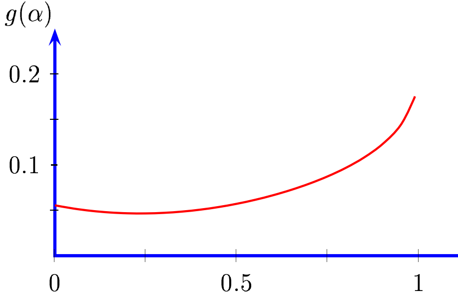
- from $m/M \to 0$ (H^-, $\alpha_1 = 0$ below) which is weakly bound
- to $m/M \to \infty$ ($\mathrm{H_2}^+$, $\alpha_1 = 1$ below) which is deeply bound.

Closer look: the relative excess of binding g as compared to the threshold

$$g(\alpha_1) = \frac{E - E_{\text{th}}}{E_{\text{th}}} \ ,$$

$$\alpha_1 = \frac{1/M}{1/M + 2/m} \ ,$$

is not monotonic. Minimum somewhere between H^- $(\alpha_1 = 0)$ and $\mathrm{Ps}^-(\alpha_1 = 1/3)$.



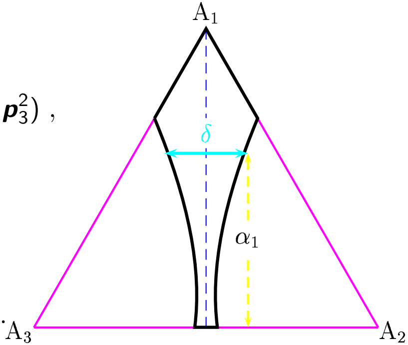
Width of the stability band

From the decomposition into symm. and antisym.

$$H(\alpha_1,\alpha_2,\alpha_3) = H(\alpha_1,\alpha_{23},\alpha_{23}) + \lambda(\boldsymbol{p}_2^2 - \boldsymbol{p}_3^2)\ ,$$
$$\alpha_{23} = (\alpha_2+\alpha_3)/2 = (1-\alpha_1)/2\ ,$$
$$\lambda = (\alpha_2-\alpha_3)/4\ .$$

one gets

$$\delta = \frac{2}{\sqrt{3}}\,(\alpha_3 - \alpha_2) \lesssim \frac{2}{\sqrt{3}}\frac{g(\alpha_1)}{1+g(\alpha_1)}(1+\alpha_1)\ .$$



Diverse developments

- Extension to three arbitrary charges $\{[m_1, +q_1], [m_1, -q_2], [m_3, -q_3]\}$ (Krikeb, Martin, R., Wu)
- Extension to four unit charges $(+, +, -, -)$, Fleck et al., Froehlich et al.
- More than four charges: exploratory numerical studies (Mitroy et al.)
- Screened Coulomb (Bressanini et al.)
- Extension to unnatural parity states,

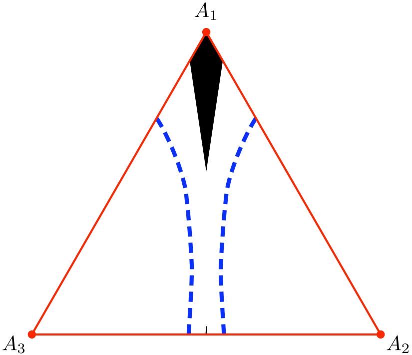
Unnatural parity states of H^- and H^--like states

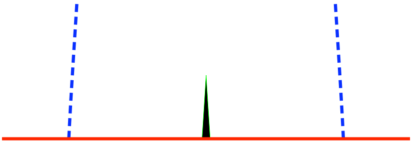
- The effective threshold is $\mathrm{H}(2p) + e^-$, i.e., $E_{\rm th} = -0.125$ in a.u.
- Found (by Norvegians) to be bound with $E \simeq -0.1253$, very close to threshold.
- Confirmed e.g. by Drake,
- This is the only bound state of unnatural parity (Grosse et al.)
- No analogue found for Ps^- by Mills
- Hence it deserves further study as a function of the masses,
- $$\Psi = (\boldsymbol{y} \times \boldsymbol{z})_j \sum_i \gamma_i \exp(-a_i\, x - b_i\, y - c_i\, z)\ ,$$

  where $\boldsymbol{x} = \boldsymbol{r}_2 - \boldsymbol{r}_3$, etc., which generalises for 1^+ the wave function used by Hylleraas and Chandrasekhar for 0^+ (axial character, explicit dependence on $x = r_{12}$, and several terms)
- Convexity and star-shape properties remain, not Hill's result about stability along the axis

Results

In the “Dalitz plot” stability of unnatural- vs. natural-parity ground state.

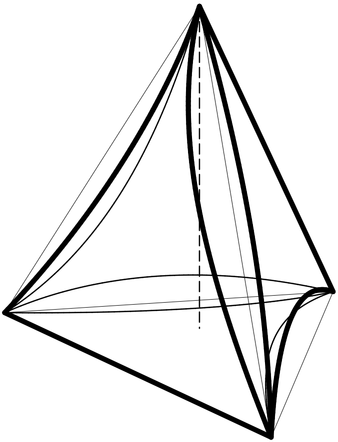




- disconnected
- very tiny in the H^- region

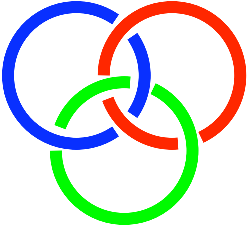
Four unit charges $(m_1^+, m_2^+, m_3^-, m_4^-)$

- The domain is now a tetrahedron,
- Less rigorous results, due to the variety of thresholds,
- If the lowest threshold is $3+1$, e.g., $(p, p, \bar{p}, e^-)$, the ground state is stable.
- If two particles are identical, e.g., $m_3 = m_4$, the ground state is always stable, $\forall m_1, m_2$
- The stability of $\mathrm{Ps}_2(e^+, e^+, e^-, e^-)$ implies a better stability of $\mathrm{H}_2(p, p, e^-, e^-)$
- The alternative symmetry breaking $\mathrm{Ps}_2 \to (M^+, m^+, M^-, m^-)$ spoils stability



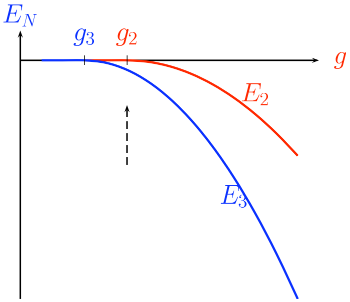
Borromean binding-1

Well-known problem, especially for this audience!



Borromean binding-2

Linked to Thomas collapse ($|E_3| \gg |E_2|$ near $g = g_2$) and Efimov effect



But this is for short-range potentials

What about long-range pot., in particular Coulomb?

Borromean binding-3

- A possibility is to play with some **mass ratio** instead of coupling constant!
- For three-charges $(M^\pm, m^\pm, m^\mp)$ stability found for

$$0.70 \lesssim \frac{M}{m} \lesssim 1.64 \;,$$

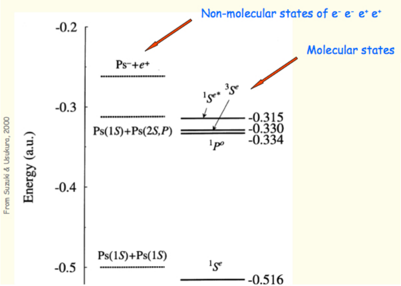
- However, at least two studies of the asymmetric four-body molecule $(M^+; m^+, M^-, m^-)$ indicates stability restricted to

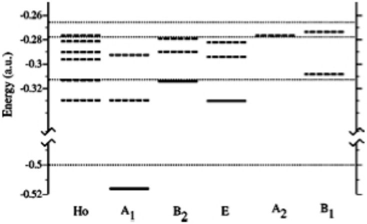
$$\frac{1}{2.2} \lesssim \frac{M}{m} \lesssim 2.2 \;,$$

- indicating a window of Borromean binding near $M/m \simeq 2$,
- feasible in a trap with $(d, p, \bar{d}, \bar{p})$ or $(p, K^+, \bar{p}, K^-)$
- Note: $\bar{d}$ production is not an issue, the problem is cooling and storage!

Excitations of Ps_2

Excitations and resonances of Ps_2 rely on the existence of an effective threshold, due to quantum numbers and overlap with the lowest states. This generalises the unnatural-parity for three-charge systems.





Conclusions

- Role of mass ratios for the stability of Coulomb systems,
- For (m_1^+, m_2^-, m_3^-) very narrow domain of stability for unnatural parity states,
- Mass ratio replaces coupling constant to determine the frontier of Borromean binding,
- It would be interesting to analyse the interplay of coupling constant and mass ratio for short-range potentials where both effects are possible,
- Many applications to hadron physics
  - Better stability of H_2 as compared to Ps_2suggests analogues in quark models with *flavour independence*: $(QQ\bar{q}\bar{q})$ more stable than $(qq\bar{q}\bar{q})$,
  - Weakly-bound states or resonances of meson–meson systems suggested as an explanation of newly-found hadron, such as $X(3872)$,
  - Unnatural-parity states of three quarks difficult to produce and identify! This led to alternative models of baryons: quark–diquark