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Atmospheric phase measurements (radio seeing)

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Introduction

While it is important to compare the opacities of possible sites for the submillimeter array, in order to ensure that the maximum time is available for observations, the phase fluctuations in the atmosphere are no less important, although they are harder to study. The available measurements of atmospheric phase effects in the radio domain are relatively few, and have been obtained mainly with two different techniques. The first, direct technique is to measure phase with a radio interferometer. This gives a measure of the r.m.s. phase fluctuations over the range of baselines of the instrument. The second is to look at pointing fluctuations with a single dish. This gives a measure of the phase slope over the single dish, which is roughly equivalent to the phase difference between two points separated by some fraction of the dish diameter. The main problem with both of these types of measurements is that they are limited to sites where suitable instruments already exist. A second problem, which is particularly severe for the second type of measurement, is that the instability of the system may limit the measurement during good atmospheric conditions.

A third technique, which is being explored by NRAO, involves the use of water-vapor radiometers to measure fluctuations in the atmospheric water vapor which dominates radio seeing. The interpretation of such measurements is more complicated than that of the first two types, because the quantity being measured is only indirectly related to the phase of an interferometer. However, the equipment is simple and easily deployed, so it is the technique most likely to provide a comparison between potential sites for the submillimeter array.

The atmospheric refraction is usually characterized in terms of a structure function of phase difference, which is the average value of the square of the phase observed by an interferometer with a given baseline. For 3D Kolmogorov turbulence, the structure function of phase has a power-law character, scaling as B^α, where $\alpha = 5/3$, which means that the phase should vary as $B^{5/6}$ (Armstrong and Sramek 1982). Such a steep function implies that the largest length scales dominate and that the seeing is nearly independent of wavelength. This corresponds to the well-known case of optical seeing, where the image diameter is proportional to $\lambda^{-0.2}$. (For the limiting case in which the structure function is proportional to B^2, the seeing would be precisely independent of wavelength since, although shorter wavelengths require shorter baselines for a given angular resolution, the path differences are smaller by the same factor.) Two-dimensional turbulence gives a similar power law, but with $\alpha = 2/3$. As the atmosphere has a finite thickness, it is expected that the 3D slope at short baselines should change to the 2D value for baselines greater than some threshhold value.

Measurements

In this discussion, we will measure phase in terms of path length, which is nearly independent of frequency since the atmosphere is nearly non-dispersive in the radio and submm range. Figure 1 shows a summary of all available data from the VLA, Nobeyama, Hat Creek, IRAM 30 m, Effelsberg 100 m, and JCMT. Obviously, of these instruments, only the JCMT is situated at one of our potential sites. These data are drawn from published papers, and from Jim Moran's report on papers presented by Downes, Ishiguru, and Sramek at the Beijing meeting. The VLA and Nobeyama data both include ranges of phase deviation which depend on the weather. In the case of the VLA, the range is approximately from typical summer day (bad) to typical winter night (good). I have also included a point representing the best VLA weather, as presented by Owen and Hogg at the Beijing meeting. For Hat Creek, I have taken the scatter plot presented by Bieging *et al.* (1984) and drawn a crude upper and lower envelope, based on the assumption that phase is roughly proportional to baseline. In the case of single dishes, I have converted the angular fluctuations to path differences at an effective separation assumed to be 2/3 of the dish diameter. The correct fraction depends on the illumination, etc., but will not be dramatically different from this value. In any case, the choice of separation is not very critical because the phase fluctuations are nearly proportional to separation, giving pointing fluctuations which depend only weakly on dish size. The interferometer phase measurements are probably due entirely to atmospheric effects, but the lower values for single dishes are only upper limits, due to instrumental tracking errors at the level of ~0.″5.

The line labelled 'optical seeing' is based on the median angular fluctuations observed by the NOAO seeing survey on Mauna Kea (Merrill and Forbes 1987), and corresponds to 0.″5 seeing. This line is not exactly comparable with the other measurements, since it represents only the dry component of the atmosphere, but it is included for comparison, and as an indication of the limiting case when the water vapor is very low. Extrapolation of this line shows the coincidence remarked on by Cornwell (1984), that the predicted fluctuations agree with those observed at the VLA. The coincidence has no deep meaning, however, and is due simply to the fact that the slope of the structure function for water decreases between the scale of optical telescopes and the scale of the VLA. It is likely that the structure function for dry air also decreases at large scales, or dry air seeing would limit the VLA resolution.

One interesting feature of this diagram is that, despite the different sites, the measurements overlap fairly well. There is an apparent break in the power-law slope at a baseline of about 100 m, though this has not been seen in any one data set, but only by comparison of several different measurements. Despite this weakness, the crucial observation is the shallow slope observed at the long VLA baselines, indicating that 3D Kolmogorov turbulence cannot continue on scales of 1km and more. This change in slope might be due to the difference between 3D and 2D turbulence in a layer with a thickness of approximately 100 m. A similar indication of the thickness of the turbulent layer comes from the measurements at the IRAM 30 m telescope (Altenhoff *et al.* 1987), which are claimed to be consistent with individual blobs with sizes of about 100 m, on the basis of the observed timescales and the lack of dependence on elevation angle. This observation is also slightly weak, since, in the case of a power-law distribution, the single-dish observations would tend to pick out scales comparable with the dish diameter.

In a layer as thin as 100 m, a humidity change of 20% is required to produce an angular deviation of 3.″5. This seems quite a large change for the atmosphere near the ground, and we may speculate that the relevant part of the atmosphere is a boundary between wet and dry layers, where there is a sharp change in humidity over a height of only ~100 m. The most dramatic such boundary is at the inversion layer and this would be consistent with the observation that the periods of bad pointing at the 30 m and the JCMT correspond to times when the inversion layer rises to the mountaintop. The measurements by Parrish *et al.* (1987) showed that, even when clouds reached the summit of Mauna Kea, the opacity remained low along clear sight-paths between cumulus towers. Under this hypothesis, bad radio seeing is like looking through the surface of a swimming pool, where the blurring is dominated by the ripples on the surface, rather than the total column of water. If this speculation is true, then the seeing at high mountain sites may be much better than any surface site, since the mountain top is above the inversion, at least at night. A preliminary indication that mountain sites do have better seeing is the measurement by Owen and Hogg (1989) that the atmospheric emission fluctuations at South Baldy are significantly lower than those observed at the VLA site.

Interferometer Performance

Figure 2 shows the same diagram of phase fluctuations as a function of baseline, with the addition of horizontal lines corresponding to the path difference which gives 1 radian of phase at various wavelengths. This is the level at which phase calibration and accurate imaging fail. The intersection of each line with the measured path fluctuations shows the maximum baseline permissible at the appropriate wavelength. Finally, the 45° lines show the angular resolution ($0.7\lambda/B$), corresponding to each baseline and wavelength. At the top of the plot, for example, the 1.3 cm line crosses the lower bound of the VLA envelope at a baseline of greater than 30 km, which corresponds to an angular resolution better than 0.″1. In other words, on a typical winter night, the VLA is capable of high-resolution imaging virtually unaffected by the atmosphere. On the other hand, on a typical summer day, the intersection is at a baseline of 2 km, and the attainable resolution is roughly 1.″5, in the absence of self-calibration. In all cases, the limiting angular resolution improves with increasing wavelength. At small baselines, where the phase deviations are nearly proportional to baseline, this improvement is small, but the available angular resolution increases rapidly with increasing wavelength, because the slope of the phase curve is so small at large baselines,.

Based on the above data, the worst case for submm seeing looks quite bad. It is clear, for example, that on many days operation on Mauna Kea will be nearly impossible at any wavelength less than 1.5 mm. Even at night, if the upper limits correspond to the actual level of phase fluctuations, then the largest baseline usable at 350 μm is only about 20 m, with a resolution of 2″.5, and the largest baseline usable at 1.3 mm is about 125 m, with a resolution of 1″.5. This is almost certainly a pessimistic assessment, however, since the NRAO data already suggest that at least one mountaintop (S. Baldy) has significantly better seeing than expected from these data. In the Yellow book, it was suggested that the phase fluctuations should scale with the total quantity of water vapor, which will typically be a factor of 5 or so lower at Mauna Kea than at the sites of present interferometers. Alternatively, if the dominant effect is due to the inversion layer, then the ground-based measurements may be nearly irrelevant to mountain sites above the inversion. If the atmospheric fluctuations are a factor of 2 better than indicated by the JCMT upper limit, then the resolution at 350 μm improves to 1″.0, at a baseline of 50 m, and the resolution at 1.3 mm is 0″.6, at a baseline of 300 m. (As a rough rule-of-thumb, the expected beamsize limit at 350 μm is 4 times larger than the pointing fluctuations measured by a single dish.)

One limiting case is given by the dry-air fluctuations which have been measured in optical seeing. If the median Mauna Kea seeing of 0″.25 is extrapolated according to a Kolmogorov spectrum, then the fluctuations would reach 1 radian at 350 μm at a baseline of 125 m, with a resolution of 0″.4. This is a large extrapolation, and is also likely to underestimate radio seeing, since it ignores water entirely, but it suggests that baselines much longer than $\sim$ 100 m may be hard to use at the shortest wavelengths, without self-calibration. Depending on the shape of the structure function, the available resolution at 1.3 mm may be as good as 0″.06 under such conditions.

The dry-air fluctuations are also important because they provide the limit if the water vapor fluctuations can be measured and corrected by radiometry. Since the total power detected by each receiver in the array provides a measure of the water vapor in the line of sight, the total power information could be used to correct the phases for the effect of that water vapor (Welch, private communication). Even if this could be done perfectly, the dry-air seeing would remain.

To summarize, presently available estimates of atmospheric stability on Mauna Kea indicate that interferometry would be impossible on many *days*. On typical *nights*, the present upper limits suggest that an angular resolution of $\sim 2''$ will be attainable, with the requisite baselines of up to 125 m. It is likely that the nights are more stable than indicated by the present upper limits and that we might make good use of baselines up to a few hundred meters. If we can measure and correct for water vapor, or if the conditions are very good, then angular resolution better than $0\farcs5$ should be possible. It is desirable to improve the accuracy of present measurements of seeing on Mauna Kea by a factor of at least two, and to find a way of making similar measurements on Mount Graham.

Measuring the phase fluctuations

It is difficult to know how to measure phase unambiguously on any site without building quite a sophisticated test interferometer. The main problem here is that the atmospheric path differences are very small, compared with systematic effects. For example, to make measurements with an interferometer whose baseline is 50 m, an accuracy of 3 μm is needed to test for $0\farcs1$ seeing. This can be compared with the temperature coefficient of just 1 m of good-quality cable, which is approximately 10 $\mu\mathrm{m\ K}^{-1}$. The other types of measurement are also subject to systematic errors. For example, the single-dish measurements are probably limited by tracking errors to pointing deviations of about $0\farcs5$ or so, at the level of the current upper limits. Radiometer measurements with the NRAO machines may underestimate the fluctuations if the level of the turbulent layer is high, since their beams are very wide. Radiometers are also inherently insensitive to dry air fluctuations.

The task is simplified, however, by the relevant timescale of the fluctuations, which is very short, so that the measuring instrument requires only short-term stability. It can be shown that the dominant physical scale of turbulence at any baseline is comparable with the length of the baseline. The corresponding timescale can be obtained from the hypothesis of 'frozen turbulence', which states that, to a good approximation, the turbulent patterns are simply blown across any given point by the wind. At a baseline of 1 km, with a typical wind speed of 7 m/s, the timescale is only ~ 150 s, and it is proportionally less at shorter baselines.

This timescale also sets a limit to the integration time available for a phase measurement, either for site testing or, ultimately, for self-calibration. Using this constraint, I have plotted in Figure 3 rough estimates of the sensitivity of several different types of measurements. The first is an interferometer formed from CSO and JCMT, observing 3C84 or 3C273 at a wavelength of 1.3 mm. The second is a hypothetical interferometer observing H_2O masers at 22 GHz, with 3 m dishes and 500 K receivers. The third is the less direct, but more practical, technique of radiometry, using the NRAO devices at 225 GHz. The sensitivity plotted is that for 1σ measurements of instantaneous phase, so the increasing sensitivity at longer baselines is due to the longer available integration time. If a long time series is used to form a power spectrum, the sensitivity will be better since the fluctuations will show up statistically as a deviation from white noise.

The lines in Figure 3 show that all three techniques have sufficient sensitivity in principle for submillimeter site testing. There are significant differences in hardware complexity, however. The CSO-JCMT interferometer would require new phaselocks and l.o. distribution, a delay line, and a correlator. The H_2O maser scheme would require tracking dishes, receivers, l.o. system, and correlator. A scheme based on satellite signals would avoid the need for tracking dishes, and delay lines, although the other elements would still be needed. By contrast, the NRAO radiometers are available right now, although the measurements are the least direct. Single-dish pointing measurements may also be useful if some way can be found to improve their accuracy.

References

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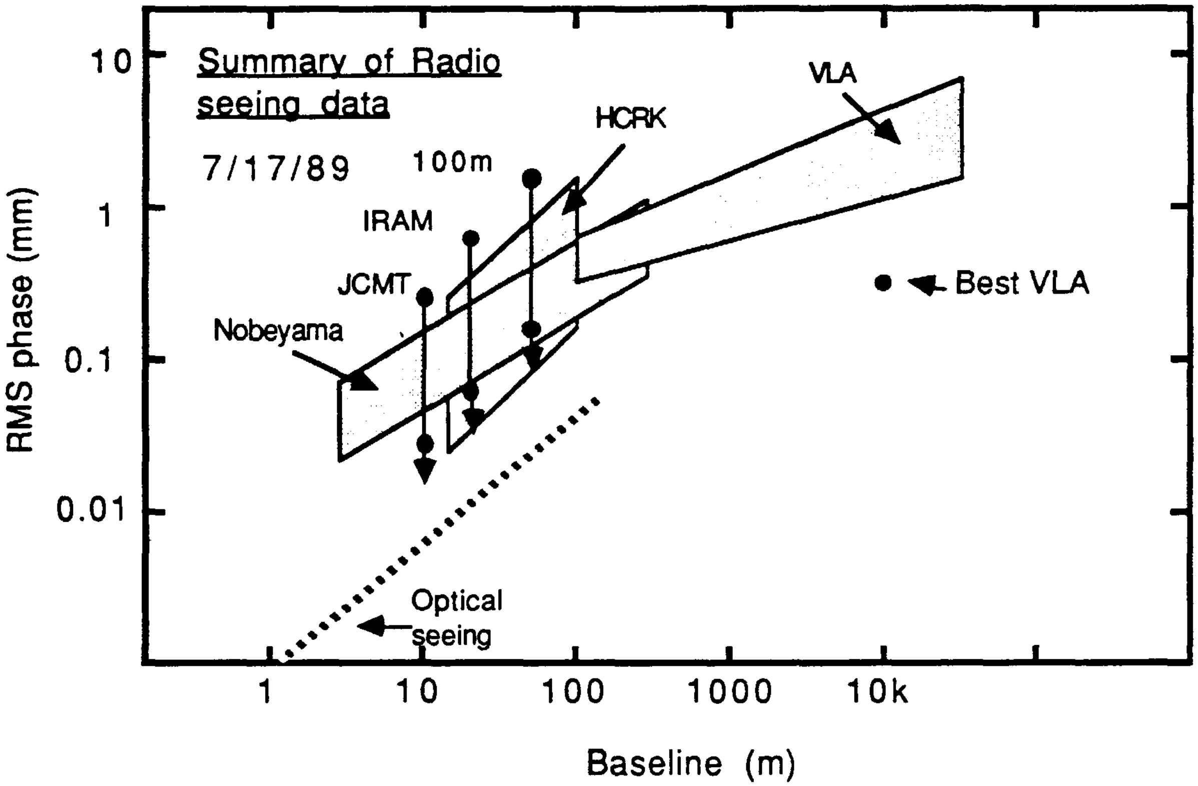

Figure 1. Summary of available radio phase data from all sources.

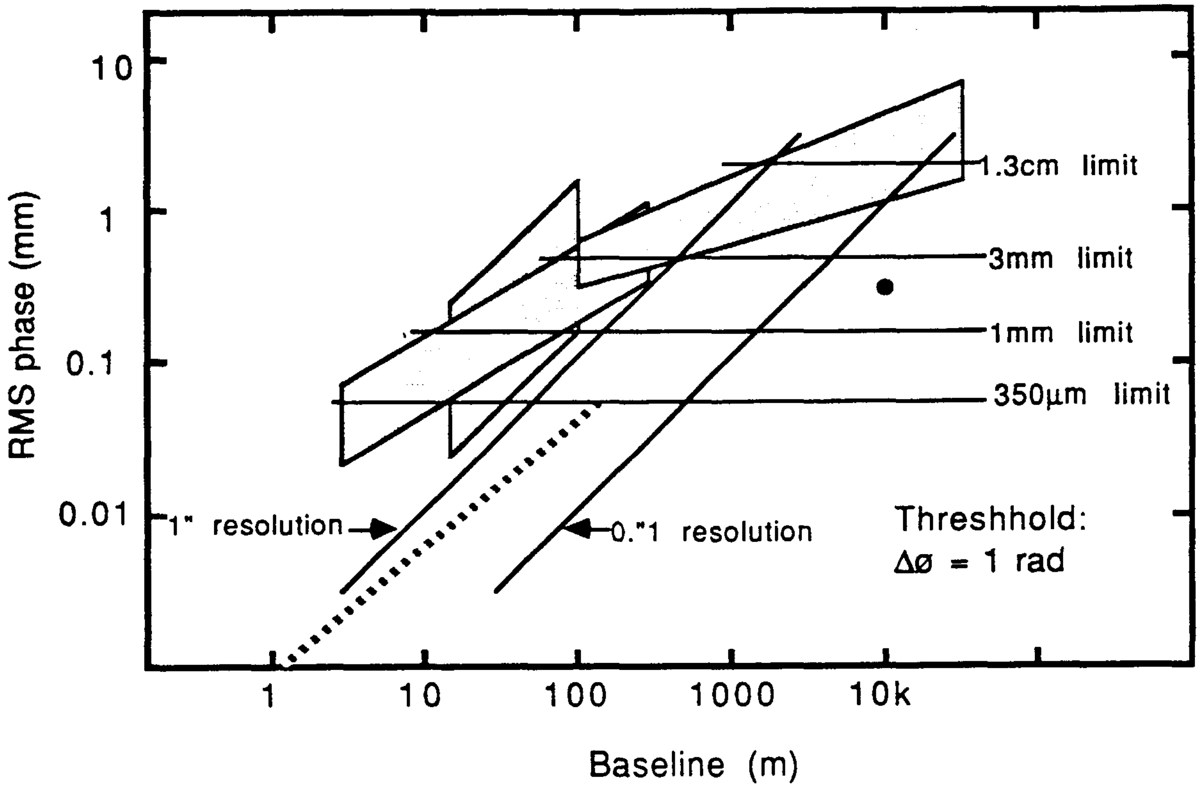

Figure 2. Summary of radio phase fluctuations with limiting baselines for various wavelengths.

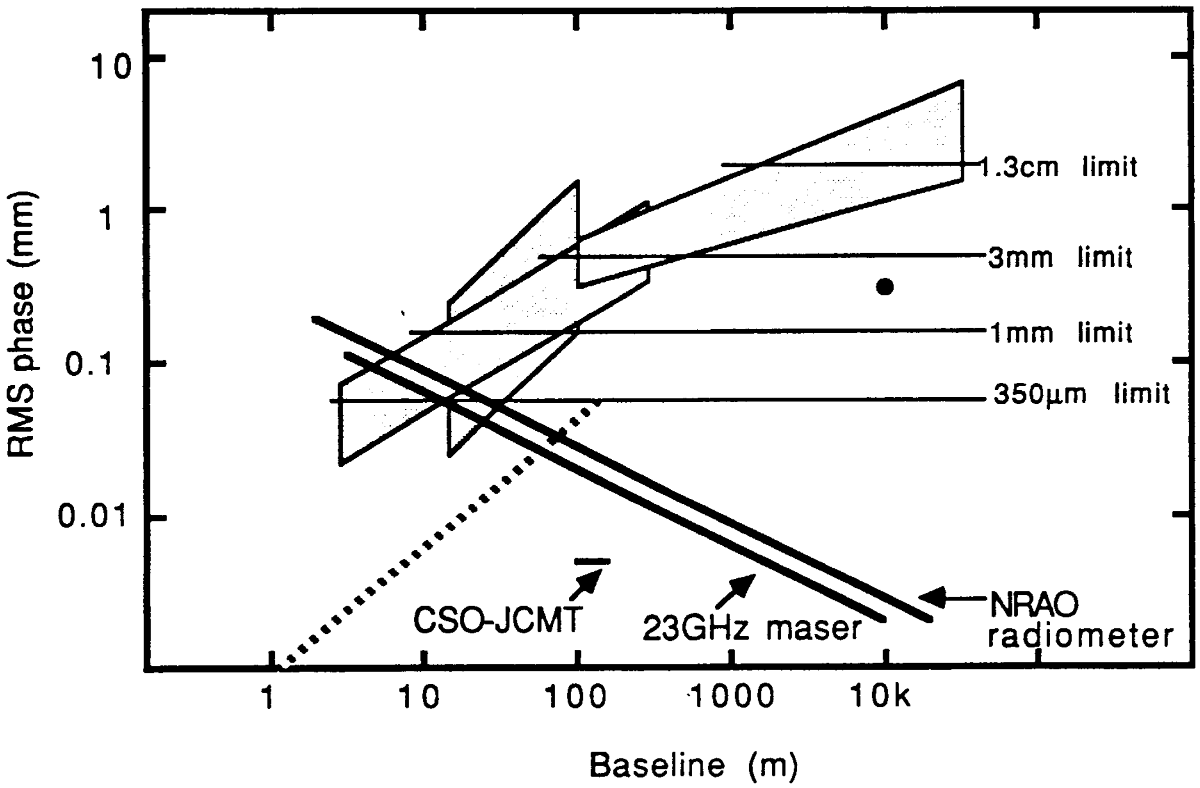

Figure 3. Sensitivity of Phase measurement.

Notes:

1) Conversions from time to distance use a wind speed of 7m/s

2) radiometer has sensitivity of 0.2K in 2 sec

3) CSO-JCMT assumes 1500K Tsys, 20 Jy, 50% eff, 1GHz bw

4) 23GHz maser assumes 500K Tsys, 2500 Jy, 50% eff, 3m dishes
70 kHz bw (1km/s)