SMA Design Study Memo 5

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Phase Calibration for the SMA

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Introduction

In previous memos I have addressed the questions of the number of available phase calibrators for the SMA, and the atmospheric phase fluctuations. In this memo, I try to bring together the information in those studies to shed light on the question of how we can calibrate the array in practice, and whether self-calibration is likely to be useful. The answers depend to some extent on unknown parameters, but I have picked plausible values and tried to show how the answers depend on those values, so that the analysis can easily be updated in the light of new information.

I) Receiver Performance in Various Atmospheric Windows

Fundamental to the question of calibration is the sensitivity of the array. I have chosen to use the best current technology, as represented by the 230 GHz and 345 GHz receivers of the CSO (Ellison *et al.* 1989), for an estimate of the system temperature although future receivers may have lower noise temperatures. The best quoted noise temperature for the 345 GHz receiver is 150 K DSB, with a typical value of 200 K. Since the array will have to be calibrated even when the system is not operating at optimal efficiency, I have adopted a conservative value of 400 K SSB at 345 GHz. To extrapolate to higher frequencies, I have assumed that the receiver temperature scales linearly with frequency, corresponding to a constant quantum efficiency. This is a very optomistic assumption for higher frequency receivers, but we shall see below that direct calibration will still be very difficult.

For the other parameters, I have assumed 5% ambient temperature spillover loss, 275 K ambient temperature, SSB receivers, and line-of-sight water vapor columns of 1 and 3 mm. I have taken the opacities (nepers/mm) from various papers. They differ slightly rom those used on page 69 of the Yellow Book. DSB receivers would perform worse in conditions of high opacity than the SSB ones assumed in the calculation. The values of system temperature above the atmosphere, T′, are given in the table below.

SSB System Temperatures above the Atmosphere

Freq GHz	T_{rx} (SSB) K	τ/mm	T′(SSB) (pwv=1mm)	T′/f	T′(SSB) (pwv=3mm	T′/f
230	200	0.05	251	1.1	306	1.3
345	400	0.17	567	1.6	908	2.6
460	600	0.80	1800	3.9	9900	22
850	1000	1.00	6800	8.0	53000	62

For purposes of phase calibration, the path length error (measured in mm) is proportional to the quantity T′/f, if the calibrator has a flux density which is independent of frequency. This is approximately true for most of the non-thermal, extragalactic objects, whose flux densities usually fall slightly with frequency. Despite the optomistic assumption made for the high frequency receivers, the values of T′/f at the lowest frequencies are dramatically better. Therefore the best phase calibration will be effected by measuring at 230 GHz, even when the observing program is at a higher frequency. For thermal calibration sources, which have rising spectra, the best frequency is probably 345 GHz in most cases, although resolution effects may make 230 GHz preferable at long baselines. As demonstrated in memo 1, thermal sources do not solve the problem of high-frequency calibration, since they do not provide enough brightness temperature for the longest baselines. In the general case, we must be prepared to calibrate with extragalactic sources.

If we choose to calibrate at a frequency different from the observing frequency, there are several implications for the system design. Care must be taken to minimize systematic phase differences between the different frequencies, and, if dual-frequency self calibration is desired, the system must be designed to operate simultaneously at two frequencies. In the calculations which follow, I will assume that this will be done, and that all calibration will be done at 230 GHz.

The highest calibration sensitivity is at 230 GHz, so I will calculate that case to determine the best we can expect. I took the numbers from p85 of the Yellow book and adjusted them for the following parameters: $T' = 309$ K, $\lambda = 1.3$ mm, $t = 300$ sec, $d = 6$ m, $\eta_A = 0.5$, 1 polarization. The resulting noise level on one baseline is 75 mJy. If we apply global fitting, the noise level in measuring the phase on any one antenna is improved by $(N/2)^{1/2}$, where N is the total number of antennas. So for $N = 6$, the global noise level for calibrating one antenna is 43 mJy.

We can also see how this noise level scales with antenna number if the total collecting area is kept constant. The area of each antenna scales as N^{-1}, and the noise level consequently scales as N. In the global fitting, the global noise level is proportional to $N^{-1/2}$, so the net effect is that the noise scales as $N^{1/2}$. Thus an array with 12 antennas would have a global noise level 1.4 times higher than one with 6, for purposes of calibration.

II) Types of phase error

Before considering the limits of phase calibration, we must consider the different types of phase error which must be corrected. These errors arise from different sources and may not all be corrected by any one type of calibration.

a) System Phase Drifts: The first type of error produces phases which are independent of position on the sky. The predominant source of such error is drift in cables, phase-locks, etc., although there is some component due to mechanical changes in the dishes. With good design, the typical path length change might be ~ 0.1 mm, with a timescale of 1 hour or more.

b) Geometry Errors: The second type of error produces phases which depend on position in the sky. The most obvious source of such errors is an incorrectly determined baseline, but there are other effects due to variations in the telescope mounts, etc. The mount errors are crudely related to pointing errors in that they are due to variations in the telescope structure, and we might expect them to scale with dish diameter. As a very rough estimate I shall assume a typical path length change of 0.1 mm, with a timescale of a few hours. Geometry errors are removed by calibration only to the extent that the calibrator is close to the program source.

c) Atmospheric Path: As discussed in Memo #4, the atmospheric path depends on baseline and the timescale is quite fast. A probable *upper* limit to the timescale is given by the 'frozen turbulence' hypothesis, that the atmospheric pattern simply moves across the array at the wind speed. For a typical wind speed, the relevant timescales are no more than 100-200 sec, even at the longest baselines.

III Types of Calibration

The requirement for calibration is that the calibrated phase should be better than 1 radian (0.05 mm at 850 GHz) for marginal imaging, or better than about 0.3 radians (0.015 mm at 850 GHz) for fairly precise (5%) imaging. We will consider three different types of calibration and their effectiveness in dealing with the three types of phase errors.

A) CLASSIC CALIBRATION This is the standard radio astronomical technique of observing a point source near to the program source at intervals of, say, 30 minutes, with an integration of about 5 minutes on source. Assume a 0.75 Jy source, with the noise levels calculated above.

(a) *System Phase:* With global calibration, the effective noise level is 43 mJy for an array of 6 × 6 m antennas, giving a s/n ratio of 17, and a path error of 0.012 mm. This is adequate for excellent imaging at any frequency.

(b) *Geometry:* Using the formula in memo #1, I estimate that there are roughly 60 sources stronger than 0.75 Jy in 4.5 steradians. The distance to the nearest calibrator is then 0.15 radians. After calibration, the residual geometric error will be $0.15 \times 0.1 = 0.015$ mm, comparable with the measurement error.

(c) *Atmosphere:* Because of the timescales, the atmospheric errors will not be helped by this type of calibration, even if the calibrator is close enough for the errors to be correlated with those of the program source.

If the atmospheric phase is negligible, then the net path error remaining after calibration is 0.02 mm, just adequate for good imaging at 850 GHz, and perfectly satisfactory for any other wavelength. This calculation depends on the unknown magnitude of the geometric error but, for the assumed numbers, 0.75 Jy is about the optimal level for a calibration source, since the residual geometric error is comparable with the noise of the measurement. Note that direct calibration at 850 GHz would worse by at least an order of magnitude in sensitivity, and good imaging would be impossible.

B) LOCAL TRANSMITTER: Here I envision a transmitter located near the array, to which all the antennas would be pointed every 30 minutes or so to measure the phase. The signal/noise ratio of the measurement would be very high, although there would be some systematic effects due to the transmitter's being in the near field.

(a) *System Phase:* Because of the high s/n ratio, the system phase could be calibrated essentially perfectly.

(b) *Geometric Error:* This would not be calibrated at all; indeed extra error would be introduced because the ground-based transmitter is far from any program source. The error is therefore increased by $2^{1/2}$ to 0.14 mm, barely adequate even for 230 GHz.

(c) *Atmosphere:* Because the atmosphere changes much faster than the calibration cycle, the phase is not improved by this technique. In addition, an extra atmospheric error is introduced by the path to the transmitter.

The local transmitter option is very valuable for testing the system, because of its high s/n ratio, but it is essentially useless for phase calibration.

C) SELF-CALIBRATION The only method which can reduce geometric errors to zero is to calibrate on a source in the program field, or on the program source itself. This is also the only method which could, in principle, correct atmospheric errors. For this estimate we will assume that the program source has a 230 GHz flux density of 1 Jy (like L1551, a very strong source), and that resolution effects can be ignored. These are obviously optomistic assumptions.

(a) *System Phase:* Since system phase drifts are slow enough that 1 hour integrations are permitted, the noise level in the measurement is very low, 0.003 mm.

(b) *Geometry:* Geometric errors are calibrated perfectly.

(c) *Atmosphere:* The crucial question for atmospheric effects is one of timescales. The phase as a function of baseline and time will be analysed later, but for the purposes of this analysis, we can calculate the minimum time in which the s/n ratio will be sufficient for good calibration, and relate this to baselines using the hypothesis of frozen turbulence. The threshhold for good imaging at 850 GHz is 0.015 mm, which corresponds to a s/n ratio of 14 on a 1 Jy source. Using global phase, this s/n will be achieved in 120 s, corresponding to 840 m at the median wind speed of 7 m/s. In other words, self-calibration will not provide precision imaging at 850 GHz for arrays of the size which we are contemplating, even with a strong, 1 Jy source. With a weaker source, say 0.25 Jy, self cal will not provide precision images at any frequency if the atmosphere is the limiting factor.

Although we have not analyzed self-cal in detail here, it is clear that it will be only of marginal use in correcting atmospheric phases, because of the limited s/n available. However, it will be useful for correcting geometric and system phase errors in some cases.

Finding Baselines

Measuring the interferometer baselines is closely related to phase calibration since it involves measuring the phases on a number of calibrators in different parts of the sky. If sufficient calibrators are available for observations, then there are sufficient for finding baselines.

IV) Conclusions

1) The most accurate calibration is obtained at the lowest frequency of the array (230 GHz), although self-cal on thermal sources may be better at 345 GHz. At this frequency we can achieve sufficient path-length accuracy for operation of the array at all frequencies up to 850 GHz. Direct calibration at the higher frequencies will not be possible in general, although it may succeed in some cases, particularly at low resolution. This has important implications for the system design, since we must provide the capability for dual frequency operation and also minimize phase drifts which are not common to all frequencies.

2) For fixed total collecting area, the calibration error is proportional to the square root of the number of antennas. Configurations with few, large antennas are therefore preferred to those with many small ones. The effect may be mitigated slightly if the geometric errors scale with antenna diameter.

3) A fixed transmitter is not very useful for calibration, although it is useful for testing.

4) None of the calibration methods will remove atmospheric phase in general, although self-cal may be of some help in favorable cases.

5) There are sufficient calibrators for an array of 6 × 6 m antennas, although the residual phase at 850 GHz will still be 0.4 radians.

References

Ellison, B. N., Schaffer, P. L., Schaal, W., Vail, D., and Miller, R. E. 1989, preprint.