To: Bill Bruckman

MEMO #24

From: Heinrich Foltz

Date: 16-30 July 1990

Subject: Nasmyth Mirrors

Summary: Section I of this memo describes the positioning tolerance for the Nasmyth mirror(s). The 10-20 micron translation and 1-4 arcsecond rotation mentioned in your last fax are well within the 1% loss tolerance limits even for a deeply curved Nasmyth mirror. However, if you intend to use the Nasmyth mirror for beam switching the losses will be higher and a more careful analysis must be done.

Section II itemizes some of the other effects associated with the Nasmyth mirrors. For flat mirrors the primary effect should be an ohmic/roughness loss of 0.9% maximum for aluminum, and a Ruze type loss if the surface is not accurate.

For curved mirrors there will be cross-polarization and illumination asymmetry, which increase as the curvature increases. For a fairly deep mirror, the cross-polar level could be as high as 1.5%, but more typically a curved system would have a cross-polar level of 0.25%. The illumination asymmetry would be less than 0.5% and typically 0.12%.

Section III has some preliminary information related to how easy it would be to maintain flat Nasmyth mirrors. The primary drawback is the requirement for fairly large receiver apertures and a larger subreflector.

I. NASMYTH POSITIONING TOLERANCE

The point matching and physical optics programs I am working on will be able to give an accurate analysis of the Nasmyth system, including the effects of mispositioning of the mirrors. What I have done in this report is try to derive some analytical expressions analagous to those in Padman's report so that you can get a rough idea of the sizes and tolerances required. I have made use of the equivalent paraboloid concept, which is known to be very accurate on axis but becomes questionable off-axis. For the very small deviations (10-20 microns) which you are anticipating, the formulas below should be adequate. If you attempt to do beam-switching with the Nasmyth, however, motions on the order of millimeters will be required. The formulas for loss, etc. below are probably still accurate in the beam switching case but I would want to use the programs (or some other calculation not based on equivalent paraboloids) for verification.

I.A. Flat mirrors

I.A.1 Translation Translations of the Nasmyth can be broken down into two components: (1) motion parallel to the Nasmyth mirror surface, and (2) motion parallel to the normal to the surface.

Component 1 should have very little effect, until the motion is large enough to cause significant spillover and edge diffraction. Therefore, the tolerance for this type of motion is on the order of 2-3 centimeters.

Component 2 is equivalent to a lateral motion of the feed in a direct Cassegrain system, by an amount 1.414 times the Nasmyth motion. This type of motion will result in a beam shift and a loss in boresight gain. An estimate of both effects can be calculated from the equivalent paraboloid, i.e. one with a focal length of Mf. The equivalence between Nasmyth motions and motions in the Cassegrain is exact, but the equivalence between the

Cassegrain and the equivalent paraboloid is only an approximation. Based on the information in reference [2] the approximation is fairly good for small displacements.

<u>Beam shift</u> The beam shift can be found from direct subsititution into equation (18) or (22) in Padman's report and should be about

$$\theta_b = \tan^{-1}(1.414\Delta / Mf)$$

where Δ is the motion in direction (2) and Mf is the focal length of the effective paraboloid. The "baseline design" in Padman's report has Mf = 60 m, but in system with a flat Nasmyth Mf would probably be higher, something like Mf = 120 m.

<u>Gain loss</u> The loss cannot be calculated directly by substituting Mf for f and 1.414 Δ for Δ in equation (20) in the report. The loss according to the formula in the report would be about

$$L = \frac{4\pi^2}{18\,(4\,Mf/D)^6} \times \text{BPLLD} \times (1.414\,\Delta/\lambda)^2$$

where BPLLD is given in Padman's report as about 0.4. For even a fairly pessimistic value of Mf/D = 10 (this would be a very short Nasmyth arm), several centimeters of motion give negligible loss; to get 1% loss requires a motion of thousands of wavelengths. This formula assumes a system limited by coma, while a system with a very large f/D ratio such as the equivalent paraboloid may be limited by astigmatism. In the worst case limit of uniform illumination I get an astigmatism loss of

$$L = \frac{1}{96}\,\frac{\pi^2}{(Mf/D)^6}\left(\frac{(1.414\,\Delta)^2}{D\,\lambda}\right)^2$$

The motion required to produce a 1% loss is on the order of hundreds of wavelengths. The final phasing type error is defocussing, which in the case where one mirror moves with respect to the other goes as

$$L = \frac{4\pi^2}{3\,(4\,Mf/D)^4}\left(\frac{1.414\,\Delta}{\lambda}\right)^2$$

The motion for a 1% loss here is 31 wavelengths for a short Mf/D (= 10), which works out to 11 mm at 800 GHz. All three of the above formulas were derived for small deviations (several wavelengths or so), so they are not completely valid when the losses are as high as 1%. Also, when the three effects are present simultaneously the total effect will be greater than the sum of the three effects taken individually. However, the formulas show that the defocussing, astigmatism, and coma losses for motions up to a centimeter are negligible.

Another source of loss in mispositioning is increased spillover around the Cassegrain subreflector. For a Gaussian taper this works out to:

$$L = \frac{10^{-T/10}}{1 - 10^{-T/10}}\,0.46\,T\,(0.46\,T - 1)\,\frac{(1.414\,\Delta)^2}{D_s^2}$$

Where D_s is the diameter of the Cassegrain subreflector and T is the edge taper in dB. For an 0.5 m subreflector and a 10 dB edge taper the 1% loss tolerance is still 2.6 centimeters.

The conclusion to all the above is that defocussing and spillover are the two limiting factors for the positioning tolerance, but that in both cases the anticipated subreflector positioning errors are such that even these factors are negligible.

Phase shift Translation of one of the two Nasmyth mirrors independent of the other one will cause a phase change of $2.818\pi\Delta/\lambda$ radians (27 degrees at 800 GHz for 20 microns shift) uniformly across the aperture in addition to the effects mentioned above. This does not make any difference in a single dish telescope, but if it occured rapidly (i.e. a vibration) it might cause a problem in an interferometer. Translation of the two mirrors together would not create this phase change.

I.A.2 Rotations Rotations of a flat Nasmyth mirror about a vertical axis (perpendicular to the Nasmyth axis and the telescope axis) and a horizontal axis (lying in the Nasmyth surface and in a plane with the Nasmyth and telescope axes) are equivalent to a lateral shift and rotation of the feed in a Cassegrain. The effective feed shift is twice the radian angle times the distance from the Nasmyth to the receiver, while the effective feed rotation is twice the rotation of the mirror.

Beam Shift The effective feed shift will cause a beam shift and losses as described in the translation section above; again, the primary effect should be a beam shift. The amount of beam shift will be

$$\theta_b = \tan^{-1}(2\,\phi\, D_{NR} / Mf)$$

where ϕ is the rotation of the mirror and D_{NR} is the distance from the Nasmyth mirror to the receiver. (This formula applies to either of the two mirrors.) For 4 arcseconds, Mf = 120 meters, and D_{NR} = 10 meters, the beam shift is 0.66 arcseconds. A 4 arcsecond mirror rotation is equivalent to 194 microns of motion of the feed if D_{NR} =10 meters, so that the rotation has a stronger effect than the 10-20 micron translation.

Gain Loss In the case of mirror rotation, the coma and astigmatism will be small as in the translation case. The defocussing caused by mirror rotation is much smaller than that caused by mirror translation:

$$L = \frac{1}{128}\frac{\pi^2}{(Mf/D)^6}\left(\frac{(1.414\,\Delta)^2}{D\,\lambda}\right)^2$$

Therefore in the rotation case the increase in spillover will be the limiting factor. For the expected subreflector motions even these losses will be very small. The spot on the subreflector moves by the subreflector-Nasmyth distance times twice the mirror rotation, with the result

$$L = \frac{10^{-T/10}}{1-10^{-T/10}}\,0.46\,T\,(0.46\,T-1)\,\frac{(2\,\phi\,D_{SN})^2}{D_s^2}$$

where D_{SN} is the subreflector to Nasmyth distance and D_s is the subreflector diameter. For 4 arcseconds rotation the loss is negligible; to get 1% loss requires a rotation of 16 arcminutes assuming D_{SN} = 4 meters, D_s = 0.5 meters, and T = 10 dB.

I.B. Curved Mirrors

Curved Nasmyth mirrors will be much less forgiving of positioning errors than flat ones, as can be seen from the $(f/D)^6$ and $(f/D)^4$ terms in the denominators of the coma, astigmatism, and defocussing error expressions given above. The equivalent paraboloid concept is not directly applicable in this case but it should still give good "order-of-magnitude" results. The defocussing tolerance starts to dominate the spillover tolerance as the mirror curvature increases.

However, motions on the order of 10-20 microns and 1-4 arcseconds should still be acceptable. Even a fairly deeply curved Nasmyth mirror would have an equivalent Mf/D > 2. For the coma loss expression above, the 1% tolerance would be 38 wavelengths; the astigmatism 1% tolerance would be 15 mm at 800 GHz; the defocussing 1% tolerance would be 1.25 wavelengths (470 microns); while the spillover tolerance would remain at the centimeter level.

II. OTHER LOSSES ASSOCIATED WITH NASMYTH MIRROR(S)

II.A. Nasmyth Surface Tolerance

Surface roughness on the reflectors has two effects: the first, caused by slow deviations with a transverse dimensions comparable to the wavelength, creates phase errors; the second, caused by deviations with transverse dimensions comparable to the skin depth, leads to increased ohmic loss. The second effect will be mentioned later in the secton on ohmic losses.

The first effect is the usual Ruze phase-error loss. An error normal to the surface causes 0.707 times as much phase shift as it would in the central area of the primary, because of the 45 degree incidence angle. If random errors in the Nasmyth and the primary are uncorrelated, they will add at "right angles":

$$\delta_{\text{effective}} = \sqrt{\delta^2_{\text{primary}} + \delta^2_{\text{Nasmyth}} + \delta^2_{\text{Coude}} + \text{etc}}$$

where δ represents phase errors, so

$$\varepsilon_{\text{effective}} = \sqrt{\varepsilon^2_{\text{primary}} + \frac{1}{2}\varepsilon^2_{\text{Nasmyth}} + \frac{1}{2}\varepsilon^2_{\text{Coude}} + \text{etc}}$$

where ε represents surface normal errors. If the r.m.s. surface normal error in a Nasmyth is 10% of the r.m.s. in the main dish then the overall r.m.s. will only go up by 0.25%. If the Nasmyth r.m.s. is 50% of the primary r.m.s. then the overall r.m.s. will go up by 6%.

At 800 GHz, with a 15 um primary, the Ruze loss with a perfect secondary is 22.3%. The losses with Nasmyth and Coude mirrors can be substantially higher unless they are much better than the primary. The following table shows the total Ruze loss for a 15 um primary at 800 GHz with various accuracies of Nasmyth and Coude mirrors.

Mirror r.m.s.	Nasmyth	Nasmyth + Coude
Perfect	22.3%	22.3%
1 um	22.4%	22.4%
5 um	23.4%	24.5%
10 um	26.6%	30.5%
15 um	31.5%	39.7%

II.B. Edge Diffraction Losses and Spillover

The losses due to diffraction and spillover at the edges of the Nasmyth mirrors should be negligible if they are made large enough (mirror diameter >4 spot sizes). There may still be diffracted power due to feed sidelobes hitting the edges, but this is power which would not have made it to the aperture anyway.

II.C. Depolarization

Coude vs. Nasmyth The suggestion in your letter that a Coude will have better polarization than a Nasmyth most likely refers to a comparison between (a) a single curved Nasmyth and (b) a curved Nasmyth plus a second curved mirror to form a Coude. In the second case the system could be designed so that the cross-polariztions cancel, at least partially, as described below.

Flat Nasmyth However, a flat Nasmyth (or a Coude made from flat mirrors) will in principle create *no* cross-polarization. One possible source of slight depolarization would be grooves left over from the machining process. Small grooves (a few microinches), if they have a tendency to lie along one direction, create differential ohmic loss and thus a preference for one polarization. This effect could be at a level of 0.45% maximum, but most likely would be much less, and could be removed by buffing so that the scratches are randomly oriented.

Curved Nasmyth A carfeul calculation of the cross-polarization due to a single curved Nasmyth mirror will require the use of the point matching and physical optics programs for specific geometries. However, I can estimate the loss either by analogy with an offset paraboloid primary or through the results in reference [3], which cover reflections of Gaussian beams from offset ellipsoids and paraboloids.

For a severe (upper limit to loss) case, one could consider an offset ellipsoid configured so that there is a focus centered between the subreflector and the Nasmyth, spaced 2 meters on either side, and then a focus behind the dish 2 meters from the Nasmyth, with a projected Nasmyth diameter of 0.5 meters.

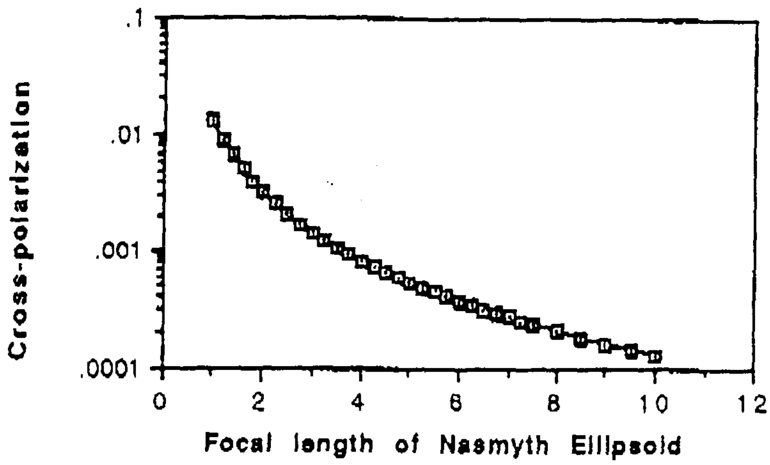
By the first method, the enitre system is equivalent to an f/D = 4 paraboloid with a 90 degree offset angle. From reference [4], figure 8, extrapolated to the 90 degree case, the cross-polar loss will be very roughly 1.5%. (Keep in mind that this is an especially severe case.) By the second method, from reference [3], equation (21), the cross-polarized power is

$$L = \frac{1}{4} \tan^2\theta \left(\frac{W_m}{f}\right)^2$$

where W_m is the spot size on the mirror (0.23 meters), f is the focal length for the ellipsoid (1 meter for the case above), and theta is the angle of incidence (45 degrees). The result by this method is 1.4%.

The cross-polarization level will drop quickly as the focal length is increased. More typical values would be f = 1.5 meters and W_m = 0.15 meters, with a cross-polarization of 0.25%.

Cross-polar losses for Nasmyth mirror with typical spot size



Two Curved Mirrors Under some conditions two curved mirrors can be chosen so that the cross-polarizations cancel. However, in your system, the first Nasmyth would rotate with the elevation of the antenna while the second Nasmyth mirror or the Coude mirror would remain fixed, so that the relative orientation would only be correct at some particular elevation. I believe the best you could do is adjust it for low cross-polarization at some "average" elevation, with only partial cancellation at elevations above or below this.

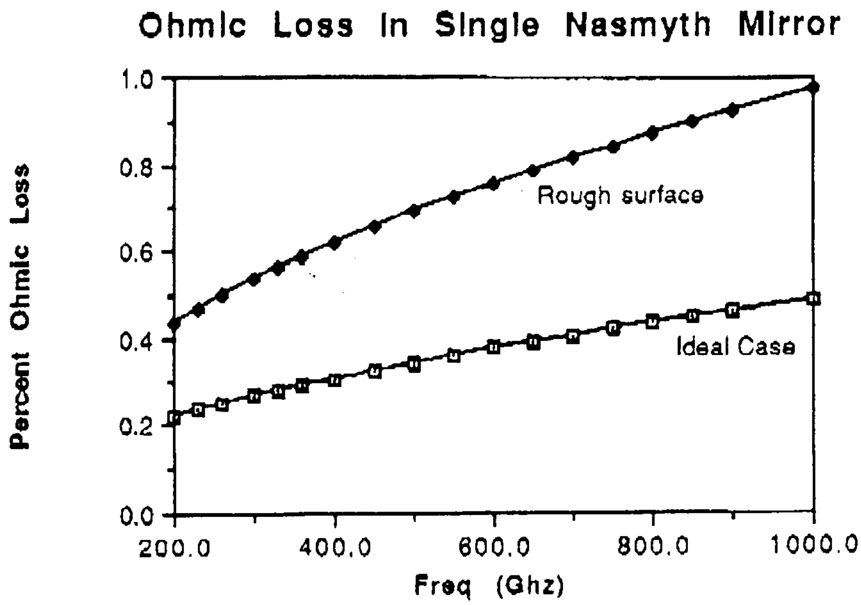
II.D. Illumination Distortion

If curved mirrors are used there will be an asymmetry created in the aperture illumination, which will lead to a gain reduction and beam asymmetry. As with the cross-polarization this can be compensated if two mirrors are used but only at one elevation.

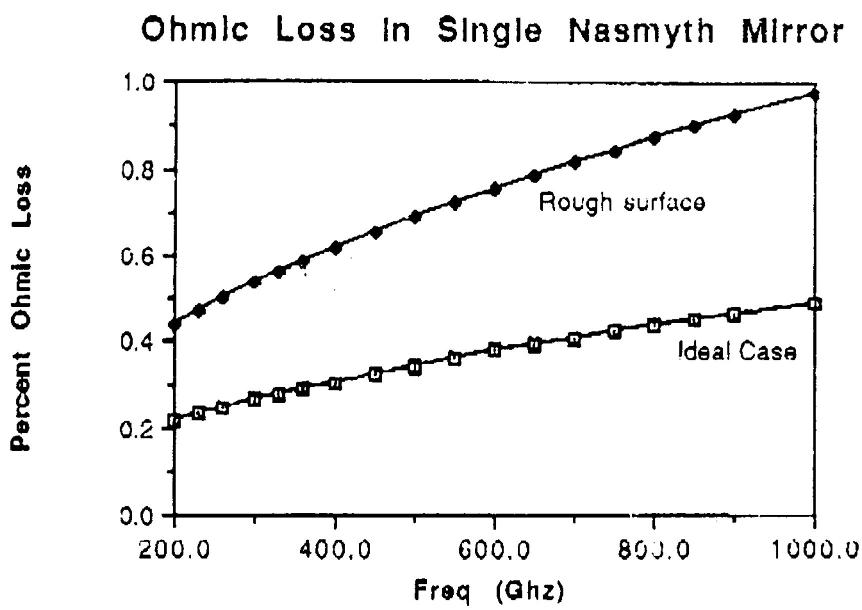
From formulas in reference [3] the loss for a deeply curved (f/D =1) Nasmyth would be about 0.5%, and a more typical value would be 0.12%. Flat mirrors would not create any distortion.

II.E. Ohmic Losses

The ohmic loss in a single reflection from an aluminum surface is given in the graph below. Microscopic surface roughness increases the ohmic loss above that for a perfectly smooth surface (see ref. [1]). (By microscopic I mean small compared to wavelength but large compared to skin depth.) A good estimate for the relative increase is the ratio of the path following the actual surface to the path along the ideal smooth surface. For a uniform covering with microscopic scratches with a square profile, this gives a 100% increase in loss, so that the true ohmic loss at 800 GHz should be about 0.9% per reflection. To reduce the loss to the ideal case would require polishing to a small fraction of the skin depth (3 microinches).



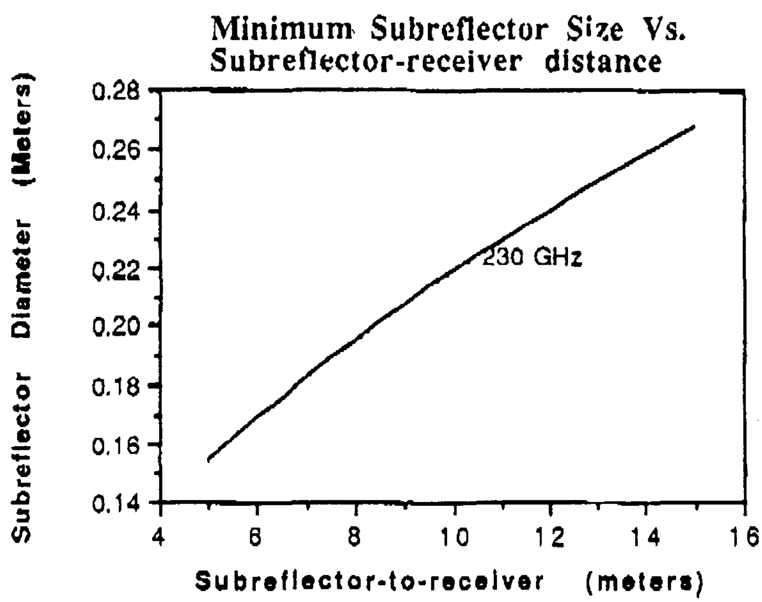
Ohmic Loss in Single Nasmyth Mirror
Percent Ohmic Loss
1.0
0.8
0.6
0.4
0.2
0.0
200.0
400.0
600.0
800.0
1000.0
Freq (Ghz)
Rough surface
Ideal Case


Ohmic Loss in Single Nasmyth Mirror
Percent Ohmic Loss
Freq (Ghz)
Rough surface
Ideal Case
1.0
0.8
0.6
0.4
0.2
0.0
200.0
400.0
600.0
800.0
1000.0

III. LIMITS ON USING FLAT NASMYTH MIRRORS

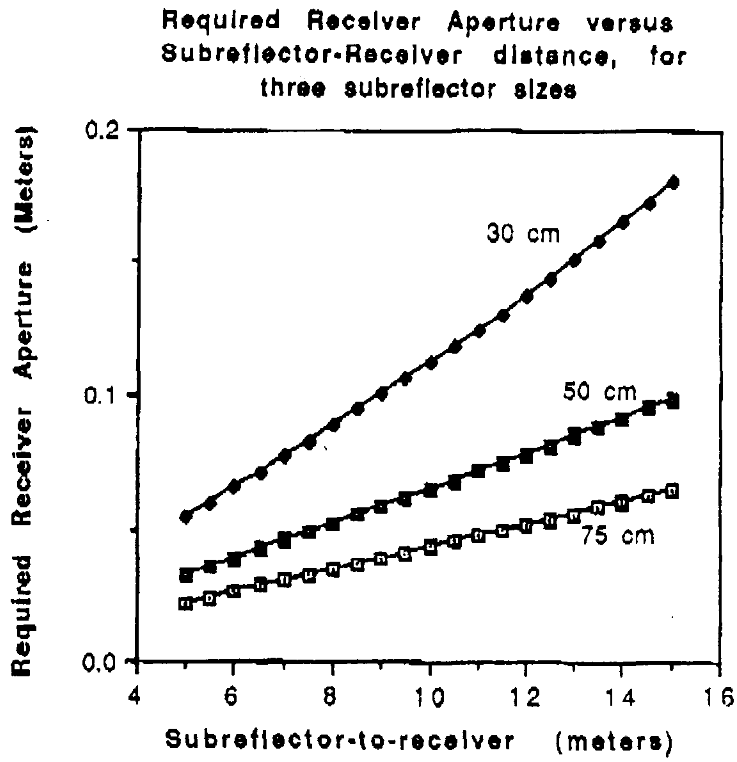
The sizes of the subreflector, the Nasmyth mirrors and the receiver (effective feed) aperture interact. The following material is based on simple zero-order Gaussian mode calculations.

Absolute Minimum Subreflector Diameter for Flat Nasmyth and Waist at Receiver
The following figure shows the *minimum* Cassegrain/Gregorian subreflector size versus the distance from the subreflector to the receiver, assuming that you want a beam waist in the vicinity of the receiver and a flat Nasmyth mirror(s), and taking the minimum frequency to be 230 GHz. (If you go to lower frequencies the size must increase as the square root of wavelength.) I assumed a subreflector 2.4 times the minimum obtainable spot size. With a subreflector as small as shown in this figure, the receiver aperture would have to be 0.707 times the subreflector size, which works out to be unreasonably large. Therefore, the subreflector will need to substantially larger (possibly x 2) than what is shown in the graph.



The subreflector 0.46 meter diameter subreflector used for the "baseline" calculation in Padman's report shoould be adequate for a flat-Nasmyth system, although you may want to increase the size further to reduce the required receiver aperture, as discussed in the next section. For a long Coude made entirely of flat mirrors a larger subreflector will probably be necessary.

Required Receiver Aperture for Flat Nasmyth/Coude System If the distance from the subreflector to the receiver is made larger or the diameter of the subreflector decreases, the necessary receiver aperture becomes larger. The following figure shows the required aperture at 230 GHz for three different diameters of subreflector, assuming a flat Nasmyth mirror. I again assumed that the subreflector diameter would be about 2.4 waist radii. The recevier aperture (diameter) was taken to be 3.25 waist radii.



It should be noted that the "required receiver aperture" does not necessarily mean that the grids have to have a clear aperture as large as shown. Additional optics could be used at the receiver to cut down the beam size. An offset pair could be designed either for low cross-polarization or low illumination distortion, but probably not both.

References

[1] S.P.Morgan, "Effect of Surface Roughness on Eddy Current Losses at Microwave Frequencies," J. Appl. Physics, vol. 20, Apr. 1949, pp.352-362.
[2] W.C.Wong, "On the Equivalent Parabola Technique to Predict the Performance Characteristics of a Cassegrainian System with an Offset Feed," IEEE Trans. Antennas and Propagat., vol. AP-21, May 1973, pp.335-339.
[3] J.A.Murphy, "Distortion of a Simple Gaussian Beam on Reflection from Off-Axis Ellipsoidal Mirrors," Int. J. Infrared and Millimeter Waves, vol.8, 1987, pp.1165-1187.
[4] J.Dijk et al, "The Polarization Losses of Offset Paraboloidal Antennas," IEEE Trans. Antennas Propagat., vol.AP-22, July 1974, pp.513-520.