Submillimeter Array Technical Memorandum

Number: 25
Date: August 21, 1990
From: Colin Masson

Phase Errors due to Non-Intersecting Axes

Summary

I describe briefly the method for calculation of phase error due to non-intersection of the antenna axes. An offset of more than about 0.25 m of metal structure will require a *temperature-dependent* correction term to correct the path length to better than 15 µm. It may not be possible to make adequate corrections for offsets of more than about 0.5m since it may not be possible to determine the temperature with sufficient accuracy (< 1K). There should be no difficulty in calibrating the phase errors produced by small (< 0.01m) offsets produced by construction tolerances. I conclude that *we should avoid designs with non-intersecting axes*, unless they have other dramatic advantages.

Calculation of excess phase path in an antenna

The effect of non-intersecting axes has been discussed by Wade (1970) and by Thompson, Moran and Swenson (1986). The diagram in Figure 1 shows the optical path through an alt-az antenna pointed at a source in the direction of the unit vector **s**. The diagram is a 2D projection, but in general the paths may lie out of the plane of the page. The vectors **p1** - **p5** represent the optical paths between reflections and the junctions between them are at mirrors. There is assumed to be a mirror on the elevation axis between **p3** and **p4**, and a further mirror between **p4** and **p5**, on the azimuth axis. This mirror on the azimuth axis is at a special location, the *first fixed point* (FFP), the first position in the incoming beam which is fixed in space, not moving as the antenna slews. The first fixed point is the effective location (phase center) of the antenna. If the axes intersected (**p4** = 0), then the FFP would be at the intersection of the axes.

The minimum possible phase path from the FFP to the source would be a straight line in the direction of **s**. (Note that displacements normal to **s** do not affect phase, since the incoming signal is a plane wave .) This minimum possible distance is independent of the orientation of the antenna. In passage through the antenna, however, the actual path is folded by the reflections and has a greater length. The excess length, Δl, is just the difference between these two:

$$\Delta l = \{ |\mathbf{p1}| + |\mathbf{p2}| + |\mathbf{p3}| + |\mathbf{p4}| \} - (\mathbf{p1} + \mathbf{p2} + \mathbf{p3} + \mathbf{p4}) . \mathbf{s}$$

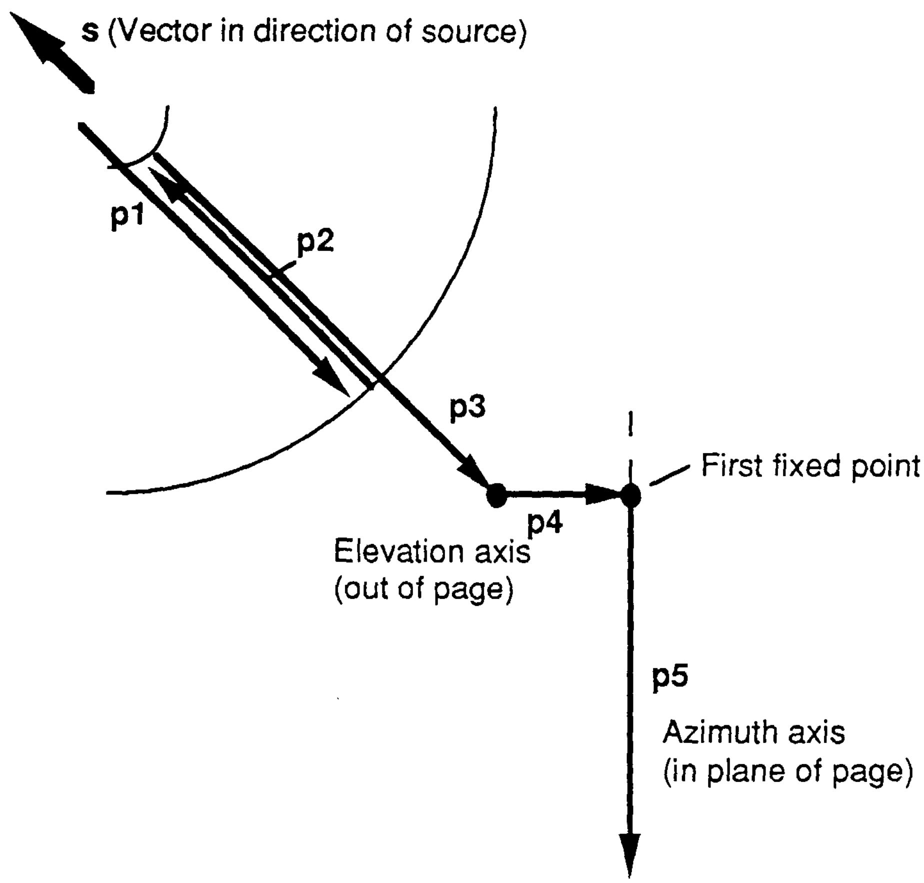

Figure 1. Optical paths in an alt-az antenna with non-intersecting axes.

This equation can be greatly simplified by noting that **p1**, **p2**, and **p3** are always aligned with **s**, since the antenna is always pointed at the source. Thus **p1**.s = |**p1**|, etc., and we have:

$$\Delta l = \{ 2 |\mathbf{p2}| + |\mathbf{p4}| \} - \mathbf{p4} . \mathbf{s}$$

The term in { } is independent of antenna orientation and is therefore just a constant offset which can be removed by calibration, leaving the variable term **p4** . s.

It is instructive to see how the analysis applies to a Cassegrain system where the beam does not physically intersect with the axes. Such a layout is shown in Figure 2. If we ignore the (fixed) path length in the receiver, the excess length can be written as :

$$l = \{ |\mathbf{p1}| + |\mathbf{p2}| + |\mathbf{p3'}| + f |\mathbf{p4'}| \} - (\mathbf{p1} + \mathbf{p2} + \mathbf{p3'} + \mathbf{p4'}) . \mathbf{s} ,$$

where f is a multiplicative factor to account for the extra electrical length of the cable.

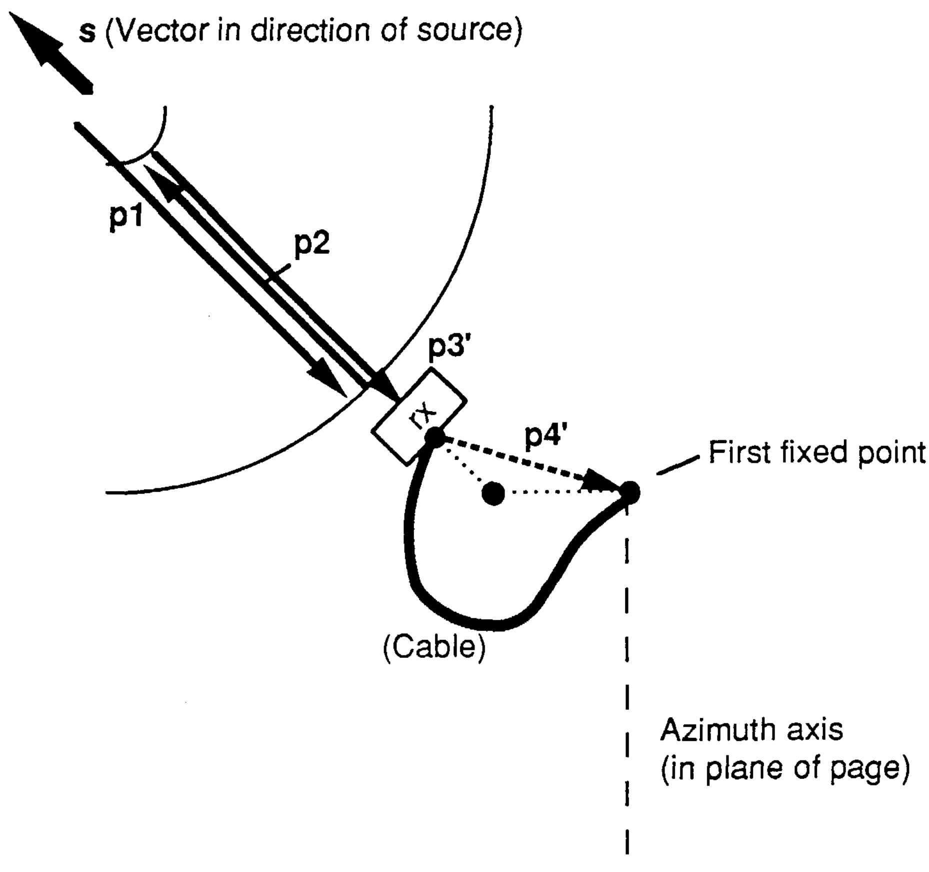

Figure 2. Optical paths in an alt-az antenna with non-intersecting axes and a Cassegrain receiver. **p4'** is the vector for the path which is actually traversed by the cable. The thin dotted lines show that **p4'** can be broken up into one part parallel to **s** and one part equal to the vector **p4** in Figure 1.

This simplifies as before to :

$$l = \{ 2\,|\mathbf{p2}| + f\,|\mathbf{p4'}| \} - \mathbf{p4'} \cdot s \;.$$

The fixed term is different from before, but the variable part is the same, as can be seen by splitting the vector **p4'** into two parts, one identical to the old **p4** and the other one parallel to s, and therefore producing a constant phase. The phase variation due to the non-intersecting axes is the same in the cases sketched in Figure1 and Figure 2. The total path through the antenna is different (and constant) in both cases, but the phase variation arises from the variable degree of folding of the path.

In general any point on the azimuth axis can be chosen as the reference, since all points on the azimuth are fixed. It is usually convenient, however, to choose that point which minimizes the length of **p4**, or the actual place where the beam intersects the axis, if there is one.

The Effect of Calibration.

During operation the telescope is calibrated at regular intervals by moving off to point-like sources which may be as much as 1 radian away from the program source. Phase contributions, such as the terms in { }, which are the same for both source and calibrator, will be removed in this process. Path length changes in the cables will be corrected by a length-measuring system.

Phase contributions which are systematically different between sources are not corrected by such calibration and must be removed by prior knowledge with a mount model for phase, rather like the pointing mount model. In the cases sketched above, we must know **p4**, both in length and in orientation, to calculate the path correction **p4.s** .

The sources of uncertainty in this correction are a) in the length of **p4**, due to thermal expansion and b) in the angle, due to tilting of the mount. If the offset length is subject to thermal expansion of 12 ppm/K (steel), and there is a temperature uncertainty of 5K, then a p-p error of 15 μm will be produced by an axis offset of as little as 0.25 m. If the expansion coefficient is lower than 12 ppm/K, then the offset could be correspondingly larger. The angular uncertainty is less critical: if the mount axis has an error of 1", then a p-p error of 15 μm would be produced by an axis offset of 3m.

Implications for the Design

We require that phase errors from the mount should total less than 20 μm. (This is greater than the surface error specification because it is not doubled by reflection, but it is reduced again to allow for the combination of errors from two antennas.) Any single source of error should be smaller than this, and should be avoided entirely if possible. We will almost certainly need a correction term for axis offset, even the small one due to construction tolerances. However, if the offset is significant (> 0.05 m) then we will be forced to make this correction term temperature dependent. If the offset is larger than 0.5 m, it may be impossible to know the temperature accurately enough to make a proper correction. These numbers assume a steel mount structure and would be less strict for more forgiving materials like invar or cfrp. *Unless there are dramatic gains to be made in other respects, we should avoid any designs which do not have intersecting axes.*

This type of analysis can be expanded in a fairly obvious way to find the effects of misalignments between the radio beam and the mount axes, and offsets between the mirrors and the mount axes. I will do this once we have settled on an optical design, but the general effect is similar. Offsets of the order of 0.01m are tolerable, but they must still be calibrated out in some sort of mount model for phase. It should be possible to align the mount axes to the order of 1mm, and the beam and mirror alignments should have similar errors, so I don't expect any serious problems to arise in operation, but we should all be aware of the need to avoid serious misalignments.

Finally, there is a substantial fixed phase term in the expression for excess length. This is temperature dependent also and must not be allowed to vary significantly during the interval between calibrators (say 20 min). This is not trivial for metal structures. A 10m excess path length in a steel structure should be held stable to 0.2 K to keep errors down to 20 μm. This may not be quite as bad as it seems, since any smooth drifts will be removed by calibration, so it may be sufficient to make the thermal time constant significantly greater than 20 min. The distance from the FFP to the ground must also be maintained stable to the same sort of accuracy

References

Thompson, A. R., Moran, J. M., and Swenson, G. W. 1986, *Interferometry and Synthesis in Radio Astronomy*. Wiley : New York.

Wade, C. M. 1970, *Precise Positions of Radio Sources. I. Radio Measurements*. Ap. J. **162**, 381.

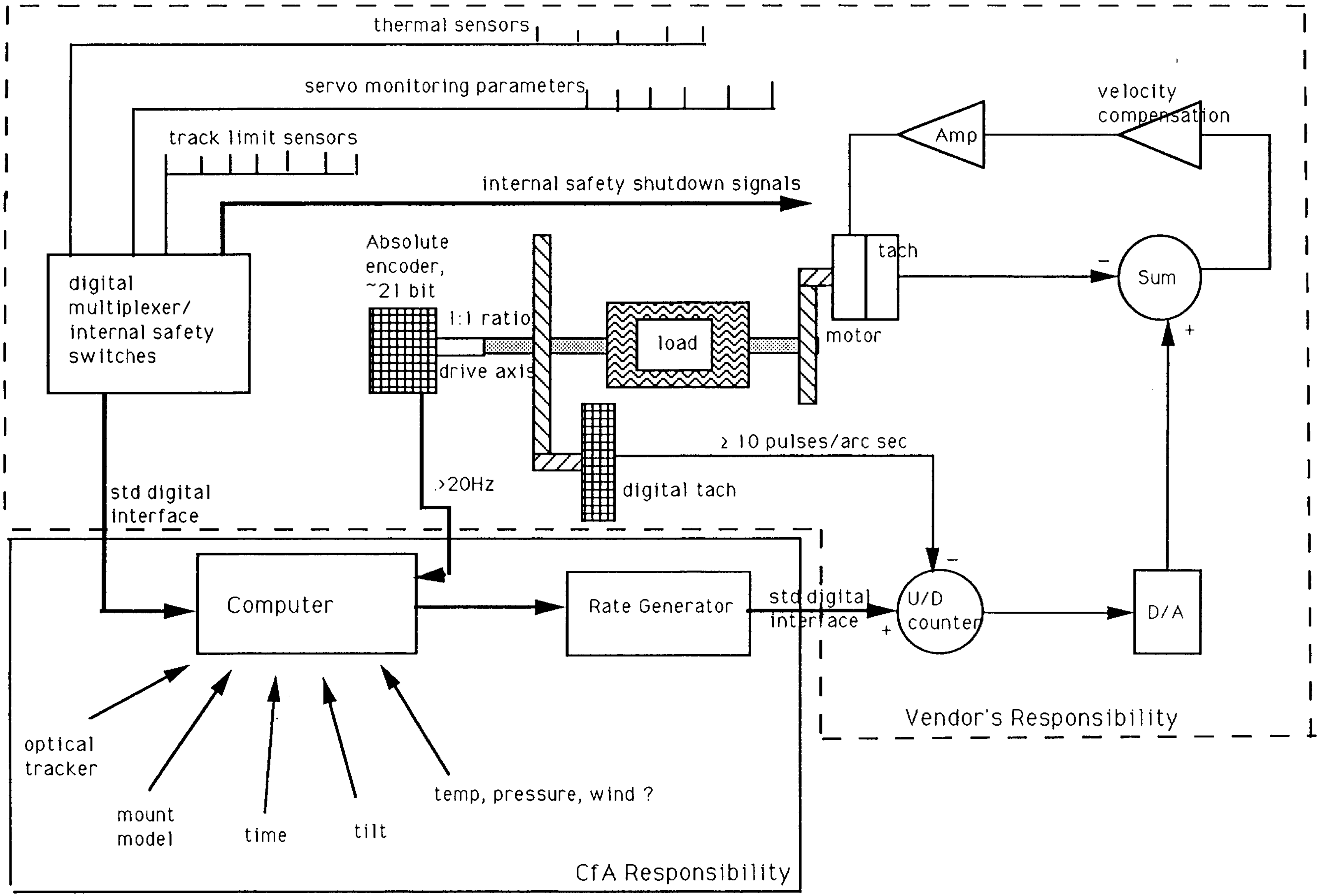

Figure 2: Schematic Representation of a one axis control system