MEMO #27

SAO SUBMILLIMETER ARRAY

TECHNICAL NOTE

(PROOF OF CONCEPT)

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1 6-meter Submillimeter-Wavelength Antenna Structure

The SAO 6-meter Submillimeter Array Antennas are high precision instruments designed to operate in open air. The operating requirements and performance specification are summarized as follows:

Operating Requirements
- operate in open air (ie. subjected to wind)
- transportable on unimproved roads
- precision operation $\leq$ 10 m/s
- degraded operation $\leq$ 25 m/s
- survival $\leq$ 75 m/s

Performance Specification
- surface accuracy $\leq$ 15 μm for precision operation
- pointing accuracy $\leq \pm 1$ arcsec each axis for precision operation
- surface accuracy $\leq$ 35 μm for degraded operation
- pointing accuracy $\leq \pm 2$ arcsec each axis for degraded operation

The high performance specification coupled with the requirement to operate in open air demand a superior antenna structure. Deflection minimization is of primary importance. There are two major sources of loading which cause antenna deflections, the self-weight of antennas and wind. The self-weight of an antenna is static, and it depends on member lengths and member sizes. The wind load on an antenna is dynamic, and it depends on wind speed, and the profile of the structure presented to the wind.

We have control over member sizes and structural configuration, both major factors governing dead load deflection. However, we have no control over wind. Therefore the challenge is to limit the effects of wind to an acceptably low level during telescope operation. There are two approaches to this end. One is to minimize wind loads imposed on the antennas. The other is to make the antennas inherently stiff such that any deflections caused by wind are kept to a minimum.

Wind load is a function of wind speed squared, and wind speed varies directly with height above ground. A low antenna profile can significantly reduce the wind load imposed on an antenna. Installation of wind fences which redirect air flow around an antenna is another method of reducing wind load. However, this alternative is not cost effective; it also complicates the transportation of antennas within an array.

The second approach is to strengthen the antenna structures thereby minimizing the effects of wind. This results in a heavier structure which is advantageous from the structural stability standpoint, but a hindrance from the transportation standpoint.

Obviously, these are the trade-offs and there are many more of them which need to be considered in a design. Rather than starting off fresh with a new design for the 6-meter Submillimeter Antenna, it is worthwhile to study the 12-meter Radio Schmidt Telescope design which we did for the Dominion Radio Astrophysics Observatory (DRAO). It would serve as a starting point even though it is for a much lower precision telescope (ie. 0.6mm rms, 22 arcsec pointing), because it embodies the trade-offs which we have carefully evaluated.

1.1 Structural Performance of the 12-meter Radio Schmidt Telescope

The performance of the 12-meter Radio Schmidt Telescope (RST) is based on operating wind speed of 9m/s as specified by DRAO. Table 1.1-1 and 1.1-2 summarize the surface accuracy and pointing accuracy of the telescope respectively.

Table 1.1-1 rms Surface Accuracy of 12-meter RST

Reflector Elevation	Gravity Alone	Gravity + Max. Wind	Panel Tolerance	Total
0.0°	0.218 mm	0.296 mm	0.500 mm	0.581 mm
45.0°	0.051 mm	0.124 mm	0.500 mm	0.515 mm
90.0°	0.186 mm	0.335 mm	0.500 mm	0.600 mm

Table 1.1-2 Pointing Accuracy of 12-meter RST

Reflector Elevation	Gravity (Systematic)	Max. Wind (Random)
0.0°	5.8 arcsec	3.8 arcsec
45.0°	0.0 arcsec	3.3 arcsec
90.0°	9.8 arcsec	16.8 arcsec

1.2 Dimensional Analysis

We can estimate the surface accuracy of a scaled down Radio Schmidt Telescope (ie. from 12m to 6m) by scaling the governing parameters for deflections accordingly. The simplest approach is to consider the entire reflector as a simply supported beam. For deflection of a simply supported beam, the following relationship holds true:

$$\Delta \propto \frac{P \cdot L^3}{E \cdot I} \qquad [1.2\text{-}1]$$

where

P = Load (N)
L = Beam span (mm)
E = Young's Modulus (Mpa)
I = Moment of inertia of beam section (mm^4)

Since dead load deflection and wind deflection are governed by different parameters, they will be discussed separately. For dead load defections, the self-weight, P_d, of an antenna is a function of material and member sizes:

$$P_d \propto \gamma \cdot \sum_{i=1,2...}^{n} (A_i \cdot L_i) \qquad [1.2\text{-}2]$$

where

γ = Unit weight of material (N/mm^3)
A_i = Area of individual reflector members (mm^2)
L_i = Length of individual reflector members (mm)

The overall moment of inertia, I_{rf}, of the reflector is a function of the depth of the reflector trusses and the area of individual members:

$$I_{rf} \propto \sum_{j=1,2...}^{N} (A_{cj} \cdot d_j^2) \qquad [1.2\text{-}3]$$

where

A_{cj} = Area of a typical reflector top/bottom chord (mm^2)
d_j = Average depth of reflector trusses (mm)

Substituting eqn[1.2-2] and eqn[1.2-3] into eqn[1.2-1]:

$$\Delta_d \propto \frac{\gamma \cdot \sum_i (A_i \cdot L_i) \cdot L^3}{E \cdot \sum_j (A_{cj} \cdot d_j^2)} \qquad [1.2\text{-}4]$$

For wind load defection, the wind load, P_w, imposed on an antenna is a function of the projected area of the reflector:

$$P_w \propto A_{rt} \qquad [1.2\text{-}5]$$

where

A_{rt} = Projected Area of the reflector (mm^2)

The overall moment of inertia, I_{rf}, of the reflector is as discussed earlier (see eqn[1.2-3]). Substituting eqn[1.2-5] and eqn[1.2-3] into eqn[1.2-1]:

$$\Delta_w \propto \frac{A_{rt} \cdot L^3}{E \cdot \sum_j (A_{cj} \cdot d_j^2)} \qquad [1.2\text{-}6]$$

The total surface deflection of the telescope is proportional to:

$$\Delta \propto \Delta_d + \Delta_w \qquad [1.2\text{-}7]$$

By using deflections of the 12-meter RST as the base for comparison, the surface accuracy of the RST for a different size can be estimated by means of scaling. For example, the dead load deflection of a RST of size *k* is given by:

$$(\Delta_d)_k = \frac{r_{\gamma k} \cdot r_{\Sigma(A_i \cdot L_i)_k} \cdot r_{L k}^3}{r_{E k} \cdot r_{\Sigma(A_{cj} \cdot d_j^2)_k}} \cdot (\Delta_d)_{12} \qquad [1.2\text{-}8]$$

where

$$r_{\gamma k} = \gamma_k / \gamma_{12}$$

$$r_{\Sigma(A_i \cdot L_i)_k} = \Sigma(A_i \cdot L_i)_k / \Sigma(A_i \cdot L_i)_{12}$$

$$r_{L_k} = L_k / L_{12}$$

$$r_{E_k} = E_k / E_{12}$$

$$r_{\Sigma(A_{cj} \cdot d_j^2)_k} = \Sigma(A_{cj} \cdot d_j^2)_k / \Sigma(A_{cj} \cdot d_j^2)_{12}$$

Similarly, the wind load deflection of a size *k* RST is given by:

$$(\Delta_w)_k = \frac{r_{A_{rl_k}} \cdot r_{L k}^3}{r_{E k} \cdot r_{\Sigma(A_{cj} \cdot d_j^2)_k}} \cdot (\Delta_w)_{12} \qquad [1.2\text{-}9]$$

where

$$r_{A_{rl_k}} = A_{rl_k} / A_{rl_{12}}$$

$$r_{L_k} = L_k / L_{12}$$

$$r_{E_k} = E_k / E_{12}$$

$$r_{\Sigma(A_{cj} \cdot d_j^2)_k} = \Sigma(A_{cj} \cdot d_j^2)_k / \Sigma(A_{cj} \cdot d_j^2)_{12}$$

The pointing accuracy of a scaled down Radio Schmidt Telescope (ie. from 12m to 6m) can be estimated in a similar fashion. The three supports for the reflector can be treated as one cantilever beam. The rotation at the end of a cantilever beam is proportional to:

$$\beta \propto \frac{P \cdot L^2}{E \cdot I} \qquad [1.2\text{-}10]$$

where

P	=	Load (N)
L	=	Length of cantilever beam (mm)
E	=	Young's Modulus (Mpa)
I	=	Moment of inertia of beam section (mm^4)

The dead load, P_d, and wind load, P_w, are given by eqn[1.2-2] and eqn[1.2-5] respectively. The moment of inertia of the supports, I_{sp}, is a function of the distance between supports and the area of individual members:

$$I_{sp} \propto \sum_{m=1,2\ldots}^{M} \left(A_{sm} \cdot L_{sm}^2\right) \qquad [1.2\text{-}11]$$

where

A_{sm}	=	Area of individual reflector support members (mm^2)
L_{sm}	=	Distance between reflector supports (mm)

By using the pointing error of the 12-meter RST as the base for comparison, the pointing accuracy of a RST of size k can be estimated by the following equations:

$$(\beta_d)_k = \frac{r_{\gamma_k} \cdot r_{\Sigma(A_i \cdot L_i)_k} \cdot r_{L_k}^2}{r_{E_k} \cdot r_{\Sigma\left(A_{sm} \cdot L_{sm}^2\right)_k}} \cdot (\beta_d)_{12} \qquad [1.2\text{-}12]$$

where

$$r_{\gamma_k} = \gamma_k / \gamma_{12}$$

$$r_{\Sigma(A_i \cdot L_i)_k} = \Sigma(A_i \cdot L_i)_k / \Sigma(A_i \cdot L_i)_{12}$$

$$r_{L_k} = L_k / L_{12}$$

$$r_{E_k} = E_k / E_{12}$$

$$r_{\Sigma\left(A_{sm} \cdot L_{sm}^2\right)_k} = \Sigma\left(A_{sm} \cdot L_{sm}^2\right)_k / \Sigma\left(A_{sm} \cdot L_{sm}^2\right)_{12}$$

Similarly, the pointing error of a size k RST is given by:

$$(\beta_w)_k = \frac{r_{Arl_k} \cdot r_{L_k}^2}{r_{E_k} \cdot r_{\Sigma\left(A_{sm} \cdot L_{sm}^2\right)_k}} \cdot (\beta_w)_{12} \qquad [1.2\text{-}13]$$

where

$$r_{A_{rl_k}} = A_{rl_k}/A_{rl_{12}}$$

$$r_{L_k} = L_k/L_{12}$$

$$r_{E_k} = E_k/E_{12}$$

$$r_{\Sigma\left(A_{s_m}\cdot L_{s_m}^2\right)_k} = \Sigma\left(A_{s_m}\cdot L_{s_m}^2\right)_k/\Sigma\left(A_{s_m}\cdot L_{s_m}^2\right)_{12}$$

1.2.1 Fully Scaled 6-meter Radio Schmidt Telescope

The simplest method of scaling the 12-meter RST down to 6-meter size is to use a ratio of 1/2 for scaling lengths, and a ratio of 1 for material properties, and member sizes.

$$r_{\gamma_6} = 1$$

$$r_{\Sigma(A_i\cdot L_i)_6} = 1/2$$

$$r_{L_6} = 1/2$$

$$r_{E_6} = 1$$

$$r_{\Sigma\left(A_{cj}\cdot d_j^2\right)_6} = (1/2)^2$$

$$r_{\Sigma\left(A_{s_m}\cdot L_{s_m}^2\right)_6} = (1/2)^2$$

Substituting all the relevant ratios into eqn[1.2.8]:

$$(\Delta_d)_6 = \frac{1\cdot 1/2\cdot(1/2)^3}{1\cdot(1/2)^2}\cdot(\Delta_d)_{12} \qquad [1.2\text{-}14]$$

Similarly, the ratios for wind load deflection are:

$$r_{A_{rl_6}} = 1/4$$

$$r_{L_6} = 1/2$$

$$r_{E_6} = 1$$

$$r_{\Sigma\left(A_{sj}\cdot d_j^2\right)_6} = (1/2)^2$$

$$r_{\Sigma\left(A_{s_m}/L_{s_m}\right)_6} = (1/2)^2$$

Substituting all the relevant ratios into eqn[1.2.9]:

$$(\Delta_w)_6 = \frac{1/4\cdot(1/2)^3}{1\cdot(1/2)^2}\cdot(\Delta_w)_{12} \qquad [1.2\text{-}15]$$

The total rms reflector surface deflection of a 6-meter RST is obtained by adding together the deflection from eqn[1.2-14] and eqn[1.2-15]:

$$\Delta_6 = \frac{1\cdot 1/2\cdot(1/2)^3}{1\cdot(1/2)^2}\cdot(\Delta_d)_{12} + \frac{1/4\cdot(1/2)^3}{1\cdot(1/2)^2}\cdot(\Delta_w)_{12} \qquad [1.2\text{-}16]$$

Similarly, substituting all the relevant ratios into eqn[1.2-12] and eqn[1.2-13], the total pointing error is given by:

$$\beta_6 = \frac{1\cdot 1/2\cdot(1/2)^2}{1\cdot(1/2)^2}\cdot(\beta_d)_{12} + \frac{1/4\cdot(1/2)^2}{1\cdot(1/2)^2}\cdot(\beta_w)_{12} \qquad [1.2\text{-}19]$$

1.2.1.1 Surface Accuracy

The reflector is supported on a three point support system. Hence, it deflections is asymmetric depending on reflector elevation. It is important to recognize that the reflector surface is adjusted to a perfect paraboloid for a specific reflector elevation under its own weight. Therefore all surface error calculations should reflect this adjustment.

Sources of reflector surface error include: panel fabrication tolerance, gravity and wind deflection. As discussed in the previous section, reflector surface deflection of a 6-meter RST can be estimated by eqn[1.2-18], based on results of the 12-meter RST study.

From the information supplied by panel manufacturers, a rms panel fabrication tolerance of $5\mu m$ can be achieved. Table 1.2.1-1 summarizes the rms surface accuracy of a 6-meter RST. Please note that the design wind speed of 10 m/s specified by SAO is greater than the 9 m/s specified by DRAO. The wind load has been adjusted accordingly (ie. $(10/9)^2$).

Table 1.2.1-1 rms Surface Accuracy of 6-meter RST

Reflector Elevation	Gravity Alone	Gravity + Max. Wind	Panel Tolerance	Total
0.0°	67.3 μm	79.4 μm	5.0 μm	79.6 μm
45.0°	15.8 μm	27.0 μm	5.0 μm	27.5 μm
90.0°	57.4 μm	80.4 μm	5.0 μm	80.6 μm

1.2.1.2 Pointing Accuracy

Antenna mechanical pointing error is the space angle between the direction in space defined by the axis readout and the actual direction of the axis of the reflector. It is caused by deflection of reflector supports. There are two types of pointing error: systematic and random. Sources of systematic pointing errors include: deflections due to gravity loads, mechanical misalignments, and error inherent in the axis angle coupling devices. These errors are usually compensated for during calibration, and are not critical to antenna performance. The other type of pointing error, random pointing error, is significant to antenna performance. The major source of random pointing error comes from wind gusts.

Pointing error can be estimated by eqn[1.2-19], based on results of the 12-meter RST study. Table 1.2.1-2 summarizes the pointing accuracy of a 6-meter RST. Please note that the design wind speed of 10 m/s specified by SAO is greater than the 9 m/s specified by DRAO. The wind load has been adjusted accordingly (ie. $(10/9)^2$).

Table 1.2.1-2 Pointing Accuracy of 12-meter RST

Reflector Elevation	Gravity (Systematic)	Max. Wind (Random)
0.0°	2.9 arcsec	1.0 arcsec
45.0°	0.0 arcsec	0.8 arcsec
90.0°	4.9 arcsec	4.2 arcsec

1.2.2 Partially Scaled 6-meter Radio Schmidt Telescope

The results from section 1.2.1 show that the surface accuracy of a fully scaled 6-meter RST falls short of the SAO specification. Referring to Table 1.2.1-1, it is clear that the majority of surface error comes from dead load deflection. For dead load deflection, the truss depth, d_j, is the most significant parameter because it is varies quadratically. If we make the truss depth ratio equal to 1, and maintain all other ratios the same as before:

$$r_{\gamma_6} = 1$$

$$r_{\Sigma(A_i \cdot L_i)_6} = 1/2$$

$$r_{L_6} = 1/2$$

$$r_{E_6} = 1$$

$$r_{\Sigma\left(A_{cj} \cdot d_j^2\right)_6} = (1)^2$$

$$r_{\Sigma\left(A_{c_m} \cdot L_{c_m}^2\right)_6} = (1/2)^2$$

Substituting all the relevant ratios into eqn[1.2.8]:

$$(\Delta_d)_6 = \frac{1 \cdot 1/2 \cdot (1/2)^3}{1 \cdot (1)^2} \cdot (\Delta_d)_{12} \qquad [1.2\text{-}20]$$

Similarly, the ratios for wind load deflection are:

$$r_{A_{rl}6} = 1/4$$

$$r_{L6} = 1/2$$

$$r_{E6} = 1$$

$$r_{\Sigma(A_{cj} \cdot d_j^2)_6} = (1)^2$$

$$r_{\Sigma\left(A_{s_m} \cdot L_{s_m}^2\right)_6} = (1/2)^2$$

Substituting all the relevant ratios into eqn[1.2.9]:

$$(\Delta_w)_6 = \frac{1/4 \cdot (1/2)^3}{1 \cdot (1)^2} \cdot (\Delta_w)_{12} \qquad [1.2\text{-}21]$$

The total rms reflector surface deflection of a 6-meter RST is obtained by adding together the deflections from eqn[1.2-20] and eqn[1.2-21]:

$$\Delta_6 = \frac{1 \cdot 1/2 \cdot (1/2)^3}{1 \cdot (1)^2} \cdot (\Delta_d)_{12} + \frac{1/4 \cdot (1/2)^3}{1 \cdot (1)^2} \cdot (\Delta_w)_{12} \qquad [1.2\text{-}22]$$

Assuming a rms panel fabrication tolerance of $5\mu m$ as in the previous section, Table 1.2.1-3 summarizes the rms surface accuracy of a partially scaled 6-meter RST. Please note that the design wind speed of 10 m/s specified by SAO is greater than the 9 m/s specified by DRAO. The wind load has been adjusted accordingly (ie. $(10/9)^2$).

Table 1.2.1-3 rms Surface Accuracy of 6-meter RST

Reflector Elevation	Gravity Alone	Gravity + Max. Wind	Panel Tolerance	Total
0.0°	16.8 μm	20.0 μm	5.0 μm	20.6 μm
45.0°	4.0 μm	6.8 μm	5.0 μm	8.4 μm
90.0°	14.4 μm	20.1 μm	5.0 μm	20.7 μm

2 Structural Analysis: Self-Weight

The foregoing discussion requires verification. For this purpose, the 6-meter RST under its own weight was analyzed using a commercial FEM analysis package, ANSYS-PC/LINEAR ver.4.3A4 by the Swanson Analysis Systems Inc., Houston, USA.

2.1 Fully Scaled 6-meter Radio Schmidt Telescope

Table 2-1 compares the RST performance based on results from structural analyses to the estimated performance.

Table 2-1 Verification of RST Performance

Reflector Elevation	ANSYS rms Error (Gravity)	ANSYS Max Pointing (Systm.)	Estm. rms Error (Gravity)	Estm. Max Pointing (Systm.)
0.0°	52.9 μm	2.4 arcsec	67.3 μm	2.9 arcsec
45.0°	12.5 μm	0.0 arcsec	15.8 μm	0.0 arcsec
90.0°	44.9 μm	5.1 arcsec	57.4 μm	4.9 arcsec

2.2 Partially Scaled 6-meter Radio Schmidt Telescope

Table 2-2 compares the rms surface error calculated from deflection of the 6-meter model to the estimated surface error.

Table 2-2 Verification of RST Performance

Reflector Elevation	ANSYS rms Error (Gravity)	Estm. rms Error (Gravity)
0.0°	20.5 μm	16.8 μm
45.0°	4.8 μm	4.0 μm
90.0°	17.5 μm	14.4 μm