To:
Bill Bruckman
Colin Masson
Harvard-Smithsonian Center for Astrophysics

MEMO #34

From:
Heinrich Foltz
University of Texas, ECE Dept.

Date:
5 October 1990

Subject:
REVISED REQUIREMENTS FOR BENT NASMYTH OPTICS
I. Outline of Optical Requirements
II. Sample Designs
III. Estimated Loss Budget

This document is a revised version of the one dated 6 Septembet 1990, "Requirements for Coude or Nasmyth Optics," with changes based on the drawings of a bent Nasmyth I received on September 27.

I. OUTLINE OF OPTICAL REQUIREMENTS

(1) The system should deliver an image of the aperture to the receiver, as opposed to an image of the sky. The axial position and size of the image should be frequency independent.
(2) The sizes of the components should be reasonably small:
  (a) Subreflector
  (b) Receiver optics and feed horns
  (c) Nasmyth mirrors
(3) The system should provide for beam-switching at a speed of 10 Hz or greater.
  (a) Ideally, the beam switching should have equal spillover and path length between the switched positions.
  (b) The beam switching should have minimum impact on the normal no-switching mode of operation.
(4) The system should provide for chopping to both room temperature and cold loads.
(5) The number of reflections, and especially the number of curved mirrors, should be kept low to minimize ohmic losses, asymmetry, and alignment difficulties.

(1) Aperture Image Feed

As described in Padman's report, with some types of feed horns it is advantageous for the feed to be located at an image of the aperture rather than at an image of the sky. The design of the receiver optics, diplexers, etc. is simplified if the location and size of this image is independent of frequency.

Number of Mirrors Required At least one curved mirror is required for frequency-independent matching of one Gaussian spot with fixed size and radius of curvature to another. If only one curved mirror is used, then both its focal length and position relative to the input and output images are forced by the desired image sizes. If two curved mirrors are used, then there are more free parameters and the positions can be specified arbitrarily while frequency independence is obtained through selection of the two focal lengths.

In the SMA case, the mirror positions are fixed by the mechanical configuration of the telescope mounting, so most likely two curved mirrors will be required.

(2) Component Sizing

(2a) Subreflector

Aperture blockage causes two separate but more or less equal loss mechanisms to occur. First, the feed sees the blocked area, so that a portion of the feed pattern is not coupled to the antenna aperture (the aperture is defined here as a plane in front of the prime focus). Second, the remaining aperture has a hole in its illumination, which leads to a decrease in peak gain and main beam efficiency. Assuming that the unblocked system would have a Gaussian aperture illumination, the relative loss in the beam efficiency and boresight gain due to the subreflector is

$$L = 1-\left(1-\frac{1-10^{\left(\frac{-T}{10}\left(D_s^2/D_p^2\right)\right)}}{1-10^{\frac{-T}{10}}}\right)^2$$

where T is the edge taper in dB, D_s is the diameter of the subreflector, and D_p is the diameter of the primary. The second term inside the parentheses is the integral over the projected area of the subreflector, weighted by the illumination, relative to the weighted integral over the whole aperture. The squaring of the whole quantity inside the parentheses is due to the two loss effects described above.

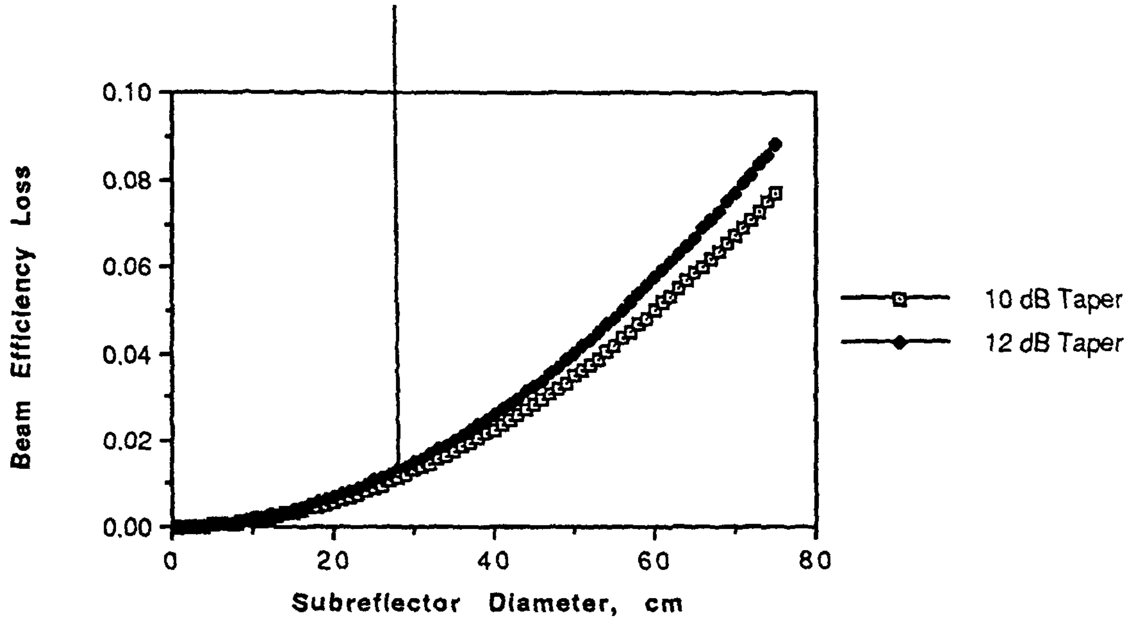


Figure 1
Loss in beam efficiency due to subreflector blockage, assuming gaussian aperture illumination. Blockage due to the support spars is not included.

Figure 1 above shows the loss in beam efficiency and peak gain due to subreflector blockage, versus subreflector diameter, for a 6-meter primary. The real SMA would not have exactly Gaussian illumination but the graph should give a fairly close idea of what the blockage losses would be. The numbers are about twice as bad as the "3 x area" figure I mentioned to Colin on my visit to Boston because I was taking into account only one of the two separate effects. It should also be noted that there could be an increase in system noise temperature because of the portion of the feed pattern seeing the blocked area (equal to roughly one half the loss given in the graph). Presumably the blocked region would be terminated with absorber to reduce the VSWR; therefore, the effect on noise temperature would be that of an ambient temperature attenuator.

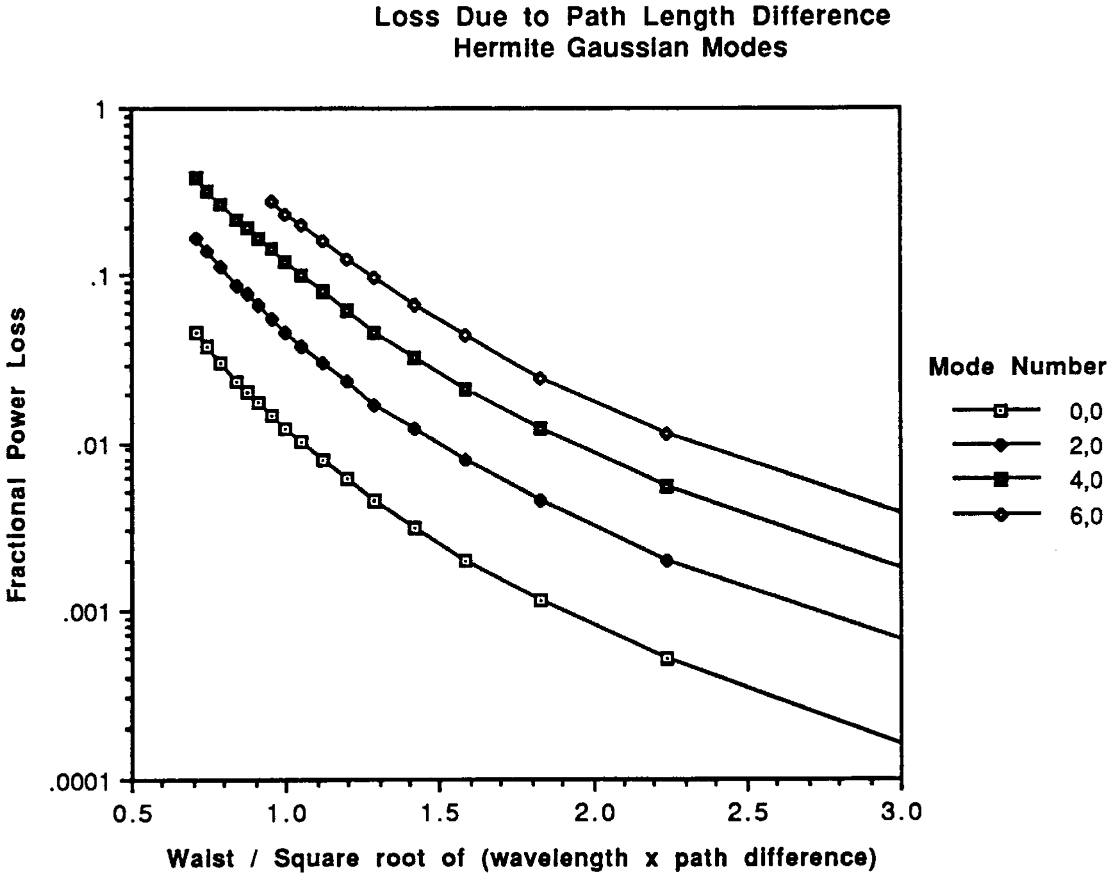
(2b) Receiver Optics and Feeds

Ray has told me that he does not want to go to a receiver clear aperture greater than 5 cm; this sets an upper limit to the size of aperture image at the receiver.

If the receivers use any form of interferometer for diplexing or SSB filtering, the difference in spot sizes in the two paths sets a lower limit to the waist size that can be used. For a single 0,0 Gaussian mode the transmission is given by:

$$T = \frac{1}{4}\left[1 + \left(1 + \left(\frac{\lambda\,\Delta}{2\,\pi\,w_0^2}\right)^2\right)^{-1/2}\right]^2$$

where Δ is the path length difference and w_o is the waist size. The reduction in transmission is due to the mismatch in spot sizes in the two interfering beams. For a 4 GHz IF an LO diplexer requires Δ = 3.75 cm; so at 230 GHz the waist should be larger than 7.2 mm for 1% or less loss. The required waist size is even larger when higher order modes are involved, as shown in the graph (Figure 2) below.



On the other hand, the mode content at the higher orders is much less. The following table, based on information given in "Gaussian Mode Analysis of Cassegrain

Antenna Efficiency," by Padman, Murphy, and Hills, gives the typical mode content for a wideband scalar horn:

Laguerre Index	Percent of Power
0	98.06%
1	0.00%
2	1.44%
3	0.19%

Laguerre mode (2) is equivalent to a summation of Hermite (0,0), (2,0), (0,2), (2,2), (4,0) and (0,4) modes, Laguerre mode 3,0 contains components up to (6,0) and (0,6). If the waist size is chosen to be 9 mm, the path difference losses should be about 0.5% total.

As described in more detail in section (2c), the presence of higher order modes also increases the necessary ratio of clear aperture to spot size. Therefore, if a Martin-Puplett or similar arrangement is used at 230 GHz, the entire 5 cm clear aperture would be needed unless an additional pair of mirrors is inserted between the receiver and the diplexers.

(2c) Nasmyth Mirrors

The minimum size of the Nasmyth mirrors is determined by two requirements: (1) Essentially all of the beam coming from the receiver should get through the optics and out of the Cassegrain hole so that the spillover is onto the sky rather than into the backup structure of the telescope, and (2) the aperture fields should be able to propagate through the optics to form a good image at the receiver.

At points in the optics where there are images of the aperture, the fields will be fairly sharply limited and thus any aperture stops can be sized even smaller than would be appropriate for a single 0,0 Gaussian mode. At other points, however, the higher-order modes have slipped in phase so that the fields have a wider extent, and the stops must be sized much larger than would be necessary for a single mode. Figure 3 (next page) shows the effects of truncation of the higher order modes by an aperture stop.

The higher order mode content for the image of an unblocked aperture is

Laguerre Index	Power
0	81.45%
1	0.00%
2	5.35%
3	3.39%
4	0.39%
all higher modes	9.42%

There is a large component at indices of 5 and higher, but the aperture efficiency increase due to feeding at an image of the aperture is due mainly to the effects of the '2' and '3' modes. Mirrors with a diameter 5.5 times the spot radius should be sufficient to keep the loss in the '2' and '3' modes down to 0.5% of the total.

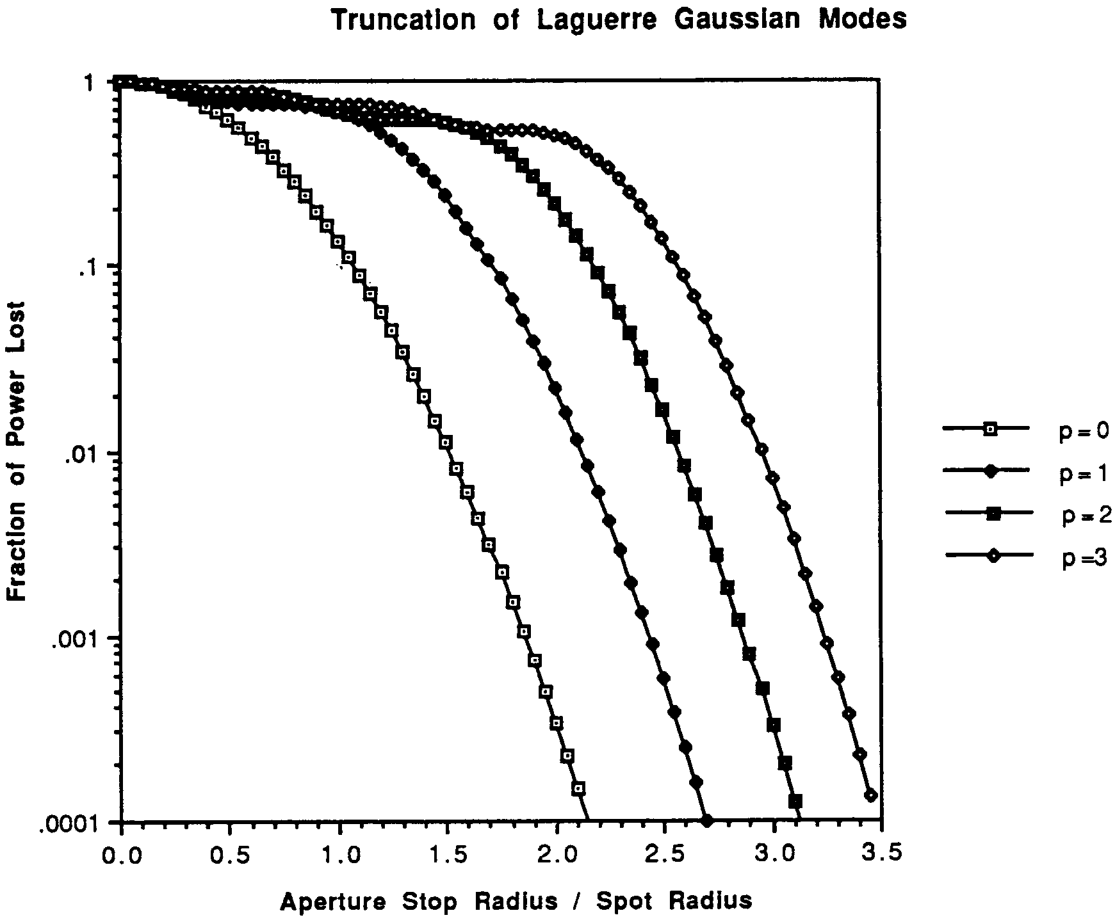


Figure 3
Loss in truncation of Laguerre-Gaussian modes by a centered round aperture stop. The curve p = 0 is for the normal zeroth order Gaussian beam. The curves p = 1 and p = 2 are for beams with an $L_1(2r^2/w^2)$ and $L_2(2r^2/w^2)$ radial variation, respectively.

(3) Beam Switching

Since "no switching" interferometry is to be the main mode of operation, chopping arrangements in which the unswitched beam is not on-axis with maximum aperture efficiency are not up for consideration. Therefore the switching would likely be either (a) two-position unbalanced, with the switched beam having less gain, more spillover, and a different path length than the on-axis beam , or (b) three-positioned, with the two switched beams symmetric on either side of the on-axis beam.

If you are willing to accept the asymmetry, option (a) is easiest to implement- a chopper in front of one of the mirrors which lies close to a focal plane. One problem is that in optics designed to give a frequency independent image of the aperture, the images of the sky (focal planes) move around quite a bit with frequency. This will mean that the tradeoff between beam throw and spillover will not be optimum.

Option (b) could be implemented with focal plane choppers but would require a few extra reflections in each of the three paths to get the path lengths to work out, and if the reflectors were not aligned/polished/sized the same the asymmetry might cancel out part of the advantage over option (a). I can think of some other arrangements using moving mirrors which should be symmetric, but they involve at least two additional reflections. Thus if symmetry is needed some sort of aperture-image plane wobbler would be best.

For optimum performance the wobbler should be at an image of the aperture. The sample designs given in section II produce an image of the aperture at the receiver, but this image is too small to be used for switching. At an image of the aperture, the feeding beam has to be tilted (rotated) to achieve beam switching; moving side to side will result mainly in spillover without beam throw. The small image has to be tilted too far off axis in the switched positions, with the result that the spots on the intermediate reflectors have movements of 10's of cm. Even if you were willing to oversize the mirrors by this much, the off-axis aberrations would be large. I think in general, (although I haven't worked out that many examples), the wobbling has to be at a fairly large image of the aperture or this will be the case.

In one of the sample designs given in Section II, it turns out that the field on the first mirror is a rough image of the subreflector, and so might be a candidate for wobbling. By adding an extra curved mirror to the system, it might be possible to force an image of the aperture to appear at a smaller size on one of the mirrors. However, as stated above, I think that smaller wobbling images are inseparable from widely varying intermediate spots.

(4) Calibration Chopping

This could be easily accomplished with a three-section rotating vane (open, reflecting, and absorbing), with the axis of rotation in the range of 20 to 30 degrees off the beam axis. The reflecting section would send the beam into a cold load and the absorbing section would act as a warm load. The best location is where the beam is small; right above the receiver "turret" is probably best. There are small beams in other parts of the optics but the waist locations are variable with frequency, so a relatively large vane would be needed.

For the sample designs given in section II of this report, the beam above the receiver turret needs a clear aperture of about 6-7 cm, so a light vane 30-35 cm in diameter would be sufficient.

II. SAMPLE DESIGNS FOR BENT NASMYTH OPTICS

The following describes some rough sample designs for a Bent Nasmyth system.

(1) Mirror Positions

The numbers I used for the mirror placements are based on the drawings faxed to me by Bill Bruckman.

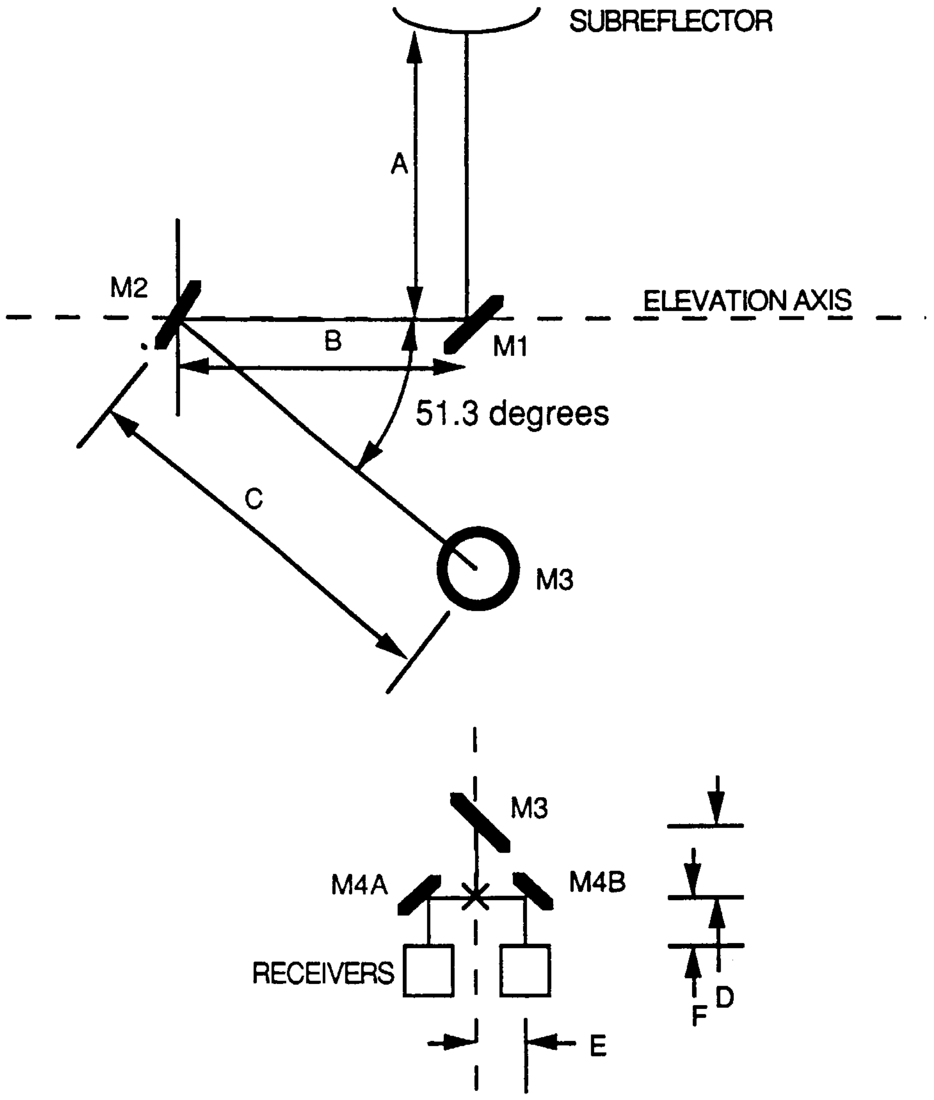


FIGURE 4

Referring to the bent Nasmyth shown in Figure 4, the dimensions are as follows:

A (Subreflector to Elevation Axis)	4.300 m
B (Distance along Elevation Axis)	0.896 m
C (Distance from M2 to M3)	1.432 m
D (Drop down to Polarization Split)	0.358 m
E + F (Path from Polarization Split to "Feed Point") (E = 0.25, F = 0.25 m)	est. 0.500 m

As discussed in section (1), if the mirror positions are already determined then at least two more variables in addition to the subreflector focal length are required in order to design a frequency independent beam guide. Therefore there will need to be at least two curved mirrors in the system.

I looked at the following possibilities, using thin lens type calculations at 230, 345, 490, 690, and 810 GHz:

(a) Curve M1 and M2
(b) Curve M1 and M3
(c) Curve M1 and M4
(d) Curve M2 and M3
(e) Curve M2 and M4
(f) Curve M3 and M4

The results are summarized in the following table:

Table 3

Case	(a)	(b)	(c)	(d)	(e)	(f)
Elev. Dependent?	Yes	Yes	Yes	No	No	No
Largest mirror dimension	Unreal (>3 m)	Unreal (>2 m)	Bad (1 m)	OK (34 cm)	Bad (70 cm)	OK (38 cm)
At least 1 mirror near aper. image?	Poor (55°)	Poor (77°)	Good (9.3°)	Bad (113°)	OK (31°)	Fair (48°)
Smallest f/w	Bad 1.4	Bad 2.2	OK 8.9	OK 12.0	Bad 2.6	OK 8.8

- Cases (a), (b), and (c) involve using a curved mirror M1. Since M1 moves with the telescope elevation while M2, M3 do not, this would lead to a small elevation dependent asymmetry and cross-polarization. It also might lead to additional complication in alignment.
- Cases (a) and (b) lead to unrealistic mirror sizes , so they can be ruled out. The 70 cm mirror in (e) is M4, so this is also unreasonable. (d) and (f) are best in this respect.
- Aperture Image: the number is the degrees of phase slippage between the 0,0 and 4,0 Hermite (or 2,0 Laguerre) modes at the best mirror at the worst frequency (230 GHz). This should be a measure of how useful the mirror is as a wobbler. I am assuming that only even numbered Hermite modes (symmetric modes) count in forming the aperture image.

- Cases (a), (b), and (e) have small ratios of focal length to spot size, which leads to higher asymmetry distortion and makes the thin-lens calculation less valid. The number is the smallest f/w ratio in the system at the worst frequency, 230 GHz.

Based on the above, it looks like (d) and (f) are the best possibilities, the only drawback being that none of the mirrors in (d) are a particularly good choice for a wobbling mirror.

(3) Selection of Image Size

As discussed in section I, part (2b), the receiver optics must be considerably larger than the aperture image in order to make sure that the receiver spillover is onto the sky. If it is assumed that an aperture of 5.5 times the horn spot size is enough, and that the aperture is 50 mm, then the horn spot size (after transformation by lenses and/or mirrors integrated into or inside of the dewar) should be about 9.1 mm. The spot size best approximating a horn is 0.64 times the transformed radius of the horn mouth, giving a mouth radius of 14.2 mm. The ratio of horn radius to image radius for highest-coupling in the unblocked case is about 1.3, thus the image radius should be about **10.9 mm**.

(4) Design for Frequency Independence

Using two-curved mirrors at fixed positions to obtain frequency independent transformation of one Gaussian beam with known spot size and radius of curvature to another is covered in a paper by Chu (IEEE Trans. on A&P, 1983). The procedure below is derived from Chu's.

I assumed that the two points where the spot size and radius were fixed with frequency were at the receiver, where we want a frequency independent image, and at the subreflector, where the diameter is fixed by the subreflector's physical size and the radius of phase curvature is fixed by the focal points chosen for the hyperboloid. Therefore the optics actually produce a flat image of the subreflector, which is in turn very close to an image of the primary aperture. It may be more accurate to use the diameter and radius of curvature for the equivalent primary; in any case the differences in the resulting design will be very minor.

The "inputs" are:

LS = path length from subreflector to first curved mirror
L = path length between the two curved mirrors
LR = path length from second curved mirror to image at receiver
h = the ratio of the image size to subreflector size.

The "outputs" are:

RS = radius of curvature of phase front at subreflector.
F1 = focal length of first curved mirror
F2 = focal length of second curved miror

There is also an intermediate image between the two curved mirrors; let

LB = distance from image to first curved mirror
LA = distance from image to second curved mirror
Note that L = LA + LB

First RS is found from

$$RS = \frac{LS}{1 + \frac{h X L}{LR} + h^{2}\frac{LS}{LR}}$$

RS is the distance from the subreflector back to the geometric Cassegrain focus. LB is found from

$$LB = \frac{LS X h X L}{LR + h X LS}$$

LA is found from LA = L - LB. Then the unknown focal lengths are found from

$$\frac{1}{F1} = \frac{1}{LB} + \frac{1}{LS}$$

and

$$\frac{1}{F2} = \frac{1}{LA} + \frac{1}{LR}$$

The numerical results are tabulated below:

Summary of Focal Length Calculations

Curved:	1,2	1,3	1,4	2,3	2,4	3,4
LS	4.300	4.300	4.300	5.196	5.196	6.628
L	0.896	2.328	2.936	1.432	2.040	0.608
LR	2.290	0.858	0.250	0.858	0.250	0.250
LS+L+LR	7.486	7.486	7.486	7.486	7.486	7.486
h	0.048444 in all cases					
RS	5.277	3.761	2.671	4.745	3.598	5.617
LB	0.0747	0.1676	1.3344	0.3248	0.7184	0.3418
LA	0.8213	2.1604	1.6016	1.1072	1.3216	0.2662
F1	0.0734	0.1613	1.0184	0.3057	0.6311	0.3250
F2	0.6045	0.6141	0.2162	0.4834	0.2102	0.1289

(5) Mirror Size Calculations, Aperture Image Verification

Regular thin-lens type analysis was applied to the designs found from the above sections; the results are appended to the end of this report. The frequency independence was verified by repeating the analysis at 230, 345, 490, 690 and 810 GHz, and the spot sizes versus frequency at each mirror were determined. The mirrors were then chosen to be 5.5 spot sizes across near images of the aperture (to get the spillover through to the sky), and 6 spot sizes across away from images of the aperture (to prevent vignetting). Of course, one dimension must be larger in each case because of the angle of incidence.

From this analysis the phase slippage between the modes versus frequency is also calculated. Note that the slippage is higher at lower frequencies, because of the nongeometric behavior. At an image of the aperture, the phase slippage is either 0 degrees or a multiple of 360 degrees. For all the designs, there is a good frequency independent image at the receiver, with the axial position varying very little. In design (f), the spot on M1 might be close enough to an image of the aperture to be used as a wobbling mirror.

(6) Asymmetry and Cross-Polarization

The complete calculations are not done, but an estimate of the asymmetry and cross-polarization loss can be found using formulas from a paper by Murphy. The asymmetry loss due to a single reflection from an offset curved surface is about

$$\frac{w^2}{8f^2}\tan^2\phi$$

while the polarization loss is about

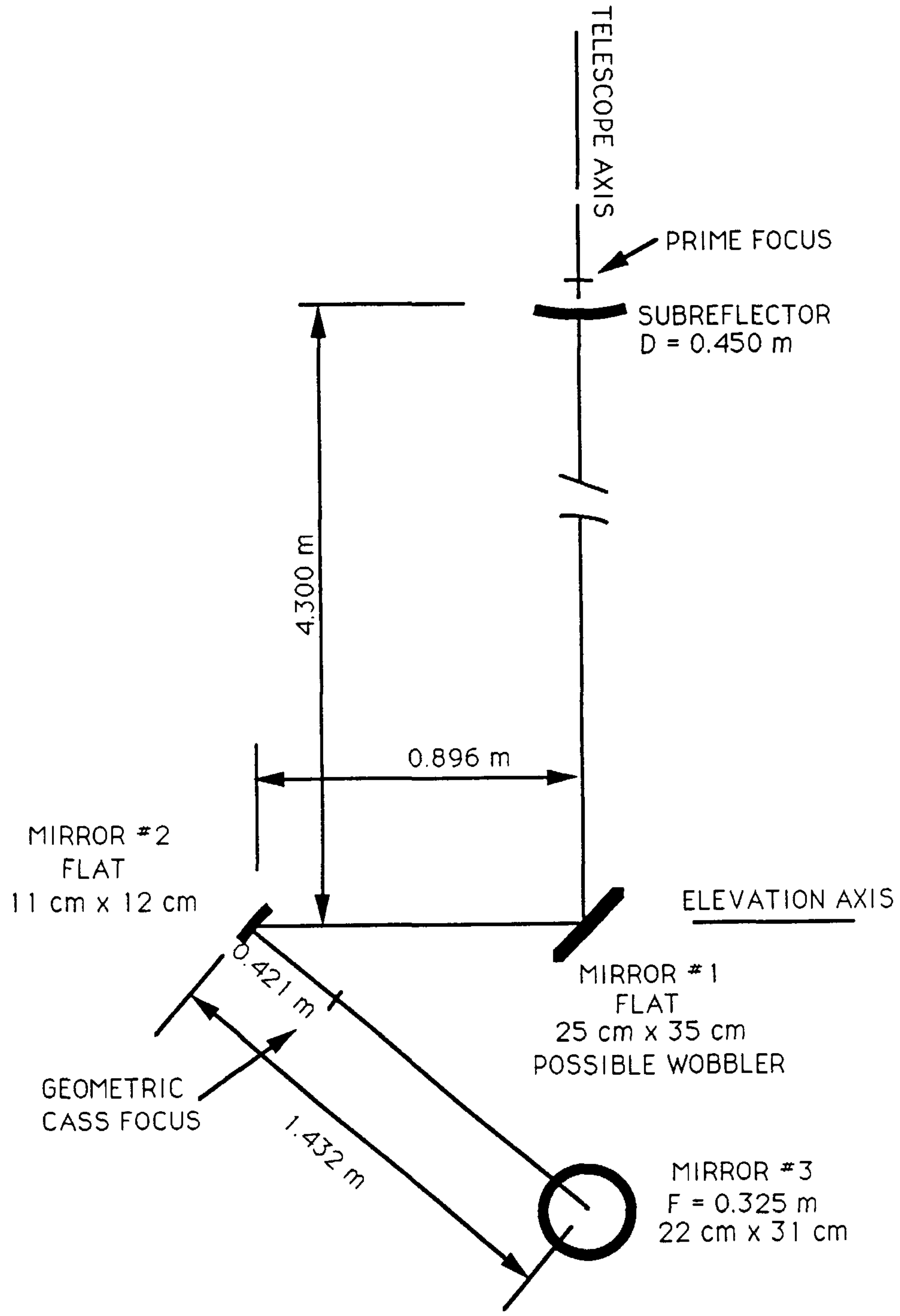
$$\frac{w^2}{4f^2}\tan^2\phi$$

where w is the spot size on the mirror and f is its focal length. In a multiple mirror system, the losses may add or cancel depending on the phase slippage. In the worst case, the total loss is the square of the sum of the square roots of the losses.

For design (c), M1 and M4 curved, the worst case losses are 0.41% for asymmetry and 0.82% for cross-polarization.

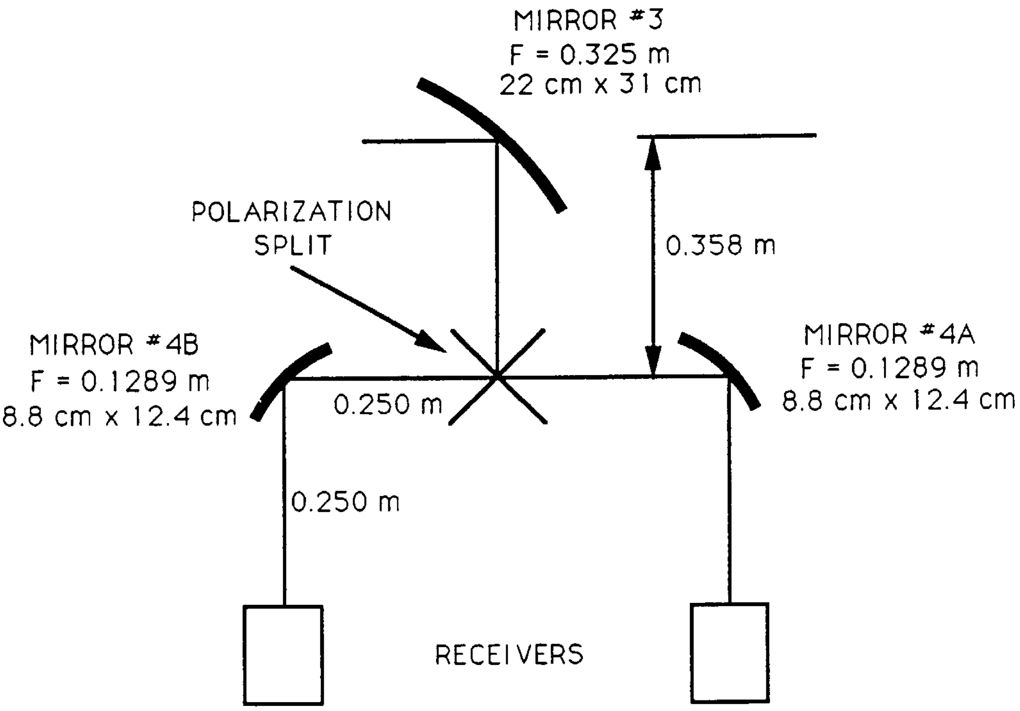
For design (d), M2 and M3 curved, the worst case losses are 0.17% for asymmetry and 0.34% for cross-polarization.

For design (f), M3 and M4 curved, the worst case losses are 0.64% for asymmetry and 1.28% for cross-polarization.



BENT NASMYTH w/ FREQ. INDEPENDENT OPTICS
FED AT FLAT IMAGE OF SUBREFLECTOR APERTURE
VERSION (f)- MIRRORS #3 AND #4 CURVED
PAGE 1

6 OCTOBER 1990 H.D.FOLTZ
E.E.R.L. - UNIV. OF TEXAS



BENT NASMYTH w/ FREQ. INDEPENDENT OPTICS
FED AT FLAT IMAGE OF SUBREFLECTOR APERTURE
VERSION (f)- MIRRORS #3 AND #4 CURVED
PAGE 2

6 OCTOBER 1990 H.D.FOLTZ
E.E.R.L. - UNIV. OF TEXAS

III. Estimated Loss Budget for SMA Telescope

The loss budget below is broken into two categories. The numbers in part "A" had at least some small basis for calculation, while the numbers in part "B" had no basis whatsoever; they are just guesses. The items in part "A" (except for illumination and spillover) are fixed losses, while those in part "B" are more dependent on how the telescope is used. Please keep in mind that only the items in part "A" have any basis in reality.

	FREQUENCY	230 GHz	810 GHz
A.	Cross Polarization due to Primary Curvature[1]	>99.9%	>99.9%
	Subreflector Blockage[2]	97.0%	97.0%
	Spar Blockage[3]	99.4%	99.4%
	Primary Surface Tolerance Losses[4]	97.9%	77.1%
	Optics Surface Tolerance Losses[5]	99.3%	91.7%
	Primary Ohmic Losses[6]	99.5%	99.1%
	Optics Ohmic Losses[7]	97.5%	95.6%
	Illumination x Spillover Efficiency[8]	86.9%	86.9%
	Nasmyth Illumination Asymmetry[9]	99.8%	99.8%
	Nasmyth Cross Polarization[10]	99.6%	99.6%
	Upper Limit on Aperture Efficiency	**78.4%**	**55.7%**
B.	"Receiver" Losses[11]	90.0%	80.0%
	Misalignment, defocussing[12]	99.5%	95.0%
	"Realistic" Aperture Efficiency	**70.2%**	**42.3%**

(1) There is cross-polarization due to other effects (blockage, Nasmyth mirrors, etc.) not included in this number.
(2) Based on 0.45 m diameter subreflector and 11-dB tapered Gaussian illumination.
(3) Based on 4 spars, 10 cm wide each, going from edge of subreflector to edge of primary, with 11-dB tapered Gaussian illumination.
(4) Based Ruze formula with 15 micron r.m.s. primary.
(5) Based on 3 further reflections at 5 micron r.m.s. each (subreflector plus two curved Nasmyth mirrors). I assumed that the r.m.s. deviations of flat reflectors would be negligible.
(6) Theory value for 25 microinch polished aluminum.
(7) Based on 5 reflections from mirrors (Subreflector plus M1 through M4), but does not include ohmic losses in polarization split or diplexers, which are lumped into the guess for 'receiver losses'. Again, these are theory values for 25 microinch aluminum.
(8) Scalar horns at appropriate feed points. Does not include next item (9).
(9),(10) I used the values for the best design under the worst conditions. This is probably a bit pessimistic.
(11) Feed horn errors, lens losses, impefect diplexers or beam-splitters, ohmic losses in diplexers or receiver optics, VSWR, etc.
(12) 810 GHz number is complete guess; 230 GHz number is based on misalignment loss going as inverse square of wavelength.

```
 Program GAU03.FOR
 B.Nas. 1,2 curved
 All dimensions in meters

Curvature of field at subreflector    5.27700E+00
Subreflector to elevation axis        4.30000E+00
Focal length- first mirror            7.34000E-02
Distance along elevation axis         8.96000E-01
Focal length- second mirror           6.04500E-01
Distance from M2 to M3                1.43200E+00
Focal length- third mirror            1.00000E+12
Distance from M3 to M4                6.08000E-01
Focal length- fourth mirror           1.00000E+12
Distance to diplexer                  2.50000E-01

Subreflector diameter                 4.50000E-01
Edge taper (dB)                       1.24600E+01

 OUTPUT:

Spot size on subreflector             1.87858E-01
Spot size on first mirror:
    230.0    3.60540E-02
    345.0    3.53523E-02
    490.0    3.50652E-02
    690.0    3.49244E-02
    810.0    3.48851E-02
Spot size on second mirror:
    230.0    4.34432E-01
    345.0    4.27398E-01
    490.0    4.24528E-01
    690.0    4.23122E-01
    810.0    4.22729E-01
Spot size on third mirror:
    230.0    1.57233E-01
    345.0    1.54504E-01
    490.0    1.53390E-01
    690.0    1.52843E-01
    810.0    1.52691E-01
Spot size on fourth mirror:
    230.0    3.95794E-02
    345.0    3.86572E-02
    490.0    3.82789E-02
    690.0    3.80931E-02
    810.0    3.80411E-02
Spot size on diplexer:
    230.0    9.12912E-03
    345.0    9.12904E-03
    490.0    9.12907E-03
    690.0    9.12910E-03
    810.0    9.12908E-03
Distance from waist to diplexer:
    230.0    4.58410E-02
    345.0    4.72941E-02
    490.0    4.79072E-02
    690.0    4.82119E-02
    810.0    4.82974E-02
```

} Due to roundoff in input values.

```
Estimate of phase slippages between modes
Phases are in degrees

Cumulative slippage at mirror 1
    N+M >     1       2       3       4       5       6
    230.0    15.3    30.5    45.8    61.1    76.4    91.6
    345.0    10.3    20.6    31.0    41.3    51.6    61.9
    490.0     7.3    14.6    21.9    29.2    36.5    43.8
    690.0     5.2    10.4    15.6    20.8    26.0    31.2
    810.0     4.4     8.9    13.3    17.7    22.2    26.6

Cumulative slippage at mirror 2
    N+M >     1       2       3       4       5       6
    230.0   193.9    27.8   221.7    55.7   249.6    83.5
    345.0   189.4    18.8   208.1    37.5   226.9    56.3
    490.0   186.6    13.3   199.9    26.5   213.2    39.8
    690.0   184.7     9.4   194.2    18.9   203.6    28.3
    810.0   184.0     8.0   192.1    16.1   200.1    24.1

Cumulative slippage at mirror 3
    N+M >     1       2       3       4       5       6
    230.0   194.4    28.8   223.2    57.6   252.1    86.5
    345.0   189.7    19.4   209.2    38.9   228.6    58.3
    490.0   186.9    13.8   200.6    27.5   214.4    41.3
    690.0   184.9     9.8   194.7    19.6   204.5    29.4
    810.0   184.2     8.3   192.5    16.7   200.9    25.0

Cumulative slippage at mirror 4
    N+M >     1       2       3       4       5       6
    230.0   196.7    33.5   230.2    66.9   263.7   100.4
    345.0   191.3    22.7   214.0    45.3   236.7    68.0
    490.0   188.0    16.1   204.1    32.1   220.2    48.2
    690.0   185.7    11.4   197.2    22.9   208.6    34.3
    810.0   184.9     9.8   194.6    19.5   204.4    29.3

 Cumulative slippage at diplexer
    N+M >     1       2       3       4       5       6
    230.0     0.1     0.1     0.2     0.2     0.3     0.3
    345.0     0.0     0.1     0.1     0.1     0.2     0.2
    490.0     0.0     0.0     0.1     0.1     0.1     0.1
    690.0     0.0     0.0     0.1     0.1     0.1     0.1
    810.0     0.0     0.0     0.0     0.1     0.1     0.1
```

```
 Program gau04.FOR
 B.Nas. 1,3 curved
 All dimensions in meters

Curvature of field at subreflector    3.76100E+00
Subreflector to elevation axis        4.30000E+00
Focal length- first mirror            1.61300E-01
Distance along elevation axis         8.96000E-01
Focal length- second mirror           1.00000E+12
Distance from M2 to M3                1.43200E+00
Focal length- third mirror            6.14100E-01
Distance from M3 to M4                6.08000E-01
Focal length- fourth mirror           1.00000E+12
Distance to diplexer                  2.50000E-01

Subreflector diameter                 4.50000E-01
Edge taper (dB)                       1.24600E+01

 OUTPUT:

Spot size on subreflector             1.87858E-01
Spot size on first mirror:
    230.0    2.85485E-02
    345.0    2.76570E-02
    490.0    2.72891E-02
    690.0    2.71080E-02
    810.0    2.70573E-02
Spot size on second mirror:
    230.0    8.81384E-02
    345.0    8.25937E-02
    490.0    8.02486E-02
    690.0    7.90805E-02
    810.0    7.87515E-02
Spot size on third mirror:
    230.0    2.74211E-01
    345.0    2.58581E-01
    490.0    2.52002E-01
    690.0    2.48733E-01
    810.0    2.47813E-01
Spot size on fourth mirror:
    230.0    8.17433E-02
    345.0    7.72984E-02
    490.0    7.54312E-02
    690.0    7.45043E-02
    810.0    7.42435E-02
Spot size on diplexer:
    230.0    2.90271E-03
    345.0    2.90267E-03
    490.0    2.90268E-03
    690.0    2.90267E-03
    810.0    2.90266E-03
Distance from waist to diplexer:
    230.0   -8.18133E-03
    345.0   -9.22441E-03
    490.0   -9.72450E-03
    690.0   -9.98831E-03
    810.0   -1.00645E-02
```

```
Estimate of phase slippages between modes
Phases are in degrees

Cumulative slippage at mirror 1
    N+M >      1       2       3       4       5       6
    230.0    160.6   321.1   121.7   282.3    82.8   243.4
    345.0    166.8   333.5   140.3   307.1   113.8   280.6
    490.0    170.6   341.2   151.8   322.4   133.0   303.6
    690.0    173.3   346.6   159.9   333.2   146.5   319.8
    810.0    174.3   348.6   162.8   337.1   151.4   325.7

Cumulative slippage at mirror 2
    N+M >      1       2       3       4       5       6
    230.0    332.1   304.1   276.2   248.3   220.4   192.4
    345.0    340.5   321.1   301.6   282.1   262.7   243.2
    490.0    346.0   332.1   318.1   304.1   290.1   276.2
    690.0    350.0   340.0   329.9   319.9   309.9   299.9
    810.0    351.4   342.9   334.3   325.8   317.2   308.6

Cumulative slippage at mirror 3
    N+M >      1       2       3       4       5       6
    230.0    333.5   307.0   280.4   253.9   227.4   200.9
    345.0    341.6   323.2   304.8   286.4   268.0   249.6
    490.0    346.8   333.6   320.5   307.3   294.1   280.9
    690.0    350.6   341.1   331.7   322.2   312.8   303.3
    810.0    351.9   343.9   335.8   327.7   319.7   311.6

Cumulative slippage at mirror 4
    N+M >      1       2       3       4       5       6
    230.0    334.1   308.3   282.4   256.5   230.6   204.8
    345.0    342.1   324.2   306.2   288.3   270.4   252.5
    490.0    347.2   334.4   321.5   308.7   295.9   283.1
    690.0    350.8   341.6   332.4   323.3   314.1   304.9
    810.0    352.2   344.3   336.5   328.6   320.8   313.0

 Cumulative slippage at diplexer
    N+M >      1       2       3       4       5       6
    230.0      0.0     0.1     0.1     0.2     0.2     0.3
    345.0      0.0     0.1     0.1     0.1     0.2     0.2
    490.0      0.0     0.0     0.1     0.1     0.1     0.1
    690.0      0.0     0.0     0.0     0.1     0.1     0.1
    810.0      0.0     0.0     0.0     0.1     0.1     0.1
```

```
 Program gau04.FOR
 B.Nas. 1,4 curved
 All dimensions in meters

Curvature of field at subreflector   2.67100E+00
Subreflector to elevation axis       4.30000E+00
Focal length- first mirror           1.01840E+00
Distance along elevation axis        8.96000E-01
Focal length- second mirror          1.00000E+12
Distance from M2 to M3               1.43200E+00
Focal length- third mirror           1.00000E+12
Distance from M3 to M4               6.08000E-01
Focal length- fourth mirror          2.16200E-01
Distance to diplexer                 2.50000E-01

Subreflector diameter                4.50000E-01
Edge taper (dB)                      1.24600E+01

 OUTPUT:

Spot size on subreflector            1.87858E-01
Spot size on first mirror:
    230.0    1.14965E-01
    345.0    1.14747E-01
    490.0    1.14659E-01
    690.0    1.14616E-01
    810.0    1.14604E-01
Spot size on second mirror:
    230.0    7.68516E-02
    345.0    7.68164E-02
    490.0    7.68022E-02
    690.0    7.67953E-02
    810.0    7.67934E-02
Spot size on third mirror:
    230.0    1.78612E-02
    345.0    1.70659E-02
    490.0    1.67344E-02
    690.0    1.65704E-02
    810.0    1.65244E-02
Spot size on fourth mirror:
    230.0    1.46710E-02
    345.0    1.19608E-02
    490.0    1.06745E-02
    690.0    9.98780E-03
    810.0    9.78754E-03
Spot size on diplexer:
    230.0    9.09823E-03
    345.0    9.09823E-03
    490.0    9.09823E-03
    690.0    9.09823E-03
    810.0    9.09823E-03
Distance from waist to diplexer:
    230.0   -2.36711E-03
    345.0   -5.39961E-03
    490.0   -1.09508E-02
    690.0   -2.17580E-02
    810.0   -2.99907E-02
```

```
Estimate of phase slippages between modes
Phases are in degrees

Cumulative slippage at mirror 1
    N+M >      1       2       3       4       5       6
    230.0    175.3   350.5   165.8   341.0   156.3   331.6
    345.0    176.8   353.7   170.5   347.3   164.2   341.0
    490.0    177.8   355.5   173.3   351.1   168.9   346.6
    690.0    178.4   356.8   175.3   353.7   172.1   350.5
    810.0    178.7   357.3   176.0   354.6   173.3   351.9

Cumulative slippage at mirror 2
    N+M >      1       2       3       4       5       6
    230.0    177.7   355.3   173.0   350.7   168.4   346.0
    345.0    178.4   356.9   175.3   353.8   172.2   350.7
    490.0    178.9   357.8   176.7   355.6   174.5   353.4
    690.0    179.2   358.4   177.7   356.9   176.1   355.3
    810.0    179.3   358.7   178.0   357.4   176.7   356.0

Cumulative slippage at mirror 3
    N+M >      1       2       3       4       5       6
    230.0    203.3    46.6   250.0    93.3   296.6   139.9
    345.0    196.0    32.1   228.1    64.1   260.2    96.2
    490.0    191.4    22.9   214.3    45.8   237.2    68.6
    690.0    188.2    16.4   204.5    32.7   220.9    49.1
    810.0    187.0    14.0   200.9    27.9   214.9    41.9

Cumulative slippage at mirror 4
    N+M >      1       2       3       4       5       6
    230.0    309.0   258.0   207.1   156.1   105.1    54.1
    345.0    320.6   281.1   241.7   202.2   162.8   123.3
    490.0    329.9   299.8   269.8   239.7   209.6   179.5
    690.0    337.6   315.3   292.9   270.6   248.2   225.8
    810.0    340.7   321.4   302.1   282.8   263.5   244.1

 Cumulative slippage at diplexer
    N+M >      1       2       3       4       5       6
    230.0      0.0     0.0     0.1     0.1     0.1     0.1
    345.0      0.0     0.0     0.0     0.0     0.1     0.1
    490.0      0.0     0.0     0.0     0.0     0.0     0.0
    690.0      0.0     0.0     0.0     0.0     0.0     0.0
    810.0      0.0     0.0     0.0     0.0     0.0     0.0
```

```
 Program gau04.FOR
 B.Nas. 2,3 curved
 All dimensions in meters

Curvature of field at subreflector   4.74500E+00
Subreflector to elevation axis       4.30000E+00
Focal length- first mirror           1.00000E+12
Distance along elevation axis        8.96000E-01
Focal length- second mirror          3.05700E-01
Distance from M2 to M3               1.43200E+00
Focal length- third mirror           4.83400E-01
Distance from M3 to M4               6.08000E-01
Focal length- fourth mirror          1.00000E+12
Distance to diplexer                 2.50000E-01

Subreflector diameter                4.50000E-01
Edge taper (dB)                      1.24600E+01

 OUTPUT:

Spot size on subreflector            1.87858E-01
Spot size on first mirror:
    230.0    2.00146E-02
    345.0    1.87210E-02
    490.0    1.81731E-02
    690.0    1.79001E-02
    810.0    1.78231E-02
Spot size on second mirror:
    230.0    2.12254E-02
    345.0    1.94255E-02
    490.0    1.86503E-02
    690.0    1.82607E-02
    810.0    1.81504E-02
Spot size on third mirror:
    230.0    4.01614E-02
    345.0    2.76185E-02
    490.0    2.04893E-02
    690.0    1.58961E-02
    810.0    1.43540E-02
Spot size on fourth mirror:
    230.0    1.45840E-02
    345.0    1.18538E-02
    490.0    1.05544E-02
    690.0    9.85927E-03
    810.0    9.65634E-03
Spot size on diplexer:
    230.0    9.10036E-03
    345.0    9.10036E-03
    490.0    9.10036E-03
    690.0    9.10036E-03
    810.0    9.10036E-03
Distance from waist to diplexer:
    230.0    3.99798E-05
    345.0    9.66191E-05
    490.0    2.00361E-04
    690.0    4.04298E-04
    810.0    5.57229E-04
```

```
Estimate of phase slippages between modes
Phases are in degrees

Cumulative slippage at mirror 1
    N+M >      1       2       3       4       5       6
    230.0     28.3    56.7    85.0   113.3   141.6   170.0
    345.0     19.8    39.5    59.3    79.1    98.8   118.6
    490.0     14.2    28.4    42.6    56.8    71.0    85.2
    690.0     10.2    20.4    30.6    40.7    50.9    61.1
    810.0      8.7    17.4    26.1    34.8    43.5    52.2

Cumulative slippage at mirror 2
    N+M >      1       2       3       4       5       6
    230.0    147.3   294.5    81.8   229.1    16.4   163.6
    345.0    156.8   313.6   110.4   267.2    64.0   220.8
    490.0    163.2   326.4   129.6   292.8    96.1   259.3
    690.0    167.9   335.8   143.7   311.6   119.5   287.4
    810.0    169.7   339.3   149.0   318.6   128.3   297.9

Cumulative slippage at mirror 3
    N+M >      1       2       3       4       5       6
    230.0    283.1   206.2   129.3    52.3   335.4   258.5
    345.0    289.2   218.4   147.7    76.9     6.1   295.3
    490.0    296.3   232.7   169.0   105.4    41.7   338.0
    690.0    304.9   249.8   194.7   139.5    84.4    29.3
    810.0    309.3   258.6   207.9   157.2   106.5    55.8

Cumulative slippage at mirror 4
    N+M >      1       2       3       4       5       6
    230.0    308.6   257.2   205.8   154.4   103.0    51.6
    345.0    320.1   280.3   240.4   200.5   160.6   120.8
    490.0    329.5   299.1   268.6   238.2   207.7   177.2
    690.0    337.3   314.7   292.0   269.3   246.7   224.0
    810.0    340.4   320.8   301.2   281.7   262.1   242.5

 Cumulative slippage at diplexer
    N+M >      1       2       3       4       5       6
    230.0    360.0   360.0   360.0   360.0   360.0   360.0
    345.0    360.0   360.0   360.0   360.0   360.0   360.0
    490.0    360.0   360.0   360.0   360.0   360.0   360.0
    690.0    360.0   360.0   360.0   360.0   360.0   360.0
    810.0    360.0   360.0   360.0   360.0   360.0   360.0
```

```
Session Name: indigo.cc.utexas.edu

 Program gau04.FOR
 B.Nas. 2,4 curved
 All dimensions in meters

Curvature of field at subreflector     3.59800E+00
Subreflector to elevation axis         4.30000E+00
Focal length- first mirror             1.00000E+12
Distance along elevation axis          8.96000E-01
Focal length- second mirror            6.31100E-01
Distance from M2 to M3                 1.43200E+00
Focal length- third mirror             1.00000E+12
Distance from M3 to M4                 6.08000E-01
Focal length- fourth mirror            2.10200E-01
Distance to diplexer                   2.50000E-01

Subreflector diameter                  4.50000E-01
Edge taper (dB)                        1.24600E+01

 OUTPUT:

Spot size on subreflector              1.87858E-01
Spot size on first mirror:
    230.0    3.78632E-02
    345.0    3.71956E-02
    490.0    3.69228E-02
    690.0    3.67892E-02
    810.0    3.67518E-02
Spot size on second mirror:
    230.0    8.42201E-02
    345.0    8.37846E-02
    490.0    8.36083E-02
    690.0    8.35222E-02
    810.0    8.34982E-02
Spot size on third mirror:
    230.0    3.31385E-02
    345.0    3.20304E-02
    490.0    3.15724E-02
    690.0    3.13468E-02
    810.0    3.12836E-02
Spot size on fourth mirror:
    230.0    8.24990E-02
    345.0    8.09841E-02
    490.0    8.03649E-02
    690.0    8.00613E-02
    810.0    7.99765E-02
Spot size on diplexer:
    230.0    4.89788E-03
    345.0    4.89788E-03
    490.0    4.89787E-03
    690.0    4.89788E-03
    810.0    4.89788E-03
Distance from waist to diplexer:
    230.0   -1.51488E-02
    345.0   -1.57977E-02
    490.0   -1.60755E-02
    690.0   -1.62144E-02
    810.0   -1.62536E-02
```

```
Estimate of phase slippages between modes
Phases are in degrees

Cumulative slippage at mirror 1
    N+M >      1        2        3        4        5        6
    230.0    165.5    330.9    136.4    301.9    107.4    272.8
    345.0    170.2    340.4    150.6    320.8    131.0    301.2
    490.0    173.1    346.1    159.2    332.3    145.3    318.4
    690.0    175.1    350.1    165.2    340.3    155.3    330.4
    810.0    175.8    351.6    167.4    343.2    159.0    334.8

Cumulative slippage at mirror 2
    N+M >      1        2        3        4        5        6
    230.0    172.2    344.3    156.5    328.7    140.8    313.0
    345.0    174.8    349.5    164.3    339.0    153.8    328.6
    490.0    176.3    352.6    168.9    345.2    161.5    337.8
    690.0    177.4    354.7    172.1    349.5    166.9    344.2
    810.0    177.8    355.5    173.3    351.1    168.8    346.6

Cumulative slippage at mirror 3
    N+M >      1        2        3        4        5        6
    230.0    339.9    319.8    299.6    279.5    259.4    239.3
    345.0    346.3    332.5    318.8    305.1    291.4    277.6
    490.0    350.2    340.5    330.7    321.0    311.2    301.4
    690.0    353.0    346.1    339.1    332.1    325.2    318.2
    810.0    354.1    348.1    342.2    336.2    330.3    324.4

Cumulative slippage at mirror 4
    N+M >      1        2        3        4        5        6
    230.0    345.2    330.3    315.5    300.7    285.9    271.0
    345.0    350.0    340.0    330.0    320.0    310.0    299.9
    490.0    352.9    345.8    338.7    331.7    324.6    317.5
    690.0    355.0    349.9    344.9    339.8    334.8    329.7
    810.0    355.7    351.4    347.1    342.8    338.5    334.2

 Cumulative slippage at diplexer
    N+M >      1        2        3        4        5        6
    230.0      0.0      0.1      0.1      0.2      0.2      0.3
    345.0      0.0      0.1      0.1      0.1      0.2      0.2
    490.0      0.0      0.0      0.1      0.1      0.1      0.1
    690.0      0.0      0.0      0.0      0.1      0.1      0.1
    810.0      0.0      0.0      0.0      0.1      0.1      0.1
```

```
 Program gau04.FOR
 B.Nas. 3,4 curved
 All dimensions in meters

Curvature of field at subreflector    5.61700E+00
Subreflector to elevation axis        4.30000E+00
Focal length- first mirror            1.00000E+12
Distance along elevation axis         8.96000E-01
Focal length- second mirror           1.00000E+12
Distance from M2 to M3                1.43200E+00
Focal length- third mirror            3.25000E-01
Distance from M3 to M4                6.08000E-01
Focal length- fourth mirror           1.28900E-01
Distance to diplexer                  2.50000E-01

Subreflector diameter                 4.50000E-01
Edge taper (dB)                       1.24600E+01

 OUTPUT:

Spot size on subreflector             1.87858E-01
Spot size on first mirror:
    230.0    4.50588E-02
    345.0    4.44993E-02
    490.0    4.42716E-02
    690.0    4.41602E-02
    810.0    4.41290E-02
Spot size on second mirror:
    230.0    1.81646E-02
    345.0    1.60245E-02
    490.0    1.50754E-02
    690.0    1.45906E-02
    810.0    1.44523E-02
Spot size on third mirror:
    230.0    3.68454E-02
    345.0    3.51927E-02
    490.0    3.45036E-02
    690.0    3.41628E-02
    810.0    3.40671E-02
Spot size on fourth mirror:
    230.0    1.45952E-02
    345.0    1.18646E-02
    490.0    1.05651E-02
    690.0    9.87000E-03
    810.0    9.66707E-03
Spot size on diplexer:
    230.0    9.09108E-03
    345.0    9.09108E-03
    490.0    9.09108E-03
    690.0    9.09108E-03
    810.0    9.09108E-03
Distance from waist to diplexer:
    230.0   -1.82122E-04
    345.0   -5.64069E-04
    490.0   -1.26281E-03
    690.0   -2.62800E-03
    810.0   -3.66455E-03
```

```
Estimate of phase slippages between modes
Phases are in degrees

Cumulative slippage at mirror 1
    N+M >     1       2       3       4       5       6
    230.0    12.2    24.3    36.5    48.7    60.8    73.0
    345.0     8.2    16.4    24.5    32.7    40.9    49.1
    490.0     5.8    11.6    17.3    23.1    28.9    34.7
    690.0     4.1     8.2    12.3    16.4    20.6    24.7
    810.0     3.5     7.0    10.5    14.0    17.5    21.0

Cumulative slippage at mirror 2
    N+M >     1       2       3       4       5       6
    230.0    39.2    78.4   117.5   156.7   195.9   235.1
    345.0    28.5    57.0    85.6   114.1   142.6   171.1
    490.0    20.9    41.9    62.8    83.7   104.7   125.6
    690.0    15.2    30.4    45.6    60.8    76.0    91.2
    810.0    13.0    26.1    39.1    52.1    65.2    78.2

Cumulative slippage at mirror 3
    N+M >     1       2       3       4       5       6
    230.0   156.6   313.2   109.8   266.4    63.0   219.5
    345.0   163.9   327.8   131.7   295.6    99.5   263.4
    490.0   168.5   337.0   145.5   314.1   122.6   291.1
    690.0   171.8   343.6   155.4   327.2   138.9   310.7
    810.0   173.0   346.0   159.0   332.0   145.0   317.9

Cumulative slippage at mirror 4
    N+M >     1       2       3       4       5       6
    230.0   308.6   257.2   205.8   154.5   103.1    51.7
    345.0   320.1   280.3   240.4   200.6   160.7   120.9
    490.0   329.6   299.1   268.7   238.2   207.8   177.3
    690.0   337.3   314.7   292.0   269.4   246.7   224.1
    810.0   340.4   320.9   301.3   281.7   262.1   242.6

Cumulative slippage at diplexer
    N+M >     1       2       3       4       5       6
    230.0     0.0     0.1     0.1     0.1     0.2     0.2
    345.0     0.0     0.0     0.1     0.1     0.1     0.1
    490.0     0.0     0.0     0.0     0.1     0.1     0.1
    690.0     0.0     0.0     0.0     0.0     0.1     0.1
    810.0     0.0     0.0     0.0     0.0     0.1     0.1
```