PRELIMINARY SERVO ANALYSIS

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MEMO #41

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1.0 SCOPE

The tracking disturbance caused by the wind is one of the largest non-systematic contributions in the SMA tracking error budget. Our simulations of a 6m dish with a first mode structural frequency of 17 Hz indicate that an antenna with standard proportional plus derivative plus integral control (PID), would have an azimuth tracking error of 0.5 arc sec in a 9m/s wind. This value is based on the assumption that nonlinearities in the system do not preclude a bandwidth as high as that modelled (11 Hz). This memo discusses the linear modelling effort to date, as well as the possible effects of the known large non-linearities.

2.0 INTRODUCTION

SAO is reviewing candidate pointing control laws as part of the overall effort to write a specification for the Submillimeter Array (SMA) antennas. The controlled system, in operation, is required to point to an accuracy of 1 arcsec when moved 10 deg or less and to track that position against disturbances and sidereal motion to within 1 arcsec. This requirement holds in winds up to 9 m/s.

In an effort to accomplish this task, it was suggested that we develop a representative, but low order system model to simulate tracking performance. This model was to be linear and was to permit rapid assessment of various compensation techniques, in light of the known disturbances. Though the true system is clearly non-linear, it was our intention to predict the performance of the system using a linear model, limiting the scope of the survey before we went onto more complex nonlinear models.

The modelling to date has all been done on the azimuth axis. The reason for this is that the control is a little more complex. The sensed quantity in the control loop is the position at the base of the yoke which, at the required system accuracies, is only loosely connected to the dish's position. This fact warrants a short discussion on definitions. The tracking performance of the modelled servo system is the performance of the closed loop portion of the system, i.e. the base of the yoke. The dish performance is reported as uncontrolled error or the line of sight (LOS) error. The pointing errors are not modelled and for the most part are not predicted by this modelling effort. One source of pointing error, windup due to the wind is presented.

3.0 SYSTEM MODEL

3.1 MODELLING APPROACH

The models for this system were based on the assumption that the gross subsystems can be modelled as single inertia/spring pairs with a natural frequency of the lowest mode predicted by the structural model. The models are mathematically rendered in a form called "State Space". The set of linear differential equations describing the system, generally second order, are transformed into a set of first order equations of the form:

$$d/dt(xi)=f(x1..xn).$$

Each of these equations governs the time history of a "state" (i.e. xi). This format permits the use of a number of computer tools for control systems design and modelling. In this case we used a package called MATLAB, written by MathWorks in Sudbury, MA.

The shortcoming of this approach is that it is inherently linear. The predicted time history of the system depends only on the initial conditions of the states and system inputs (e.g. commanded pointing locations, model disturbance torques). Purely state dependent behavior such as friction and saturation cannot be modelled directly. In order to gauge their effects on the system, representative impulse or step functions must be applied to the model. This gives an idea of the magnitude of the system response, though it does not yield any detailed understanding of the interaction between the nonlinearity and the system dynamics; hence, it will not predict limit cycle behavior.

On the other hand the models are simple, robust and flexible. They lend themselves to rapid modification and are easy to troubleshoot. In a general way they predict system behavior and provide a surprising degree of insight into the control system design. The predicted behavior tends to be borne out by observation thus, this method provides a fast easy approach to limiting the scope of inquiry to a few promising options, and a maximum control gain (i.e. Bandwidth).

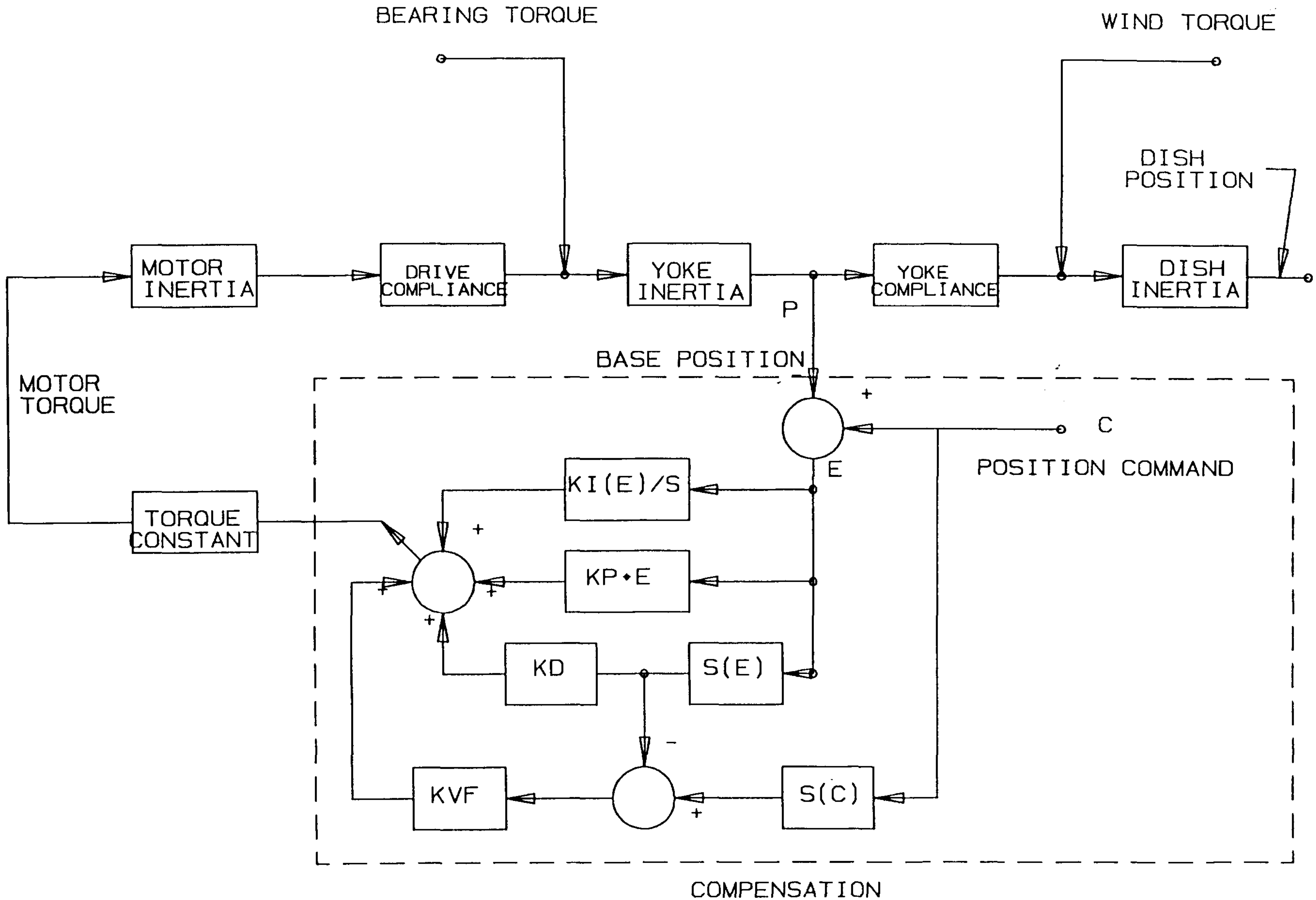
Once the field has been limited to the most promising arrangements, the effects of the non-linearities can be modelled directly. This is a very specific and detailed undertaking that is not carried out in a general way.

3.2 ANTENNA MODEL

The antenna model is very simple, it consists of three inertias connected by rotary springs. The inertias represent, in a general way, the motor and drive, the yoke and cabin, and the dish. The rotary springs model the compliance of the drive and the yoke/dish sub-system. The drive inertia and stiffness represent calculated values for a candidate drive. The dish inertia is also a calculated value. The yoke/cabin inertia is taken from TIW information that has been adjusted for the design differences between their system and ours. The second spring constant, modelled as the compliance between the motion of the yoke and the dish, was selected to set the natural frequency of the dish/spring system to 17 Hz. To the extent that this first mode is a whole body twisting mode this model is valid, in any event it is a conservative approach. The worst case is when a mode is the first natural frequency, as far as predicted performance is concerned. However, whatever the shape of the 17 Hz mode, the bandwidth has to be sufficiently far in frequency from it, to keep from exciting it. The strength of the true system response may be lower then the predicted behavior because if it is not a whole body mode there will be less mass involved.

3.3 COMPENSATION DESIGN

Though the modelling uses techniques generally called "modern control theory", the type and design of the control law is right from classic control theory. The central portion of the law is a proportional plus derivative plus integral (PID) controller. Simply stated the compensation signal is constructed by taking the difference between the commanded and actual position, sometimes called the error signal (see figure 1), determining its derivative and integral with respect to time, scaling each of these three signals separately and combining them. The proportional signal (KP) acts like a spring, forcing the system to the commanded position at a torque level that is proportional to the error. The derivative signal (KD) produces damping, removing overshoot from the system. Finally, the integral signal (KI) eliminates "hang off", the tendency for a system to stop at a point other than the desired value because of friction etc. This will happen if the friction torque equals the proportional feedback signal and their is little motion left. The integral signal will get larger, the longer the error remains, until the "hang up" is overcome. This is very useful in real systems, but the addition of an integrator tends to slow the response down and promote limit cycles if the nonlinearities are large. Note



BLOCK DIAGRAM OF AZIMUTH SERVO MODEL

Figure 1

that S is a derivative operator, S(E) is the time derivative of E while 1/S is the integral.

We added a single compensation beyond these standard ones, that is velocity feed forward (KVF). We take the difference between the time derivative of the commanded position and the system velocity and feed that to the torque signal. This is intended to reduce the tracking errors and is one of the primary methods used in the MMT tracking loop to achieve the high tracking performance that they report. There are other approaches to achieve this end, but they adversely affect the system dynamics.

In the model, as in the actual system, the azimuth loop is fed with the difference between the yoke position and the command position. To the extent that the dish follows the yoke, the dish position is controlled. The dish/spring system can be viewed as an uncontrolled mass spring trading energy with the controlled system below. Furthermore, to the extent that torques are applied to the dish and into system (as with wind), the dish will have a persistent offset from the position controlled within the loop. This behavior is seen in the linear model, and is representative of what will happen in reality to the extent that the dish compliance models the actual yoke compliance.

The tracking performance of the system is the value of the error signal and is therefore the difference of the yoke position from the command signal. The motion of the dish is the uncontrolled tracking error. The only contribution to the pointing error is the dish wind up caused by the wind.

3.4 SELECTION OF FEEDBACK CONSTANTS

The proportional and derivative feedback constants were selected by assuming the desired closed loop system dynamics and that the whole system motion was described by an equation of the form:

$$\ddot{\Theta} + 2\omega_n \zeta \dot{\Theta} + \omega_n^2 = 0 \tag{1}$$

where ζ is the damping coefficient

ω_n is the closed loop undamped natural frequency.

The natural frequency is set equal to the desired bandwidth of 11 Hz and the damping is selected for the shortest setting time, =0.7. Using the added equation:

$$I \ddot{\Theta} + B \dot{\Theta} + K \Theta = T(t) \quad (2)$$

with I equal to the entire telescope inertia about the azimuth axis, the values of k and b can be determined. These values were applied to the model and the resulting modelled bandwidth is 9 Hz, with damping of 0.7. The discrepancy results from the initial assumption that the whole system was a single mass/spring combination. The existence of the uncontrolled dish slows the response of the closed portion of the model.

The constants for the velocity feedforward and the integral feedback where determined within the model by trial and error.

3.5 OPTIMIZATION

The feedback constants are based on an educated guess of the most aggressive dynamics that is possible with a 17 Hz first mode, they do not represent an optimized control law. A computer routine was written to vary these values over preselected ranges in search of improved predicted performance. This produced a root locus of sorts from which we selected the values that gave us a balance of improved bandwidth and small loss of damping. Simulations of these optimized models showed a small improvement in performance that is, smaller errors, shorter settling times. The likely reason for only a small improvement in performance was a collateral increase in the dish dynamics, implying that the controlled base and the dish were transferring energy back and forth and doing less dampening than might be expected.

3.6 DISTURBANCE MODEL

The model includes a number of factors as disturbances simply because this is the easiest way to gauge their effect. Chief among this category is the friction of the bearings. The wind is a true disturbance and can be modelled as precisely as our data will permit.

The wind was modeled as described in memos of July 9 and August 22, 1990. This process yielded 1/40th second information about the applied torque. The model is as accurate as we are likely to get in the absence of actual measured data. There are no compromises here because of the linear nature of the model.

The bearing friction is a little more complicated though, it is a nonlinear function of the states, by modelling it as a disturbance we are able to get some idea of its effect but not its character. Past efforts have supported the view that if the bandwidth is sufficiently removed from the first system modes, the system response predicted in this way will bracket the actual behavior, in spite of the nonlinearities. The key question becomes what "sufficiently removed" is, and the answer depends on the system. In this case the bandwidth that we are suggesting is so high that some nonlinear behavior is likely, suggesting that we need a nonlinear analysis of the best control designs to fully assess the likely system behavior.

The bearing effects are gauged by placing torque pulses into the model at the appropriate nodes. The impulse levels were selected to represent the varying bearing torque, assuming that the static (or D.C) torque level will have no effect on pointing. This is not a valid assumption but it is the best that can be done with a linear system.

4.0 RESULTS

There are several performance criteria that are important when comparing the merits of a set of control laws. In this case we are interested in:

1) the ability of the servo to remove the effects of the disturbances

2) the ability of the servo to follow the command signal

3) the residue windup caused by the wind, both in the control loop and at the dish.

Table 1 shows the predicted performance under various initial assumptions.

Table 1

SERVO CONTROL LAW	RMS ERROR ARC SEC		MEAN OFFSET ARC SEC		ERROR FOLLOWING TRACKING SIGNAL ARC SEC
	CONTROL LOOP	LOS	CONTROL LOOP	LOS	CONTROL LOOP
Proportional (PD) + Derivative	0.5	1.7	0.6	4.5	0.26
PD + (PDV) Velocity Feedforward	0.5	1.7	0.6	4.5	0.003
PDV + Integral (PID + V)	0.44	1.4	0.06	4.5	0.003
PID + V "Optimized"	0.34	1.7	0.03	4.0	0.003

The effects of varying friction are smaller than the wind and not reported.

The guiding equations, in matrix form that were used in the simulation are:

$$[\ X_i\ ] = A * [\ X_i\ ] + B * [U_i\ ] \quad (3)$$

$$[\ U_{FB}\ ] = K * [\ X_i\ ] + N * [\ U_i\ ] \quad (4)$$

The A matrix is the telescope structural model, without the control system. The B matrix represents the distribution of the effects of the control and the various outside disturbances. The K and N matrices represent the control law, the K is the coefficients of the states that are fedback or forward, and the N matrix is the coefficients of the inputs that are used in the control law. The input that affects the control law is the pointing command. The actual values used in the model are shown in Figure 2 with a description of each state in Table 2.

Table 2

X_1:	Position of Drive Motor
X_2:	Velocity of Drive Motor
X_3:	Position of Yoke - Controlled State
X_4:	Velocity of Yoke
X_5:	Position of Dish
X_6:	Velocity of Dish
X_7:	Computational states related to velocity feed forward and integral feed back
X_8:	
U_1:	Control
U_2:	Wind Torque
U_3:	Bearing Torque
U_4:	Ramp Signal - sidereal rate

Figure 3 shows the noisy 9 m/s wind induced torque signal that is used in the simulations. Figures 4 and 5 show system response, figure 4 is the control loop performance, that is the base and yoke while figure 5 is the dish performance and is somewhat representative of the telescope line of sight.

$$A = \begin{matrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -50000000 & 0 & 750000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4800 & 0 & -75600 & 0 & 3600 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 13000 & 0 & -13000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \end{matrix}$$

$$B = \begin{matrix} 0 & 0 & 0 & 0 \\ 0.4167 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0000 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0.0001 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$$K = \begin{matrix} 0 & 1000 & 200000000 & 3500000 & 0 & 0 & -200000000 & 200000000 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$$N = \begin{matrix} 200000000 & 0 & 0 & 3500000 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

Figure 2 CONTROL MODEL MATRICES

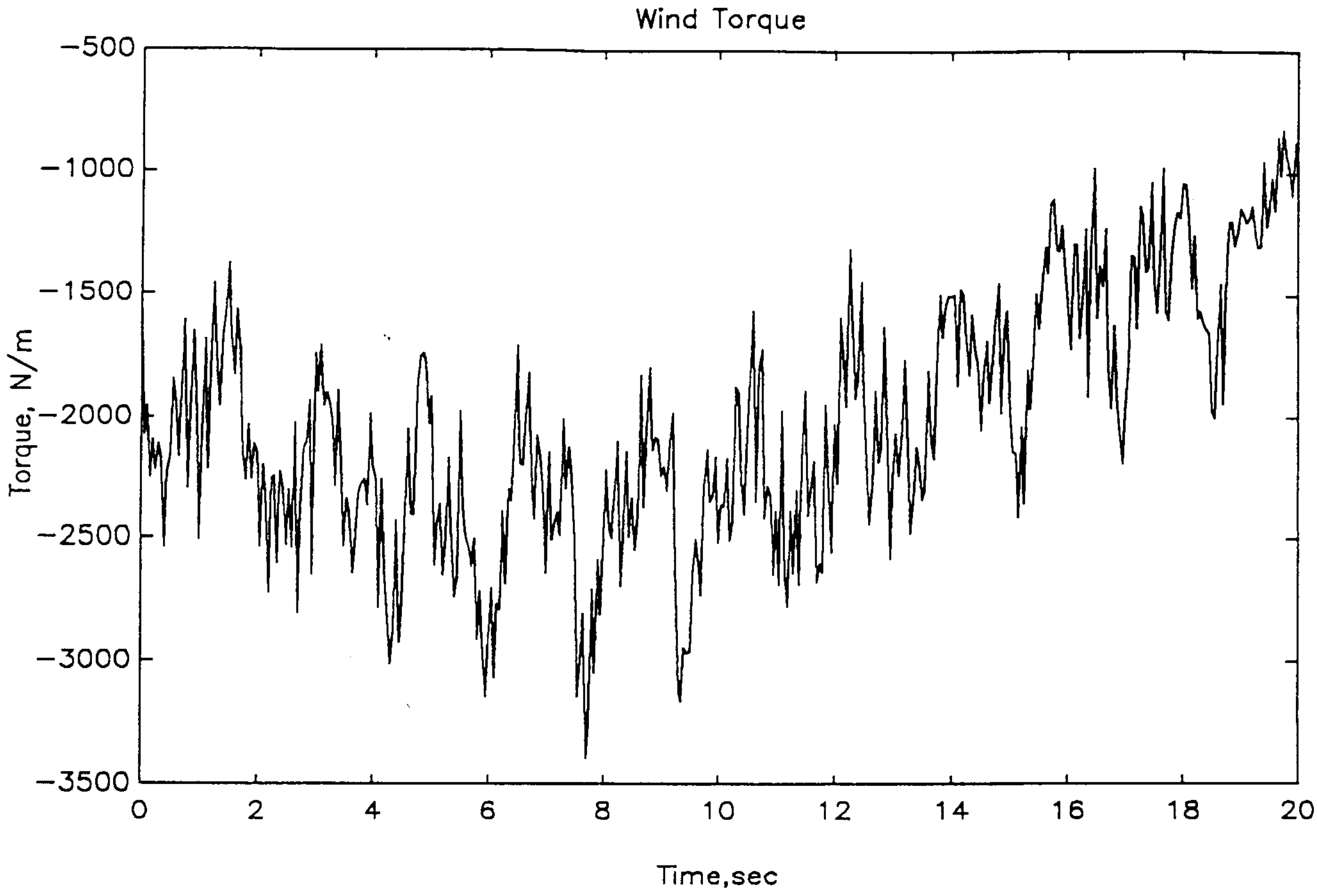


Figure 3 Torque from the 9 m/s Wind

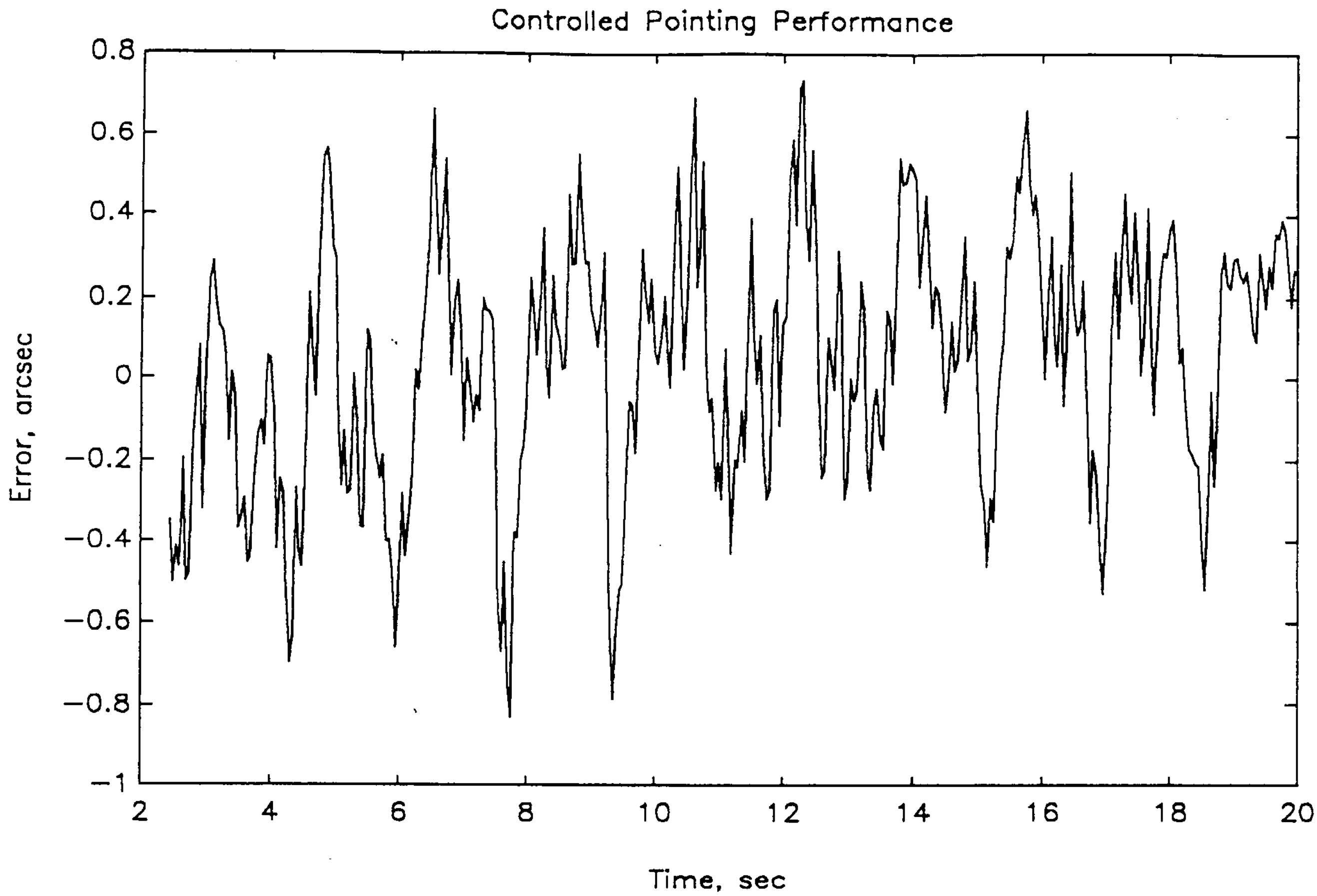


Figure 4 Performance of the Controlled Portion of the Structure Subject to Simulated 9 m/s Wind

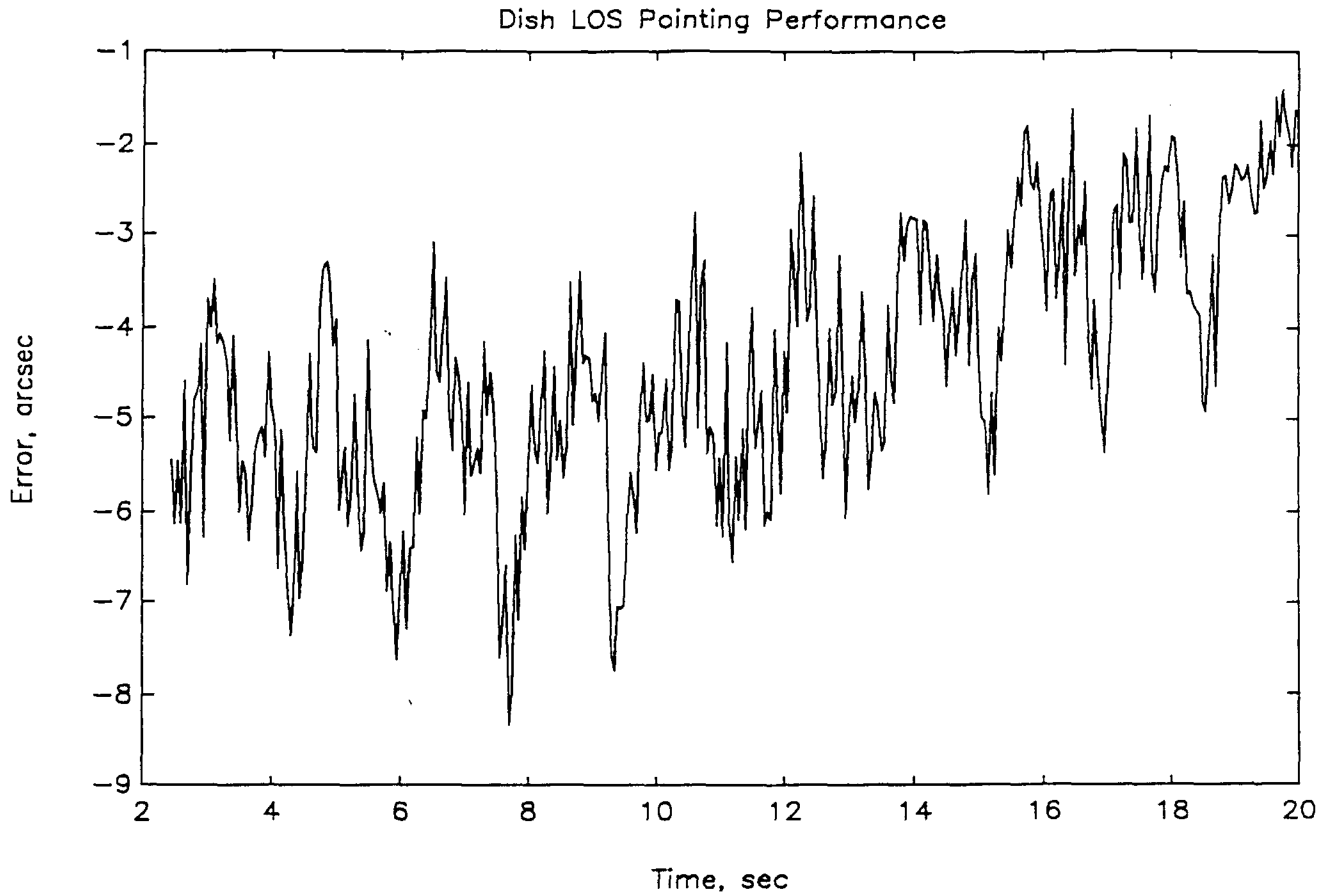


Figure 5 Performance of the Dish LOS Subject to Simulated 9 m/s Wind

5.0 DISCUSSION AND RECOMMENDATIONS

There are several issues that suggest that a nonlinear analysis is in order. First the combination of the wind and the fairly large static torque indicate that some limit cycle behavior might occur. Second the results of the optimization attempt indicate that the model is hard against a mathematical transition of sorts, small changes in the control coefficients cause large migrations in poles. The existence of nonlinearities could accentuate this, resulting in unpredictable, and possible sizable tracking errors.

The calculation that resulted in the dish wind up prediction was based on the assumption described above, essentially that a purely rotational spring constant represented the first mode. If this is not true, that is that the first mode is not the whole dish twisting, then the expected wind-up would be lower. Thus this has to be re-examined in this light. It would also be a good idea to look at some methods to actively reduce the wind-up if it turns out that it is real.

Finally the wind up should be viewed for what it is, a contribution to the pointing error budget. To the extent that we cannot measure and/or compensate for it, we will have a pointing error that varies with wind speed and direction.