Problem Set 6

Astronomy 208

Distributed: December 10, 1996; *Due:* January 14, 1997

MASERS

This problem set is based on the lecture presented on 12/5/96 by Dr. Lincoln Greenhill (lgreenhill@cfa.harvard.edu).

Two reference books for topics related to masers are, "Astronomical Masers," by Elitzur (1991, Kluwer); and "Astronomical Masers," by Reid & Moran, in *Galactic and Extragalactic Radio Astronomy,* Verschuur & Kellermann eds., (1988, Springer-Verlag), Chap. 6. However, the assigned problem can be completed by referring to the class notes alone.

For this multi-part problem, use the approximation that the angle averaged specific intensity of radiation in a maser, J_ν, is $\frac{I_\nu \Omega_b}{4\pi,}$ where Ω_b is the solid angle into which maser emission is beamed. Remember that radiative transfer of maser radiation operates in the Rayleigh-Jeans limit. You will also need certain molecular constants. To simplify calculations, just consider the characteristics of water, where $\Gamma \approx 1 \text{ s}^{-1}$, and $A = 2 \times 10^{-9} \text{ s}^{-1}$.

1. Consider a filamentary maser of length ℓ, diameter d, at a distance D from earth. The observed flux density of the maser is F_ν.

   a. Write an expression for the brightness temperature of the maser, and a condition for whether or not it is saturated, in terms of F_ν.

   b. Now consider two cases in which a maser "cloudlet" amplifies a background source but the cloud is sufficiently far from us that by itself the maser is too weak to see. The diameter of the cloudlet on the sky is d_m and the diameter of the background source is d_n. The cloudlet and background are separated by a distance $D_{nm} \gg d_n, d_m$ and $D_{nm} \ll D$.

      For $d_m > d_n$, what can you say about the perceived diameter of the detected maser emission with respect to d_m and d_n? The solid angle of the maser emission is no longer set by ℓ and d, as it was before. Give an approximate expression for Ω_b.

      For $d_m < d_n$, what is the diameter of the detected maser emission? Is Ω_b larger or smaller than in the previous case?

   c. Consider a maser filament of length ℓ in isolation. If ℓ is not so very large then the maser will be unsaturated. Otherwise the maser naturally generates enough specific intensity (I_ν) that it saturates toward its two ends. Symmetry arguements lead one to hypothesize the presence of an unsaturated core at the middle of the filament's length. Now a background source is placed along the axis of the maser and its flux ramps up from zero. (Assume it has the same diameter as the maser.)

      What happens to the location of the unsaturated core? Eventually the unsaturated core disappears. At this point how much greater has I_ν increased due to the background source? (Hint: Think about how radiation grows in a saturated maser.)

d. *Herrnstein needs help with his thesis!* As he writes that the NGC4258 masers are saturated he has second thoughts. But could they be too bright to be saturated as he assumes? (This problem applies to disks around protostars as well as those around blackholes.)

For an edge-on thin disk strong maser emission is visible in projection along the diameter perpendicular to the line of sight. (Call this diameter the "midline" of the disk). Consider a central mass M and a mass-less disk of outer radius $R_\circ$, at which the rotation velocity $v(R_\circ)$ is $V_\circ$. The maser linewidth is Δv. What is the line-of-sight path length of the maser emission as a function of projected radius along the midline? Divide the maser emission along the midline among many parallel filaments whose lengths you just computed. What is the filament diameter as a function of projected radius? (Hint: Consider the effects of velocity shear in the disk, with respect to Δv.)

Now address Herrnstein's fright. What is an expression for the maximum flux density of saturated maser filaments expressed as a function of projected radius along the midline? The observed maser lines are $0.1-0.6$ Jy. Evaluate the order of magnitude of this limit using the following assumptions: a distance of 7 Mpc, a central mass of 4×10^7 $\mathrm{M}_\odot$, $\Delta v = 1$ km s^{-1}, $\mathrm{n}_{H_2} = 10^{10}$ cm^{-3} uniformly through the disk, $\frac{n_{\mathrm{H_2O}}}{n_{H_2}} = 3\times10^{-5}$, pump efficiency of 1%, and that 1 % of the water molecules are in the upper and lower maser levels together. (This is appropriate for typical temperatures of about 400 K.)