MEMORANDUM
To: Dan Fabricant
From: Warren Brown
Date: 23 May 2002
Subject: Binospec Thermal Analysis VI: Thermal Stresses in Binospec Lenses

1. INTRODUCTION

In this memo I calculate the temperature profile and the thermal stress in a lens suddenly immersed into a different temperature environment. The motivation is to understand the thermal shock a lens will experience, for example, when a spectrograph is opened and exposed to a cold or hot environment. I look at two lens materials with high CTE: CaF_2 and S-FPL51Y.

2. TEMPERATURE PROFILE

2.1. Set-up

I model the lens as a cylinder with the rough mass and dimensions of the Binospec CaF_2 elements: radius = 0.15 m, thickness = 0.05 m, and mass ≈ 12 kg. I assume that the top, bottom, and sides of the lens are exposed to the environment. Because we are interested in temperature gradients near the surface of the lens, I make a detailed finite element model of the lens. Figure 1 shows the layout of the finite element model. There are 11 radial divisions and 6 to 24 angular divisions in the lens. The divisions are chosen to keep the finite element masses the same to within 25%. There are 10 axial divisions, of height 0.005 m, for a total of 2110 elements.

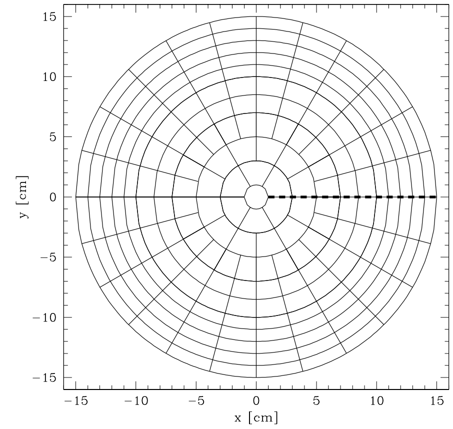


Fig. 1.— Layout of the finite element model. The heavy dashed line marks the slice used for the temperature profiles in Figures 2 to 5.

The calculations assume that the lens starts with a uniform temperature and is suddenly placed into 10 °C colder environment. I use two environments: air, with convection coefficient $h = 6$ W m^{-2} K^{-1}, and, for comparison, water. When calculating the conduction rate from the water environment, I set the conductance length equal to the finite element height. Table 1 summarizes the material properties I used in the calculations.

TABLE 1
Material Properties

Material	ρ kg/m^3	k W/m K	C J/kg K	E N/m^2	CTE m/m K	ν	Reference
CaF_2	3180	9.71	854	75.8×10^9	18.9×10^{-6}	0.26	Optovac
S-FPL51Y	3640	0.780	640	71.6×10^9	13.4×10^{-6}	0.30	Ohara
Air, 2500m	0.95	0.024	1007				NRAO
Water, 275K	1000	0.574	4211				Incropera & DeWitt

2.2. Results

Figures 2 - 5 plot the temperature profiles for the thermal models. Figures 2 and 3 are for a lens immersed into a 10 °C colder air environment; Figures 4 and 5 are for a lens immersed into a 10 °C colder water environment.

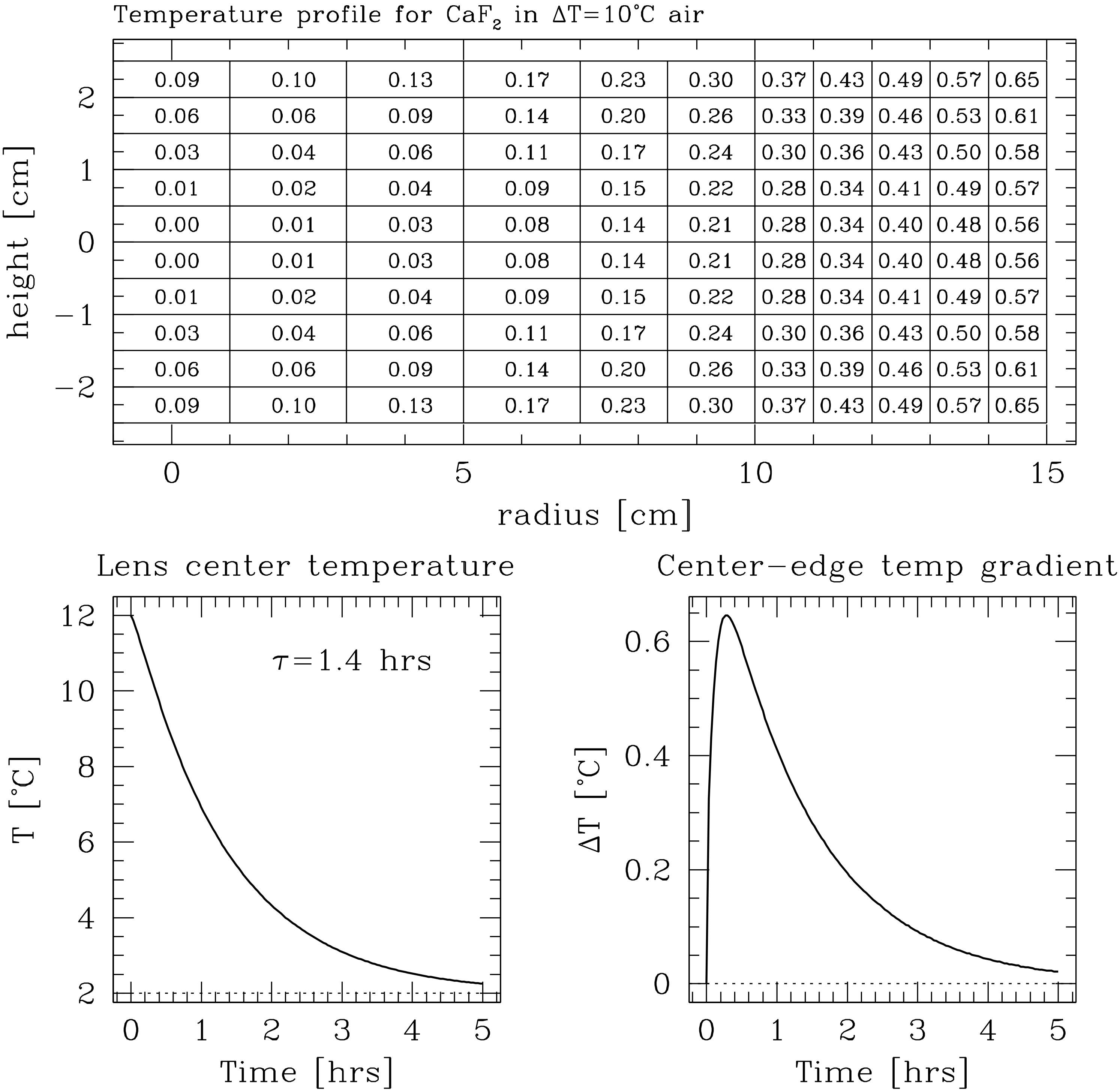
The top panels in Figures 2 - 5 show the finite element temperatures (°C) for the lens cross-section marked in Figure 1. The finite element temperatures are shown at the time of the maximum center-to-edge temperature gradient. The lower right panel of each Figure plots the center-to-edge temperature gradient with time. The time and the magnitude of the maximum center-edge temperature gradients are summarized in Table 2.

Because S-FPL51Y is 12 times less conductive than CaF_2, heat propagates more slowly through the S-FPL51Y lens and the center-to-edge temperature gradient is many times larger than for CaF_2. This is a general result true for all of the Ohara I-line glasses in comparison to CaF_2. I point out that a convex lens will produce more extreme center-to-edge temperature gradients than the cylinder model studied here.

The lower left panel in Figures 2 - 5 plots the temperature of the center node with time. The *e*-folding time constants for the lens centers to equilibrate with the environment are summarized in Table 2. Again, because the Ohara glass is less conductive than the CaF_2 crystal, the time constants are longer for S-FPL51Y.

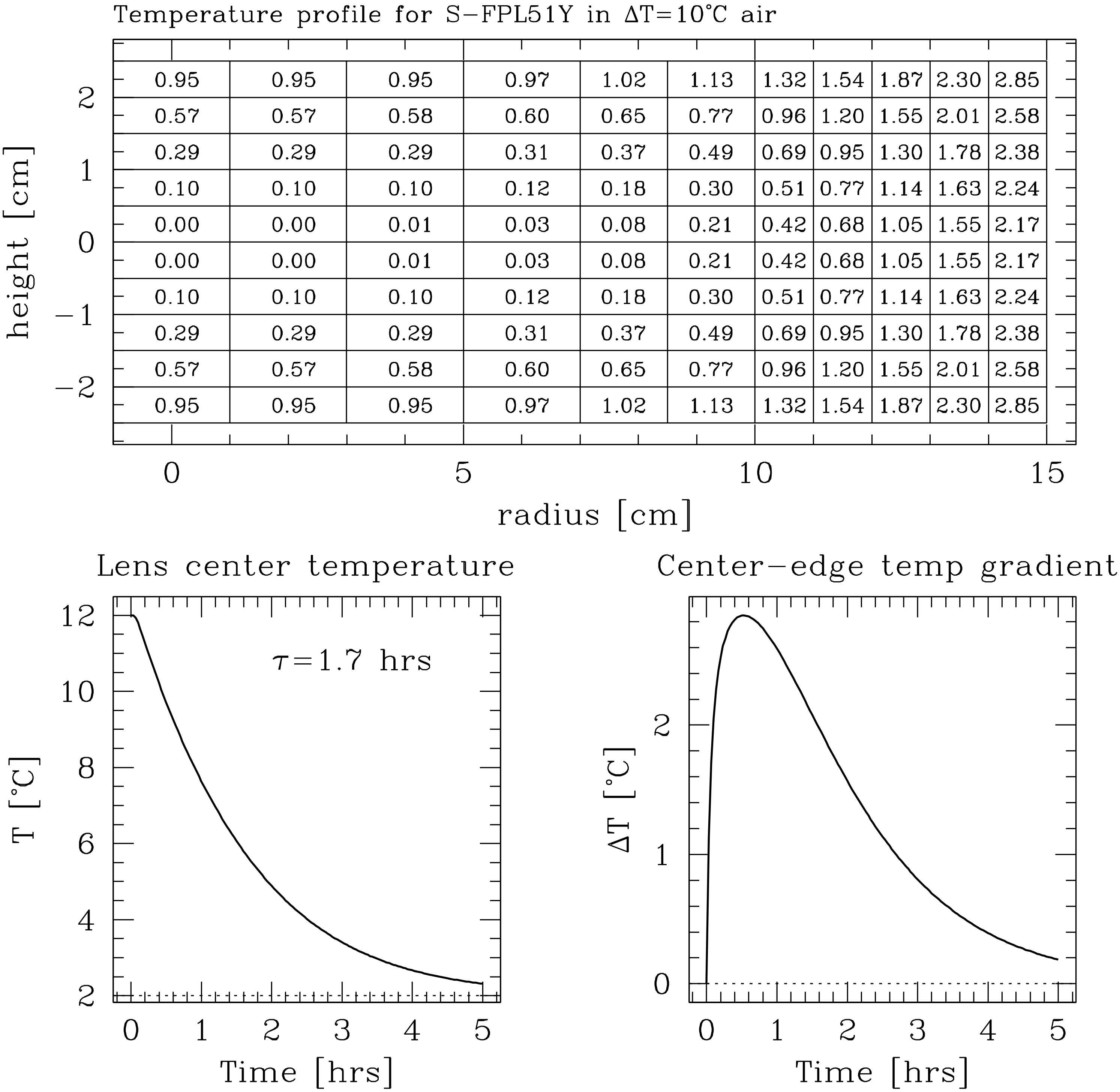
TABLE 2
Temperature Profile Results

Environment	Lens	$\Delta T_{\max}$ °C	$t(\Delta T_{\max})$ min	τ_{center} hr
air	CaF_2	0.65	16	1.4
	S-FPL51Y	2.9	32	1.7
water	CaF_2	2.7	4	0.2
	S-FPL51Y	8.0	4	0.3



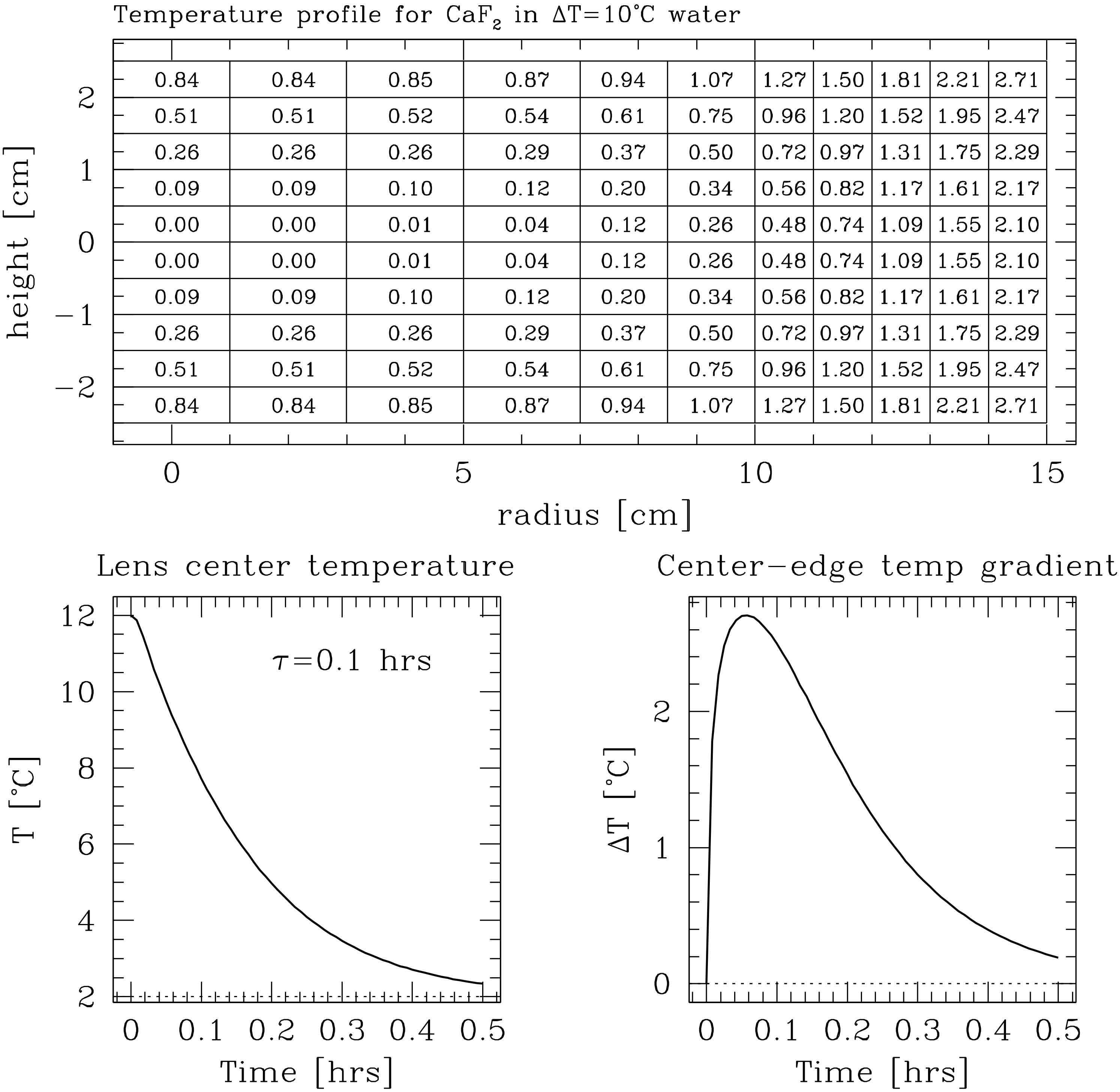
0.09	0.10	0.13	0.17	0.23	0.30	0.37	0.43	0.49	0.57	0.65
0.06	0.06	0.09	0.14	0.20	0.26	0.33	0.39	0.46	0.53	0.61
0.03	0.04	0.06	0.11	0.17	0.24	0.30	0.36	0.43	0.50	0.58
0.01	0.02	0.04	0.09	0.15	0.22	0.28	0.34	0.41	0.49	0.57
0.00	0.01	0.03	0.08	0.14	0.21	0.28	0.34	0.40	0.48	0.56
0.00	0.01	0.03	0.08	0.14	0.21	0.28	0.34	0.40	0.48	0.56
0.01	0.02	0.04	0.09	0.15	0.22	0.28	0.34	0.41	0.49	0.57
0.03	0.04	0.06	0.11	0.17	0.24	0.30	0.36	0.43	0.50	0.58
0.06	0.06	0.09	0.14	0.20	0.26	0.33	0.39	0.46	0.53	0.61
0.09	0.10	0.13	0.17	0.23	0.30	0.37	0.43	0.49	0.57	0.65

Fig. 2.— CaF$_2$ lens placed in a 10°C colder air environment. The top panel presents temperatures (°C) for each element in the lens cross-section, at the time of the peak center-edge gradient. The lower left panel plots the temperature of the center node with time. The lower right panel plots the center-to-edge temperature gradient with time.



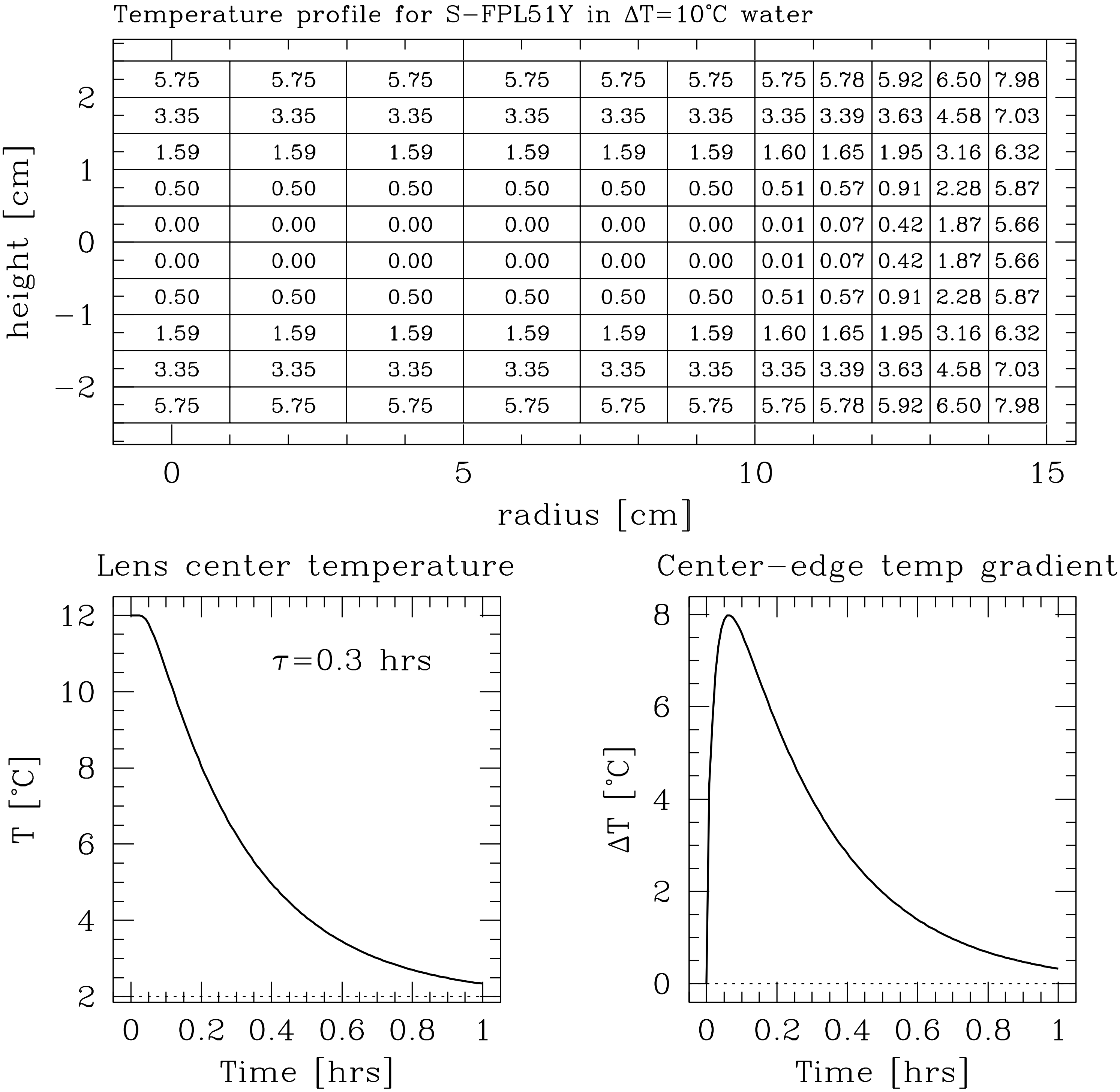
0.95	0.95	0.95	0.97	1.02	1.13	1.32	1.54	1.87	2.30	2.85
0.57	0.57	0.58	0.60	0.65	0.77	0.96	1.20	1.55	2.01	2.58
0.29	0.29	0.29	0.31	0.37	0.49	0.69	0.95	1.30	1.78	2.38
0.10	0.10	0.10	0.12	0.18	0.30	0.51	0.77	1.14	1.63	2.24
0.00	0.00	0.01	0.03	0.08	0.21	0.42	0.68	1.05	1.55	2.17
0.00	0.00	0.01	0.03	0.08	0.21	0.42	0.68	1.05	1.55	2.17
0.10	0.10	0.10	0.12	0.18	0.30	0.51	0.77	1.14	1.63	2.24
0.29	0.29	0.29	0.31	0.37	0.49	0.69	0.95	1.30	1.78	2.38
0.57	0.57	0.58	0.60	0.65	0.77	0.96	1.20	1.55	2.01	2.58
0.95	0.95	0.95	0.97	1.02	1.13	1.32	1.54	1.87	2.30	2.85

Fig. 3.— S-FPL51Y lens placed in a 10°C colder air environment. The top panel presents temperatures (°C) for each element in the lens cross-section, at the time of the peak center-edge gradient. The lower left panel plots the temperature of the center node with time. The lower right panel plots the center-to-edge temperature gradient with time.



0.84	0.84	0.85	0.87	0.94	1.07	1.27	1.50	1.81	2.21	2.71
0.51	0.51	0.52	0.54	0.61	0.75	0.96	1.20	1.52	1.95	2.47
0.26	0.26	0.26	0.29	0.37	0.50	0.72	0.97	1.31	1.75	2.29
0.09	0.09	0.10	0.12	0.20	0.34	0.56	0.82	1.17	1.61	2.17
0.00	0.00	0.01	0.04	0.12	0.26	0.48	0.74	1.09	1.55	2.10
0.00	0.00	0.01	0.04	0.12	0.26	0.48	0.74	1.09	1.55	2.10
0.09	0.09	0.10	0.12	0.20	0.34	0.56	0.82	1.17	1.61	2.17
0.26	0.26	0.26	0.29	0.37	0.50	0.72	0.97	1.31	1.75	2.29
0.51	0.51	0.52	0.54	0.61	0.75	0.96	1.20	1.52	1.95	2.47
0.84	0.84	0.85	0.87	0.94	1.07	1.27	1.50	1.81	2.21	2.71

Fig. 4.— CaF_2 lens placed in a 10°C colder water environment. The top panel presents temperatures (°C) for each element in the lens cross-section, at the time of the peak center-edge gradient. The lower left panel plots the temperature of the center node with time. The lower right panel plots the center-to-edge temperature gradient with time.



5.75	5.75	5.75	5.75	5.75	5.75	5.75	5.78	5.92	6.50	7.98
3.35	3.35	3.35	3.35	3.35	3.35	3.35	3.39	3.63	4.58	7.03
1.59	1.59	1.59	1.59	1.59	1.59	1.60	1.65	1.95	3.16	6.32
0.50	0.50	0.50	0.50	0.50	0.50	0.51	0.57	0.91	2.28	5.87
0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.07	0.42	1.87	5.66
0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.07	0.42	1.87	5.66
0.50	0.50	0.50	0.50	0.50	0.50	0.51	0.57	0.91	2.28	5.87
1.59	1.59	1.59	1.59	1.59	1.59	1.60	1.65	1.95	3.16	6.32
3.35	3.35	3.35	3.35	3.35	3.35	3.35	3.39	3.63	4.58	7.03
5.75	5.75	5.75	5.75	5.75	5.75	5.75	5.78	5.92	6.50	7.98

Fig. 5.— S-FPL51Y lens placed in a 10°C colder water environment. The top panel presents temperatures (°C) for each element in the lens cross-section, at the time of the peak center-edge gradient. The lower left panel plots the temperature of the center node with time. The lower right panel plots the center-to-edge temperature gradient with time.

3. THERMAL STRESS

We calculate thermal stresses using formulas from Roark & Young's *Formulas for Stress and Strain.* Bob Fata provided the integrated stress formulas for a disk with a uniform *radial* temperature gradient:

$$\sigma_r = 1/3 \text{ CTE } E \ \Delta T \ (R - r)/R \tag{1}$$

$$\sigma_t = 1/3 \text{ CTE } E \ \Delta T \ (R - 2r)/R. \tag{2}$$

σ_r and σ_t are the stresses in the radial and tangential directions, CTE is the coefficient of thermal expansion, E is Young's modulus, ΔT is the radial temperature gradient, and R is the outer radius. Note that both the radial and tangential stresses are a maximum in the lens center ($r = 0$). Bob Fata has verified the accuracy of Equations 1 and 2 with a finite element model.

The stress of a disk with a uniform *axial* temperature gradient, constrained around the edge, is:

$$\sigma = 1/2 \text{ CTE } E \ \Delta T/(1 - \nu), \tag{3}$$

where σ is the total stress and ν is Poisson's ratio. Note that if there is no edge constraint around a lens, then there is no stress from an axial temperature gradient (only free expansion).

Table 3 summarizes the thermal stress for the 4 temperature profiles shown in Figures 2 - 5. ΔT_{z} and ΔT_{r} are the maximum axial and radial temperature gradients, respectively, experienced by the lenses. $\sigma_{\mathrm{z\ max}}$ and $\sigma_{\mathrm{r\ max}}$ are the maximum stresses in the lens for these temperature gradients. In the radial temperature gradient case, $\sigma_{\mathrm{r\ max}}$ is the sum of σ_r and σ_t at $r = 0$. There are both axial *and* radial temperature gradients in the thermal profiles, so for my models the net stress is the sum of the axial and radial σ_{max}'s.

According to Bob Fata, the maximum stress allowed on a CaF_2 lens is 100 psi. The maximum stress allowed on an Ohara I-line lens is 500 psi. The % columns in Table 3 show the percentage of the maximum allowable stress the σ_{max}'s represent. Interestingly, even though S-FPL51Y experiences larger thermal stresses than CaF_2, the S-FPL51Y thermal stresses are a smaller percentage of the overall stress budget than for CaF_2.

TABLE 3
Thermal Stress

Environment	Lens	ΔT_{z} °C	$\sigma_{\mathrm{z\ max}}$ psi	%[a]	ΔT_{r} °C	$\sigma_{\mathrm{r\ max}}$ psi	%[a]
air	CaF_2	0.1	14	14	0.6	84	84
	S-FPL51Y	1.1	110	20	2.8	260	50
water	CaF_2	1.0	140	140	2.7	370	370
	S-FPL51Y	5.7	570	110	8.0	740	150

[a] Percentage of the maximum allowable stress for a CaF_2 lens (100 psi) and a S-FPL51Y lens (500 psi).

4. SUMMARY

I have calculated the temperature profile and thermal stress for a cylindrical lens suddenly immersed into a 10 °C colder environment. If the environment is air with convection coefficient $h = 6$ W m^{-2}, I find that a CaF$_2$ lens will equilibrate with a time constant of 1.4 hours. The peak 0.65 °C center-to-edge temperature gradient results in $\sim$100 psi of thermal stress, which is the allowable stress for a CaF$_2$ lens. A S-FPL51Y lens in the same environment will equilibrate with a time constant of 1.7 hours. Its peak 2.9 °C center-to-edge temperature gradient results in $\sim$400 psi of thermal stress, which is 80% of the allowable stress for an S-FPL51Y lens. All other Ohara I-line glasses have $\sim$1/2 the CTE value of S-FPL51Y, and so would experience half the thermal stress of the S-FPL51Y lens.

When I immerse the lenses into a 10°C colder water environment, the time constants of equilibration are reduced to $\sim$15 minutes. Correspondingly, the temperature gradients and thermal stresses increase $\sim$5 times over those for the air environment. These thermal stresses exceed the allowable stress on the CaF$_2$ lens and the S-FPL51Y lens many times over.

If I were to alter the boundary condition of the thermal model and assume the lens sides were protected from the environment (i.e. by the surrounding bezel), the radial temperature gradients would disappear. The value of the maximum axial temperature gradients, however, will remain very similar to the values listed in Table 3. Thus the total thermal stress would simply be $\sigma_{\rm z\ max}$ for a lens with protected sides.

4.1. Conclusion

Binospec-sized lenses take a couple hours to equilibrate to their environment. Temperature changes should be limited to a few °C to avoid exessive thermal stresses.