Ay 192 **Principles of Astronomical Measurement**

External Signal Degredation

I. The Atmosphere

(a) Absorption

(b) Dispersion/Refraction

(c) Emission

(d) Scattering, Polarization

(e) Turbulence - "Seeing," Image Degredation, Phase Delays

Units — Absorption	OIR	magnitudes
	Radio	dB (decibels)

(1 db = 0.25 magnitude = 10 × log = 4.34 nepers (optical depth))

per airmass or pathlength or mm of H_2O.

1 Neper = factor of 1/e.

II. Airmass

(a) Plane-parallel approximation

x z 1

0

$$Airmass = X = \frac{1}{cos\ z} = sec\ z$$

(b) Corrected for the fact that the Earth is round, with $A = (sec\ z - 1)$

$$X = sec\ z - 1.8167 \times 10^{-3} A - 2.875 \times 10^{-3} A^2 - 8.083 \times 10^{-4} A^3$$

— more than you'll ever need. X also depends on barometric pressure, temperature, etc.

$$sec\ z \;=\; (sin\ \phi \;\; sin\ \delta + cos\ \phi \;\; cos\ \delta\ cos\ h)^{-1}$$

where ϕ is the observers latitude, δ is the objects declination, and h is the objects hour angle.

III. Refraction, Differential Refraction (or Dispersion)

(a) Refraction in the atmosphere can be calculated if we know the index of refraction of air. For dry air at 15°C, p = 760 Torr,

$$(\eta - 1) \times 10^6 \;=\; 64.33 + \frac{2.95 \times 10^4}{146 - (1/\lambda)^2} + \frac{255.4}{41 - (1/\lambda)^2}$$

with λ in μ. Refraction also depends on pressure:

$$(\eta_{T,P} - 1) \;=\; (\eta_{15,760)} - 1)\Big[\frac{P[1 + (1.049 - 0.0157T) \times 10^{-6}P}{720.883(1 + 0.003661T)}\Big]$$

where T is in °C, P in Torr. H_2O *reduces* refraction in the optical. At radio wavelengths,

$$(\eta_{T,P,f} - 1) \times 10^6 \;\approx\; 78P/t + 3.9 \times^5 f/T^2$$

where P is now in millibars, T in °K, and f is the atmospheric H_2O content (partial pressure) in millibars. 1 mb = 1.02 g/cm^2; 760 Torr = 1013.25 mb. These corrections are extremely important for the pointing of modern large telescopes (with small fields of view).

(b) Differential Refraction

1. "Spatial" effects — for large area detectors (eg. photographic plates on Schmidt telescopes) you can get elongated images because as you guide (integrate), the airmass at the top and bottom of the plate changes by different amounts. This is not yet a problem for CCD's, but does show up on 6° field Schmidt plates taken at large airmasses.

Atmospheric refraction is usually quoted as

$$R \;=\; z_t \;-\; z_a \;\approx\; R_0 tan z_a - 0.''067 tan^3 z_a$$

z_t is the true zenith distance in degrees and z_a is the refracted zenith distance in degrees.

$$R_0(760\ Torr, 0°C) \;=\; 60.''3 = (n^2 - 1)/2n^2 \;\; \text{at V}$$

An approximation formula for the effects of temperature an pressure is

$$R_{T,P} \;=\; R_0\Big(\frac{P}{760 + 2.9T}\Big)$$

There is also some dependence on H_2O content.

Atmospheric refraction is along a line from the zenith through the object to the horizon, which is generally only along a line of celestial coordinates (declinatiion) when the object is on the meridian and the refraction is aligned north-south. Otherwise, the line of refraction makes and angle with the celestial coordinates known as the parallactic angle, η,

$$\sin\eta \; = \; \frac{\sin h \, \cos\phi}{[1 - (\sin\phi\sin\delta \; + \; \cos\phi\cos\delta\cos h)^2]^{1/2}}$$

where h and δ are the object's hour angle and declination and ϕ is the observers latitude (as above). (See A. Filippenko 1982, PASP 94, 715).

2. Dispersion effects

The index of refraction is a function of wavelength.

Table 1 $\mathbf{R_0}$ versus λ

λ	R_0
3000 Å	63."42
3500 Å	62."22
4500 Å	60."94
5500 Å	60."38
6500 Å	60."07
7500 Å	59."86
8500 Å	59."72

In the radio, the frequency dependence is a strong function of the partial pressure of H_2O. For f = 10 mm, R_0 = 73."2.

IV. Scattering

There are several differenct scattering processes that occur in the atmosphere as a result of the mixture of sizes of particles which range from molecules to particulates, ice crystals and water droplets.

Rayleigh scattering occurs when $\lambda >>$ particle size.

$$\sigma_R \; = \; (4\pi^2 N V^2/\lambda^4)\frac{(n^2 - n_D^2)^2}{(n^2 + 2n_D^2)^2}$$

where

$$N \; = \; \text{particle density (cm}^{-3})$$
$$V \; = \; \text{volume of particle (cm}^3)$$
$$\lambda \; = \; \text{wavelength of light (cm)}$$

$$n_D \;=\; \text{refractive index of medium}$$

$$n \;=\; \text{refractive index of particles}$$

and σ is in units of "scattering" per cm

Mie scattering occurs when $\lambda \sim$ particle size, and depends on the particle cross-section ka^2,

$$\sigma_M \;=\; Nk\pi a^2,$$

where a is the particle radius in cm and k varies between 0 and 4 (usually ~ 2).

Nonselective scattering occurs when $\lambda <<$ particle size, and is the combination of reflection, refraction and diffraction. The scattering process goes as:

λ^{-4} for Rayleigh

λ^{-4} for Mie (small particles)

λ^{-2} for Mie (d $=\lambda$)

λ^{-1} for Mie (d $=\frac{3}{2}\lambda$)

λ^{0} for Mie (d $=2\lambda$)

V. Absorption

Absorption in the optical and IR is mostly due to molecules and some particulates (ice). The most commmon molecules seen in telluric bands are O_2, O^+, H_2O, CO_2, and O_3. Molecular absorption is responisble for the Fraunhofer A and B bands. Continuum absorption increases dramatically when there is significant volcanic activity in the latitude zone of your observatory, and also when the inversion layer rises above the observatory altitude bringing smog and other heavy particulates. Water vapor absorption is primarily low lying (0-15,000 ft) which is why a premium is placed on altitude for IR observing (Mauna Kea, balloons, satellites).

Absorption in the radio is caused by free electrons, ions and neutral particles in the ionosphere. Radio propagation is affected by refraction, phase delays, group delays, polarization and absorption in the **F** regions. The **D** region, which is mostly neutral, causes absorption.

Ultraviolet radiation below 3200 Å is essentially completely absorbed in the Ozone layer (20-30 km). X-rays and γ-rays are also completely absorbed, even at mountain altitudes, but Cosmic ray particles do make it through.

Absorption due to atomic or molecular lines follows tha same behavior as seen for lines in astrophysical processes (eg. stellar atmospheres). The line profile is a combination of the natural Lorentzian profile,

$$k_\nu \;=\; \frac{S}{\pi}\frac{\alpha}{(\nu-\nu_0)^2+\alpha^2}$$

plus doppler broadening,

$$k_\nu \ = \ \frac{S}{\Delta\nu}\Big(\frac{ln2}{\pi}\Big)^{1/2} \ exp\Big[-\frac{ln2}{\Delta\nu_D^2}(v-v_D)^2\Big]$$

where

$$\alpha \ = \ \text{line natural half width}$$
$$\nu_0 \ = \ \text{line center wave number}$$
$$S \ = \ \text{total line intensity}$$
$$\Delta\nu_D \ = \ \frac{\nu_0}{c}(\frac{2kT}{m}\ ln2)^{1/2} \ = \ \text{doppler line width,}$$

and m = the particle (molecule) mass.

Note that this combination of Doppler core plus Lorentzian wings to describe absorption over a finite band produces the familiar and useful "Curve of Growth"

$A\Delta\nu \ \propto \ 2\pi\alpha\ x$ for small x (path length or line strength)

$A\Delta\nu \ \propto \ log\ x$ for saturated cores, and

$A\Delta\nu \ \propto \ \sqrt{x}$ when wings begin to grow.

Structure of the Ionosphere

Height (km)		Zone
1000		
		F2
180		
		F1
120		
	x-rays, Lyβ on O_2	**E**
90		
	Mostly Neutral	**D**
30		
	Ozone Layer	
20		

Absorption in the radio behave essentially like absorption in a plasma.

$$L(in\ db) \approx \frac{1.16 \times 10^{-6}}{(f \pm f_L)^2} \int_0^{\infty} n\nu \frac{dh}{cos\ i}$$

where

$$\begin{aligned} n &= \text{electron density,} \\ \nu &= \text{collision frequency, and} \\ f &= \text{the plasma frequency } \tfrac{w_p}{2\pi} \approx 8.9\sqrt{n}. \end{aligned}$$

At the zenith, at 100 MHz, L is approximately 0.05 db during the day, and 0.005 db at night.

Remember, the total atmospehric "extinction" is the combination of absorption and scattering. Absorption dominates in the IR, while scattering dominates in the UV.

VI. Atmospheric Emission

The sky brightness in the optical is a combination of emission and scattering from a variety of sources. The base component, which we can't get away from, *even* in earth orbit, is the Zodiacal Light. This is scattering from dust in the solar system and is mostly concentrated in the ecliptic plane. There is an especially bright spot called the Gegenschein or counterglow, which is forward scattering of sunlight by particles in the ecliptic directly opposite the Sun. The Zodiacal light is polarized by up to 23%.

In addition to the Zodiacal light, the night sky continuum also contains scattered (and direct!) starlight — diffuse emission from the galactic plane plus individual nearby stars. The first is concentrated in a band of about $\pm\ 10°$ around the galactic plane, the second is fairly isotropic. Other major components of the continuum is light from the Sun that is directly scattered in the atmosphere (multiple scatterings around the earth), moonlight when the Moon is up, and scattered terrestrial light sources, although nowadays city light (from streetlighting) is mostly concentrated in certain bands.

The optical and infrared night sky is also filled with emission lines and bands, some of which are auroral (due to recombination in the upper atmosphere) and some of which are man made. The strengths of these lines are both position and time dependent — that is to say, the auroral lines decrease in intensity to lower geographic latitude and increase with solar activity, the intensity of man-made lines depends on where the nearest cities are and what is their major nighttime light source. A partial list of prominent optical night sky lines is given below.

Further into the near infrared, the emission quickly becomes dominated by OH bands (in particular the Meinel bands), and is extremely variable both temporally and spatially. That is to say, fluctuations of several % on timescales of seconds and spatial scales of arc-minutes are not uncommon. There are a few papers by

Osterbrock and collaborators detailing the use of the OH emission line spectrum to serve as a wavelength calibration (PASP 108, 277 (1996); PASP 109, 614 (1997); and PASP 110, 1499 (1998)).

Even further into the IR, past $\sim 2\mu$, which is normally refered to as the *thermal* IR, the sky background is composed of both thermal emission (since the atmosphere is not perfectly transparent, it ansorbs and emists radiation) and molecular band emission — in particular due to H_2O.

Table 2a Components of the Light of the Night Sky

Component	Origin	Function of:
Integrated Starlight	Galactic Stars	Galactic Coords
Zodiacal Light	Scattered Sunlight from Interplanetary Dust	Ecliptic Coords
Airglow	Ambient Excitation of upper atmospheric atoms and moleculres	Time, Latitude, Season, Altitude, Solar Activity
Aurora	Excitation of upper Atmosphere by particles Solar Wind, etc.	Magnetic latitude, Solar Activity, Magnetic Activity
Diffuse galactic Light	Scattering off Interstellar Dust	Galactic Coords
Cosmic Background	High-z Galaxies, Dust, ++	Galactic Coords

After Roach and Gordon (1973)

Estimates of the above backgtound components are given in the following table.

Table 2a Components of the Light of the Night Sky

Component	Typical Zenith	Minimum
Integrated Starlight	105	30
Zodiacal Light	125	85
Airglow	45	45
Diffuse galactic Light	20	10
Cosmic Background	1	0
Total	315	175

In S_{10} units in the V band.

Table 3 Night Sky Emission Lines

λ	Species
3654.8	Hg I
3888.7	Ne
4046.6	Hg I
4099.9	N I
4358.4	Hg I
5460.7	Hg I
5577.4	[O I]
5769.6	
5790.7	Hg
5890.4	Na I
6300.2	[O I]
6363.8	[O I]
6829.9	
6862.4	

The sky background longwards of about 7200 Å**is** significantly better in space than on the ground:

Table 4 Sky Brightness

Place	U 3600Å	B 4400Å	V 5500Å	R 7000Å	I 8200Å	I' 1 μ
Ground	22.0	22.6	21.8	21.0	20.1	18.4
Space	$\sim$ 24.0	$\sim$24	24.2	23.7	23.3	$\sim$23.0

in magnitudes per □".

A good reference for this is Roach and Gordon. Remember for conversion purposes, 1 $S_1 0 = 4.4\times10^{-3}$ Rayleighs per Å $= 1.3\times10^{-9}$ ergs cm^{-2} s^{-1} ster^{-1} Å^{-1}. And finally, 21 m$_V$ /□" $\sim$ 2 Rayleighs/Å(Visible magnitudes per square arcseconds). A more recent reference is Leinert et al. 1998, A&AS Supp 127, 1.

Much further to the infrared, one also needs to worry about atmospheric and Zodiacal emission. Good numbers for the "Zodi" emission in the IR come from IRAS and COBE maps. The IRAS data show that the mean temperature of the solar system dust is about 225°K, and that its well fit by a blackbody between 10 and 100 microns. The mean thermal Zodi emission is about 10^{-21} Watts/cm^2/μ/sq-arcsec at 25 microns.

VII. Atmospheric Turbulence

"Seeing" is produced by scintillation plus atmospheric turbulence. It is affected by the presence of surface turbulence (on the ground, on the surfaces of the dome and telescope, etc), the height of the atmospheric inversion layer, high altitude winds, etc. The selection of a "good" astronomical site has become a combination of both science and art.

In the old days, seeing was measured by eye — that is to say, the astronomer at the telescope estimated the size of the images of reasonably bright stars. This definition probably corresponded to 90% encircled energy. With the advent of modern, linear detectors like CCD's and vidicons, the definition has changed to measurement via Gaussian profile, usually the FWHM. This is a bit of a cheat (since seeing measurments all of a sudden got much better!) but is also likely to be more accurate.

Under normal conditions, the profile of a point source affected by seeing can be modeled as a Gaussian. The Gaussian *point-spread function* is

$$P(r) \; = \; \pi^{-1} \, r_c^{-1} \, e^{-(r/r_c)^2}$$

where the core radius, $r_c \; = \; \sqrt{2}\,\sigma \; = \; 1.44\sigma$. The modulation transfer function, $M(f)$, and the encircled energy function, $C(r)$, are useful descriptors and for a Gaussian are given by

$$M(f) \; = \; e^{-(\pi \; r_c \; f)^2}$$

$$C(r) \; = \; 1 \; - \; e^{-r/r_c}$$

FWHM $= \; 1.67 r_c \; = \; 2.36\sigma$. The 90% encircled energy radius $r_{90} \; = \; 2.303 r_c \; = \; 3.26\sigma$. The MTF frequency, $f_c \; = \; \frac{1}{\pi r_c}$.

A simple description of seeing treats the atmosphere as a collection of lenses or cells with variations in their indices of refraction. The number of such cells in front of the telescope is a measure of the effect of seeing on an image. Fried introduced a measure of the atmospheric turbulence that depends on the integral of the modulation transfer function (MTF) and is called the Resolution, R.

$$R \; = \; \int_{-\infty}^{+\infty} MTF(f) \; df$$

$$R_m ax \; = \; \frac{\pi}{4} \left(\frac{r_0}{\lambda f} \right)^2$$

r_0, Fried's parameter, is a measure of the size of such atmospheric cells; the larger the cell size, the better the seeing. At a good astronomical site, r_0 is typically about 10 cm in the optical. At a spectacular sight, r_0 might be as large as 40 cm in the optical. By definition, telescopes with apertures smaller than that are hardly affected by seeing.

Seeing is wavelength dependent, $r_0 \propto \lambda^{6/5}$, so the seeing is *better* in the infrared. Unfortunately, the telescope resolution is also wavelength dependent, $\theta = 1.22\lambda/D$, and under normal circumstances, the crossover occurs in the thermal IR.

Seeing also really depends on where in the atmosphere the turbluence occurs. The major contribution to the seeing can occur very low, near to the surface or induced in the dome, often due to thermal variations between the telescope structure or on the surface of the mirror. Modern telescopes and domes are designed to minimize these effects by providing rapid thermal equalization (lots of air flow) or eben active thermal control. Domes are kept airconditioned to the average nighttime temperature. The major contribution can also occur at high altitude, in which case the only recourse is adaptive optics.

A "complete" mathematical treatment of astronomical seeing is given in Coulman (1985) (MTYNTK - more than you need to know).

There are two other critical definitions w.r.t. seeing, the isoplanatic patch (or angle) and the isokinetic patch. The isokinetic patch is defined by the motion of images in the telescope focal plane — it is the area in the focal plane over which images move together with the seeing. The isoplanatic patch is that area which is roughly in phase — where the wavefronts remain parallel. Usually the isoplanatic patch is much smaller than the isokinetic patch.

References:

Allen, *Astrophysical Quantities* (soon to be Cox!)

Coulman, C. E. 1985, *Ann. Rev. Astro. and Ap.* **23**, 19.

Meeks, M. L. Methods of Experimental Physics, Part B.

Kitchin *Astrophysical Techniques*

Wolfe and Zissis *The Infrared Handbook*

U. S. Standard Atmosphere Supplement

Roach and Gordon, 1979 *The Light of the Night Sky*