Lecture 2 Ay 192 Principles of Astronomical Measurement

Signal-to-Noise, DQE, NEP

The object of today's lecture is to learn about the ways in which detectors and other systems can be characterized w.r.t noise and about some noise sources, including "source" noise. There are several ways to characterize detector and telescope+detector systems, with different ones used in different wavelength regions and with detectors of different characteristics. Typical figures-of-merit include detective quantum efficiency, DQE, at short wavelengths (x-ray-near IR), noise equivalent power, NEP, in the thermal IR, and Antenna Temperature, T_A, in the radio.

In this lecture, we will go through discussions regarding DQE, types and sources of detector or system noise, NEP, T_A, detector classification w.r.t. noise sources, and photon statistics. In later lectures we will discuss external signal degredation and ways to minimize the effects of noise.

General References:

Davenport and Root, *Intro. to the Theory of Random Signals and Noise*

Blackman and Tukey, *Measurement of Power Spectra*

Johnson, H. L. 1948, *Ap. J.* **107**, 34.

Fellgett, *Vistas in Astronomy* **1**, 475.

Dereniak and Crowe, *Optical Radiation Detectors.*

I. Detective Quantum Efficiency

DQE is one of the best measures of the effectiveness of a system, and a more useful one than the raw or relative quantum efficiency (RQE or QE) generally quoted by detector manufacturers. For detectors alone, it is a measure which combines RQE and noise characteristics. When evaluated in a real experimental situation, DQE is a measure of the effectiveness of the whole system including all sources of noise. DQE as a figure of merit for a detector or system is most often quoted at optical and near-IR and shorter wavelengths.

$$DQE = \left[\frac{(S/N)_{output}}{(S/N)_{input}} \right]^2$$

Note that DQE may also be a function of *time*. See below.

II. Types and Sources of Noise

"In general, an undesired time function..." (Blackman and Tukey)

(a) Types of Noise

1. Statistical $=$ $\sqrt{n}$ **fluctucations**

Photon

Poisson

Random

Sometimes this type of noise is referred to as "Radiation Noise" since it is a statistical property of the incoming radiation.

2. Thermal — Johnson or Nyquist Johnson Noise is due to due to random (Brownian) motions of particles in a resistor. The Johnson noise power is:

$$P_N = kT\Delta f$$

$$= 4.14 \times 10^{-21} \left(\frac{T}{300^\circ K}\right)\Delta f \text{ Watts}$$

where k is Boltzman's constant, T is the temperature (say, of the detector) and Δf is the sampling bandwidth in Hertz. Johnson noise can also be expressed as a voltage, or more commonly, a current:

Voltage $V_N = (4kTR\Delta f)^{1/2} = 0.13(\frac{T}{300^\circ K})R$

Current $i_N = (\frac{4kT}{R}\Delta f)^{1/2}$

where R is the resistance of the detector or detector system.

3. Shot Noise

Shot noise in photoelectric detectors is due to random fluctuations due to discreteness of charge:

$$i_{SN} = \sqrt{2q\bar{i}\Delta f}, \quad \text{where} \quad \bar{i} = \frac{Nq}{\Delta t}$$

and where Δt is an integration time, and q is the unit charge.

It originally was used to describe thermionic emission from photocathodes, a.k.a. dark current or dark counts in a photon counting system. Its form is derived from considering the variance of the current (the shot noise current squared).

4. Other Noise Sources

Generation-Recombination Noise (hole-electron "pair-production" in semiconductors)

Phonon Noise — statistical interchange of heat between the detector and its surroundings

Preamplifier or electronic noise

Modulation Noise (resistance fluctuations in a semiconductor)

Contact Noise or "microphonics"

Temperature Noise (fluctuations in the temperature of a detector not due to the signal)

Flicker Noise

Digitization Noise (discreteness effects in counters/amplifiers, etc.)

(b) Other useful definitions

If $R(f)$ is the power spectrum of the noise, then:

White Noise $\Rightarrow \quad R(f) = c$

1/f Noise $\Rightarrow \quad R(f) = c/f$

White noise contains equal power at all sampling frequencies, while 1/f noise decreases to higher sampling freqencies.

The spectrum of the noise can take many forms.

II. Noise Equivalent Power (NEP)

(a) The NEP is the minimum intensity of radiant power that will produce a signal equal to the noise, normalzed to a 1 Hertz sampling bandwidth. NEP is measured or quoted in **Watts per root Hertz**.

The NEP is the required power incident on a detector to produce a signal output equal to the r.m.s. noise output. For an ideal detector, this is just determined by photon statistics and the quantum efficiency. For a real detector, this will be a function of the sum of the noise sources in the detector and signal and the quantum efficency. For a real astronomical observation, you also must account for the external-signal-degredation.

NEP is most often used as a figure of merit for detector and telescope systems from the IR to the radio, primarily because at these wavelengths, detection is not of single photons but rather incident power.

(b) NEP as a Figure of Merit

For an Ideal Detector (no internal noise, RQE = 1.0)

$$NEP = \left[P_\lambda \frac{hc}{\lambda}(1 - e^{-\frac{hc}{\lambda kT}})^{-1}\right]^{1/2}$$

in units of watts per root Hertz (i.e. proportional to $\sqrt{bandwidth}$), and T is the characteristic temperature of the source (or the background). The "ideal" NEP is derived from Bose-Einstein photon statistics. The number of photons per unit time for a given incident power is given by:

$$N_\lambda \;=\; \#\; of\; photons \;=\; \frac{\lambda}{hc} P_\lambda$$

Then the statistical noise in the incident signal is just the *Variance,* σ_{N_λ} in the photon rate:

$$\sigma_{N_\lambda} \;=\; \sqrt{N_\lambda}(1 - e^{-\frac{hc}{\lambda kT}})^{-1/2}$$

where the second term is the Bose-Einstein correction factor since photons are Bosons. Thus the noise power (in the variance) is:

$$P_N \;=\; \frac{hc}{\lambda}\sqrt{N_\lambda}(1 - e^{-\frac{hc}{\lambda kT}})^{-1/2} \;=\; \sqrt{P_\lambda \frac{hc}{\lambda}}(1 - e^{-\frac{hc}{\lambda kT}})^{-1/2}$$

$$= \left[P_\lambda\; \frac{hc}{\lambda}(1 - e^{-\frac{hc}{\lambda kT}})^{-1}\right]^{1/2}$$

For a real detector of quantum efficiency, RQE $= \eta$,

$$True\;\; NEP \;=\; \eta^{-1/2}\; NEP_{Ideal}$$

Note that the relevant statistics in different wavelength regimes are given by evaluating $hc/\lambda kT$.

$$hc/k \;=\; \frac{6.63 \times 10^{-27} erg\; s \;\times\; 3 \times 10^{10} cm/s}{1.381 \times 10^{-16} erg/^\circ K}$$

thus

$$\frac{hc}{\lambda kT} \;=\; \frac{1.44 cm^\circ K}{\lambda T}$$

In the Optical and IR, $\lambda \sim 1\mu = 10^{-4}$ cm, T $= 300^\circ K$, implies

$$\frac{hc}{\lambda kT} \;\simeq\; 50, \quad \text{so} \quad e^{-hc/\lambda kT} \simeq 10^{-20}$$

and the BE correction is essentially irrelevant.

In the Radio, at 1 cm and 300°K,

$$\frac{hc}{\lambda kT} \;\simeq\; .005, \quad \text{so} \quad e^{-hc/\lambda kT} \simeq 0.995$$

and the BE correction factor is about 14.

III. IR Background, Optical Throughput

Two other concepts important at this stage of the discussion are the *infrared background* and the *optical throughput* of a system. In a real infrared telescope, the noise is alsmost always dominated by the telescope/sky background — thermal emission from the sky and/or telescope. The power in this background is given by:

$$P_\lambda \;=\; \frac{1}{\pi}\, F_\lambda\ \epsilon(\lambda)\ A\Omega\ \Delta\lambda\ T_{FIL}$$

where F_λ is usually taken as the blackbody law for the telescope, $\epsilon(\lambda)$ is the emissivity of the telescope, $\Delta\lambda$ is the bandpass, T_{FIL} is the filter transmission, and $A\Omega$ is the optical throughput or *etendue*. In the special case of a diffraction limited beam (which infrared observers love for obvious reasons),

$$A\Omega \;=\; (0.61\ \pi\ \lambda)^2$$

which is *independent* of the telescope aperture. That is because, as the diameter of the telescope increases, the size of its diffraction limited beam decreases, i.e. as the collecting area increases, the solid angle of a diffraction limited beam decreases.

IV. System Temperature

In radio telescopes, astronomers usually define a system temperature (following Krauss), and use this as a figure of merit:

$$T_{sys} \;=\; T_A \;+\; T_{LP}[1/\epsilon - 1] \;+\; (1/\epsilon)T_R$$

where T_A is the antenna temperature, T_{LP} is the transmision line temperature, ϵ is the transmission line efficiency, and T_R is the reciever temperature. Then the minimum detectable source temperature and, equivalently, the minimun detectable flux density are:

$$\Delta T_{min} \;=\; \frac{K_s T_{sys}}{\sqrt{\Delta\nu t}}$$

where

$$\begin{aligned} \Delta\nu \;&=\; \text{the bandwidth in Hz} \\ t \;&=\; \text{the integration time} \\ K_s \;&=\; \text{the `sensitivity' factor} \sim 1. \end{aligned}$$

and the minumum detecable flux density is

$$\Delta F_{min} \;=\; \frac{2k}{A_e}\frac{K_s T_{sys}}{\sqrt{\Delta\nu t}}$$

where $k \;=\; 1.38 \times 10^{-23}$ Joules $°\mathrm{K}^{-1}$, and A_e is the effective aperture in m^2. The minimum brightness, (in W m-2 Hz^{-1} Ster^{-1}) is:

$$\Delta B_{min} \;=\; \frac{2k}{\lambda^2}\frac{K_s T_{sys}}{\sqrt{\Delta\nu t}}$$

V. Detector Classifications

The simplest classification of detectors (and there are many) is based on their noise properties. Noise sources fall into two very broad categories — time dependent noise and time independent noise.

(a) Class I Detectors — are those with time dependent noise, e.g. phototubes, radio antennae, proportional counters, in general, most detectors that operate by counting individual events. For Class I detectors:

$$NEP_I \;=\; (4\sigma T^4 kT)^{1/2}\left(\frac{A}{t}\right)^{1/2}$$

where the first term is the power, A is the effective senisitve area per channel or pixel and t is the integration time or time constant.

(b) Class II Detectors — are those with time independent noise, e.g. photographic emulsions, bolometers, thermocouples, and CCDs which are characterized by "readout" noise, which is the same no matter how long the integration time is. For Class II detectors:

$$NEP_{II} \;\approx\; 3\times 10^{-12}\left(\frac{A}{t^2}\right)^{1/2}$$

Nowadays, Class I detectors are usually called (or lumped into) Quantum Detectors, while Class II detectors are called Thermal or Classical detectors (except for CCDs and photographic plates, which are read noise dominated but are still quantum detectors).

VI. Statistical Distributions

Before proceeding to examples in photon statistics, its important to very quickly review some basics about probability distributions. The most commonly described distribution is a Gaussian, also known as the Normal distribution.

(a) Gaussian Distribution

$$P(x,\mu,\sigma) \;=\; \frac{1}{\sigma\sqrt{2\pi}}e^{\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

A Gaussian can be described in terms of its sigma, but also is

$$\text{Half Width, } \Gamma \;=\; 2.354\sigma$$

$$\text{Probable Error, P. E.} \;=\; 0.6745\sigma$$

Gaussians have many wonderful properties including symmetry and scaling. Perhaps most important is a theorm we'll get back to much later in the course called the Central Limit Theorem, which, in one line, states that the limiting distribution of distributions is a Gaussian. However, *photon statistics are not Gaussian,* especially for small count rates. The probability distribution that describes incoming photons is the Poisson distribution.

(b) Poisson Distribution

$$P(x, \mu) \ = \ \frac{\mu^x}{x!} \ e^{-\mu}$$

where the dispersion measure equals the mean, $\sigma^2 \ = \ \mu$

This distribution was derived by S. D. Poisson (1781-1840) from his study of Criminal and Civil Judgements, essentially recidivism, in 19th century Paris. It has the important property of always being positive (there are no "negative" counts of photons), and having its dispersion equal to its mean. In the limit of large numbers (a large mean), the Possion distribution very closely resembles a Gaussian.

(c) Other Distributions

The probability density distribution of real data can take many forms, the most common of which are the binomial, exponential, Lorentzian, χ^2, Student's T, and F. These will be discussed in more detail later.

VII. Photon Statistics

Many if not most of the detectors and data taking systems one encounters from high energy to the near infrared are photon-counting systems. We'll talk about the differeces between these and integrating or coherent systems later in the course, but for now lets consider the examples below.

Consider two detectors, an optical CCD and an optical photomultiplier. The first is a class II detector while the second is class I. That is to say, under ordinary operations the noise inherent in the CCD is dominated by readout noise, while the noise in the photomultiplier is dark counts or dark current. We can calculate the signal to noise from the simplest real observations of a point source against the sky, for example, an aperture centered around a star to be measured in the focal plane of a telescope — either the entrance aperture of a photomultiplier or an aperture in your favorite image processing language analysing a CCD frame:

Table 1. Two Detectors

	Photomultiplier Class I	CCD Class II
Noise sources		
	dark counts $= \sqrt{Dt}$	Readout $=$ R
	$\sqrt{(sky)} = \sqrt{Bt}$	$\sqrt{(sky)} = \sqrt{Bt}$
	$\sqrt{object} = \sqrt{Ot}$	$\sqrt{object} = \sqrt{Ot}$
Quantum efficiency		
	20%	60%

if B = sky counts per second, O = object counts per second, D = dark counts per second and R = readout noise in counts (equivalent photons), then for our photon-counting Class I detector we have:

$$(S/N)_I = \frac{Ot}{(Ot + Bt + Dt)^{1/2}} = \frac{O}{(O + B + D)^{1/2}}\, t^{1/2}$$

as the three noise sources are assumed to be independent and thus add in quadrature. Remember that the aperture collects or contains not only the flux from the star but also from the sky background. This needs to be determined and subtracted, and this subtraction adds noise to the measurement because of its uncertainty — the $\sqrt{Bt}$ term.

Then for the integrating CCD detector with readout noise, we have:

$$(S/N)_{II} = \frac{Ot}{(Ot + Bt + R)^{1/2}}$$

which has a more complicated dependence on integration time. Note that the readout noise as quoted for most detectors is infact the squareroot of the "R" in our above equation — that is a readout noise of 10 e^- should be translated into an effective count of 100.

Now, we should note that telescopes and detectors operate essentially in two different modes: (1) *background or sky noise limited*, or (2) *detector noise limited*. Detector noise limited implies R >> both Nt and Bt (Class II), or D >> B and O (Class I).

For **background limited** observations, Bt >> R or B >>D,

$$S \propto N \propto \Delta\lambda A t$$

both signal and noise are proportional to the bandpass, the effective collecting area and the integration time. Then

$$(S/N)_I = \frac{Ot}{(Ot + Bt)^{1/2}} = (S/N)_{II}$$

$$= \frac{O}{(O+B)^{1/2}} \, t^{1/2} \, \Delta\lambda^{1/2} \, A^{1/2}$$

For **detector limited** observations (normally due to either a poor detector, a poor telescope or instrument, or a high spatial or spectral resolution observation) we have either R

$$(S/N)_I \;=\; \frac{Ot}{(Ot+Dt)^{1/2}} \quad usually \;\rightarrow\; \frac{O}{\sqrt{D}}\, t^{1/2}$$

$$(S/N)_{II} \;=\; \frac{Ot}{(Ot+R)^{1/2}} \quad usually \;\rightarrow\; \frac{Ot}{\sqrt{R}}$$

so for a detector with time dependent noise, even when detector noise limited, the S/N grows only with the square-root of time, while for a detector dominated by time independent noise (CCD's, photographic plates, etc.)

$$(S/N)_{II} \;\propto\; t \; \Delta\lambda \; A \qquad !$$

so signal-to-noise grows linearly with time, bandpass and effective aperture. Note also that in both detector and background noise limited regimes, the signal to noise depends linearly on the signal strength (O) and as the square root of the major noise source (either R or D). Also, for future reference, remember that the "R" in the above equations is the noise in the readout of the CCD, which is usually defined as the RMS (root mean square) number of electrons, so for a readout noise of E = 10 e^-, R = $E^2 = 10^2 = 100$!

One doesn't usually want to be in the detector noise limited regime, but if you are, increased bandpass, bigger telescopes and longer integration times all help you a lot.

Note that **Background Limited** *versus* **Detector Noise Limited** *versus* **Source Limited** are three important regimes of observations. These will appear over and over again in the course and in your lives as astronomers!

Reading Assignment:

Week 1 — Bevington, Chapters 1+2