STEADY-STATE SOLUTIONS TO THE OBLIQUE CR-MHD JUMP-SHOCK CONDITIONS

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CR-MHD JUMP-SHOCK CONDITIONS

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Abstract

Almost everything in the universe is plasma, including stars and the interstellar medium. One of the best tools for understanding plasma dynamics in the cosmos is **magnetohydrodynamics** (MHD) which assumes, among other things, charge neutrality – an excellent approximation to plasma dynamics on relatively large distance scales. In addition, cosmic rays (CR) play an important role in astrophysics and in particular they are fundamental to the structure of astrophysical shocks. Because of their diffusive properties, CR's can transmit information in an astrophysical plasma faster than the shock itself, thereby creating a so-called "CR foot" leading the shocks.

The equations governing CR–MHD shocks in one dimension consist of 13 equations and 13 unknowns. These can be solved analytically to determine the post- and interior-shock conditions from the initial conditions. Comparison between these analytical solutions and solutions determined numerically will be made.

Along with providing a means of obtaining steady-state solutions, this thesis will provide a CR-MHD version of the well-known Brio and Wu (1988) shock tube test for CR–MHD computer programs. There are runs that contain the original Brio and Wu test (no CR), a similar test with the CR-fluid only, and a run with equal parts of both the ambient gas fluid and the cosmic-ray fluid.

Contents

Chapter 1

Background

Shocks play a very important role in almost every astrophysical phenomenon that are of interest to astronomers. They are vital to the understanding of phenomena such as supernovae and supernovae remnants, stellar winds, expanding HII regions, molecular outflows (jets), accretion disks, spiral density waves, cloud/galaxy collisions, *etc.* They are important because of the dramatic changes they induce in density, velocity and temperature when they pass through a fluid.

Without a magnetic field, a shock in a fluid can be described by plain hydrodynamics. If we cannot neglect the ambient magnetic field, which is often the case, we must use magnetohydrodynamics, (MHD). Magnetic fields can modify the way shocks behave by "stiffening" the equation of state, thus making the shocks less compressive. In addition, unlike non-magnetized fluids which can support only longitudinal waves, MHD fluids can support transverse waves as well. These properties can not be treated with ordinary hydrodynamics (HD).

Another component in the interstellar medium that can not be neglected is cosmic rays (CR) which interact with the ambient thermal gas. Drury and Völk (1981) showed that the addition of CR's can dramatically alter the structure of HD shocks by introducing a "CR-foot" (see Fig. 1). Because of the diffusive properties of the CR's, they are allowed to pass in front of the shock front and excite the pre-shock gas. This results in an increase in pressure before the shock front passes through that region, creating the "CR-foot", also referred to as the "smooth region". When the shock comes in contact with the upstream gas that has been partially excited by the CR's, it creates a "gas subshock". It is called a sub shock because it does not make up the entire transition as it would in the MHD case. Just upwind of the gas subshock is the smooth region. The subshock has not reached this region yet and so there are no discontinuities here. If the conditions are right there is a gas subshock or discontinuity just beyond the smooth region, (labeled with a "1" in Fig. 1), that connects it with the downstream region (labeled with a "2" in Fig. 1).

There are a variety of mathematical ways to model the CR's. The simplest of these methods is called the "two-fluid" approach. This expresses the interaction between the CR's and the ambient plasma as a scalar pressure, and assumes an isotropic mixture of the CR's and the ambient thermal plasma. These assumptions allow the CR's to be treated as a second fluid and an equation can be added to the existing MHD equations that takes into account the diffusive characteristics of the CR's. An equation of state for the CR fluid must also be added. Although this model is rather simple to implement, it does have its drawbacks. For example, there must be some assumed value for γ_c, the ratio of specific heats for the CR fluid. This value is not easily obtained through observation and usually covers a range of values, $(4/3 \leq \gamma_c \leq 5/3)$, depending on the ratio of ultrarelativistic and non-relativistic particles. Another drawback, shown by Drury and Völk (1981), is that the diffusion coefficient, κ, cannot be calculated for the CR fluid because the CR momentum distribution cannot be derived from the dynamics of the flow. As it turns out, however, the value of κ does not affect the 1-D steady-state solutions.

Another method of handling the cosmic rays originally proposed by J. Skilling (1975) involves only the assumptions that the momentum of the CR's is isotropic and the bulk flow of the CR's is sub-relativistic. Skilling's equation can be manipulated into a form similar to the other conservation equations used in MHD with the exception that it is in four dimensions (3 spatial, 1 momentum). It turns out to be a more realistic representation but in practice is much more difficult to implement into a numerical computer program because of the need for the fourth momentum direction.

There are a variety of techniques used to solve the equations of CR-MHD. One method involves converting the differential equations that govern CR-MHD into time-dependent difference equations that can be solved using a variety of numerical methods that are beyond the scope of this thesis. One of the computer programs at our disposal is the **ZEUS-3D** code developed by D. A. Clarke that solves the equations of CR-MHD (Clarke 1995) and treats CR's as a second fluid (Jun, Clarke and Norman 1994). One way to ensure that these computer programs are working properly is to compare their results with those obtained from steady-state analytical calculations.

At present, the **ZEUS-3D** code uses the two-fluid model for CR's only. The implementation of the Skilling equation is in progress. **ZEUS-3D** was used to compute the time-dependent solutions to the 1-D CR-MHD shock problem and these are discussed in the next chapter. Recently, Frank, Jones and Ryu (1995) have shown that using the Skilling approach only affects the transient (time-dependent) features and has no effect on the steady-state solutions for 1-D shocks. Thus, the steady-state solutions determined numerically in chapter 2 are quite general, independent of whether a two-fluid or Skilling model is used.

Until now, there has been no analytical steady-state solution available for oblique CR-MHD 1-D shocks. Jun, Clarke and Norman (1994) presented a steady-state solution for perpendicular 1-D CR-MHD shocks, (field perpendicular to shock normal). Dursi (1994) presented a way of finding steady-state solutions for the case where there is no discontinuity present but was not able to consider the general case which includes a subshock. Frank, Jones and Ryu (1995) used the steady-state solutions provided by Webb, Drury and Völk (1986) but these were presented only in graphical form and are of limited practical use.

In the next chapter, the general 1-D steady-state CR-MHD solutions are presented. Other than the assumptions of steady-state in one dimension, no other limitations are imposed.

Chapter 2

Steady-State Solutions

2.1 Equations and Analytical Solutions

The equations of ideal MHD used in this work are (*e.g.*, Clarke 1995)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \tag{2.1}$$

$$\frac{\partial \vec{s}}{\partial t} + \nabla \cdot (\vec{u}\vec{s}) = -\nabla(P_c + P_g) + \frac{1}{4\pi}(\nabla \times \vec{B}) \times \vec{B} \tag{2.2}$$

$$\frac{\partial e_g}{\partial t} + \nabla \cdot (e_g \vec{u}) = -P_g(\nabla \cdot \vec{u}) \tag{2.3}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) \tag{2.4}$$

where ρ is the gas density, $\vec{u}$ is the gas velocity, $\vec{s}$ is the gas momentum ($=\rho\vec{u}$), P_g is the thermal gas pressure, P_c is the CR pressure, $\vec{B}$ is the magnetic field, and e_g is the internal energy density of the thermal gas.

Equations (2.1) - (2.4) represent conservation of mass, conservation of momentum, conservation of energy, and the induction equation (governs the evolution of the magnetic field) respectively. In the two-fluid model, the CR "fluid" is governed by the diffusive energy equation, namely (*e.g.*, Jun *et al.* 1994)

$$\frac{\partial e_c}{\partial t} + \nabla \cdot (e_c \vec{u}) = -P_c(\nabla \cdot \vec{u}) + \nabla \cdot (\kappa \nabla e_c) \tag{2.5}$$

where e_c is the internal energy density for the CR-"fluid", $\nabla \cdot (\kappa \nabla e_c)$ is the diffusion term for CR and κ is the diffusion coefficient — in general, a tensor.

Finally we have the equations of state for the two fluids:

$$P_c = (\gamma_c - 1)e_c \tag{2.6}$$

$$P_g = (\gamma_g - 1)e_g \tag{2.7}$$

where the subscript c refers to the CR fluid, and the subscript g refers to the thermal gas.

For the steady state, 1-D (*e.g.,* in the x-direction in a Cartesian coordinate system), oblique shock problem, both parallel (B_x) and perpendicular (B_y) components of the magnetic field are required. The Lorentz forces from such a configuration will generate a perpendicular component of the velocity (u_y) and thus, unlike the case of a perpendicular shock (Jun *et al* 1994), two components of the momentum equation

(2.2) are required. The assumed symmetry gives us:

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0$$

and

$$u_z = B_z = 0$$

Equations (2.1) through (2.5) may be integrated to obtain the jump conditions across a CR–MHD transition. Across the smooth region, the 1-state and the 0-state (Fig. 1) are related as follows:

$$\rho_1 u_{x_1} = \rho_0 u_{x_0} \equiv I \tag{2.8}$$

$$I u_{x_1} + P_{c_1} + P_{g_1} + \frac{B_{y_1}^2}{8\pi} = I u_{x_0} + P_{c_0} + P_{g_0} + \frac{B_{y_0}^2}{8\pi} \equiv J_1 \tag{2.9}$$

$$\rho_1 u_{x_1} u_{y_1} - \frac{1}{4\pi} B_{y_1} B_{x_1} = \rho_0 u_{x_0} u_{y_0} - \frac{1}{4\pi} B_{y_0} B_{x_0} \equiv J_2 \tag{2.10}$$

$$B_{x_1} = B_{x_0} \equiv L_1 \tag{2.11}$$

$$u_{x_1} B_{y_1} - u_{y_1} B_{x_1} = u_{x_0} B_{y_0} - u_{y_0} B_{x_0} \equiv L_2 \tag{2.12}$$

$$P_{g_1} \rho_1^{-\gamma_g} = P_{g_0} \rho_0^{-\gamma_g} \equiv M \tag{2.13}$$

where I, J_1, J_2, L_1, L_2 and M are constants of integration and where the adiabatic equation of state is used explicitly (smooth transition).

Across the gas subshock the following expressions relate the 1- and 2-states

$$I u_{x_1} + P_{g_1} + \frac{B_{y_1}^2}{8\pi} = I u_{x_2} + P_{g_2} + \frac{B_{y_2}^2}{8\pi} \tag{2.14}$$

$$\rho_1 u_{x_1} u_{y_1} - \frac{1}{4\pi} B_{y_1} B_{x_1} = \rho_2 u_{x_2} u_{y_2} - \frac{1}{4\pi} B_{y_2} B_{x_2} \equiv J_2 \tag{2.15}$$

$$\frac{1}{2} I u_1^2 + G_g P_{g_1} u_{x_1} + u_{x_1} \frac{B_{y_1}}{4\pi} - \frac{B_{x_1}}{4\pi}(u_{x_1} B_{x_1} + u_{y_1} B_{y_1})$$

$$= \frac{1}{2} I u_2^2 + G_g P_{g_2} u_{x_2} + u_{x_2} \frac{B_{y_2}}{4\pi} - \frac{B_{x_2}}{4\pi}(u_{x_2} B_{x_2} + u_{y_2} B_{y_2}) \tag{2.16}$$

$$B_{x_1} = B_{x_2} \equiv L_1 \tag{2.17}$$

$$u_{x_1} B_{y_1} - u_{y_1} B_{x_1} = u_{x_2} B_{y_2} - u_{y_2} B_{x_2} \equiv L_2 \tag{2.18}$$

$$\rho_1 u_{x_1} = \rho_2 u_{x_2} \equiv I \tag{2.19}$$

$$P_{c_1} = P_{c_2} \tag{2.20}$$

where $G_g = \frac{\gamma_g}{\gamma_g - 1}$, $G_c = \frac{\gamma_c}{\gamma_c - 1}$ and where γ_g and γ_c are assumed to remain constant. Note equation (2.20) states explicitly that P_c is continuous across the subshock.

Because both upstream and downstream values exist on the hyperbola $\frac{\partial P_c}{\partial x} = 0$

($i.e$, where the profile of $P_c(x)$ is flat), we have:

$$\frac{1}{2}Iu_0^2 + G_gP_{g0}u_{x0} + G_cP_{c0}u_{x0} + u_{x0}\frac{B_{y0}}{4\pi} - \frac{B_{x0}}{4\pi}(u_{x0}B_{x0} + u_{y0}B_{y0}) =$$

$$\frac{1}{2}Iu_2^2 + G_gP_{g2}u_{x2} + G_cP_{c2}u_{x2} + u_{x2}\frac{B_{y2}}{4\pi} - \frac{B_{x2}}{4\pi}(u_{x2}B_{x2} + u_{y2}B_{y2}) \equiv K \quad (2.21)$$

where K is an additional constant of integration.

Equations 2.8 through 2.20 represent a system of 14 equations in 14 unknowns (namely the variables at the 1- and 2- states). These are solved as follows: After substituting ρ_1 from (2.8), P_{g_1} from (2.13), and B_{y_1} from (2.12) into (2.9), and rearranging (2.20) and (2.16) to get P_{c_1} as a function of B_{y_1} and P_{g_1} (which can in turn be expressed as the constants of motion and u_{x_1} only), a little algebra gives us u_{x_2} as a function of u_{x_1}, namely:

$$u_{x_2} = \left[Ku_{x_1}^{2+\gamma_g} - \frac{I}{2}\left(u_{x_1}^{4+\gamma_g} + u_{y_1}^2u_{x_1}^{2+\gamma_g}\right) - G_gMI^{\gamma_g}u_{x_1}^3 - \frac{L_2^2}{4\pi}u_{x_1}^{1+\gamma_g} - \frac{L_1L_2}{4\pi}u_{x_1}^{1+\gamma_g}u_{y_1}\right]$$

$$\times\left[G_c\left(J_1u_{x_1}^{2+\gamma_g} - Iu_{x_1}^{3+\gamma_g} - MI^{\gamma_g}u_{x_1}^2 - \frac{L_2^2}{8\pi}u_{x_1}^{\gamma_g} - \frac{L_1L_2}{4\pi}u_{x_1}^{\gamma_g}u_{y_1} - \frac{L_1^2}{4\pi}u_{x_1}^{\gamma_g}u_{y_1}^2\right)\right]^{-1} \quad (2.22)$$

Following a similar procedure, and substituting P_{g_2} from (2.20), equation (2.14) can be manipulated to:

$$u_{x_2}^2\left(\frac{1}{2} - G_g\right)I + u_{x_2}G_g\left[Iu_{x_1} + M\left(\frac{I}{u_{x_1}}\right)^{\gamma_g} + \frac{1}{8\pi}\left(\frac{L_2^2 + 2L_1L_2u_{y_1} + L_1^2u_{y_1}^2}{u_{x_1}^2}\right)\right]$$

$$-\frac{G_g}{8\pi u_{x_2}}\left(L_2^2+2L_1L_2u_{y_2}+L_1^2u_{y_2}^2\right)+\frac{L_2}{4\pi}\left(\frac{L_2+L_1u_{y_2}}{u_{x_2}}\right)+\frac{I}{2}u_{y_2}^2$$
$$-\left[\frac{I}{2}u_{x_1}^2+\frac{I}{2}u_{y_1}^2+G_gMI^{\gamma_g}u_{x_1}^{1-\gamma_g}+\frac{L_2}{4\pi}\left(\frac{L_2+L_1u_{y_1}}{u_{x_1}}\right)\right]=0 \tag{2.23}$$

which is a function of u_{x_2}, u_{x_1} and constants only.

Substituting equation (2.22) into (2.23) to eliminate u_{x_2} yields the final expression containing only constants and u_{x_1}.

$$\left[Ku_{x_1}^{2+\gamma_g}-\frac{I}{2}\left(u_{x_1}^{4+\gamma_g}+u_{y_1}^2u_{x_1}^{2+\gamma_g}\right)-G_gMI^{\gamma_g}u_{x_1}^3-\frac{L_2^2}{4\pi}u_{x_1}^{1+\gamma_g}-\frac{L_1L_2}{4\pi}u_{x_1}^{1+\gamma_g}u_{y_1}\right]^3\times$$
$$\left(\frac{1}{2}-G_g\right)I$$
$$+G_cG_g\left[Ku_{x_1}^{2+\gamma_g}-\frac{I}{2}\left(u_{x_1}^{4+\gamma_g}+u_{y_1}^2u_{x_1}^{2+\gamma_g}\right)-G_gMI^{\gamma_g}u_{x_1}^3-\frac{L_2^2}{4\pi}u_{x_1}^{1+\gamma_g}-\frac{L_1L_2}{4\pi}u_{x_1}^{1+\gamma_g}u_{y_1}\right]^2$$
$$\times\left[J_1u_{x_1}^{2+\gamma_g}-Iu_{x_1}^{3+\gamma_g}-MI^{\gamma_g}u_{x_1}^2-\frac{L_2^2}{8\pi}u_{x_1}^{\gamma_g}-\frac{L_1L_2}{4\pi}u_{x_1}^{\gamma_g}u_{y_1}-\frac{L_1^2}{8\pi}u_{x_1}^{\gamma_g}u_{y_1}^2\right]$$
$$\times\left[Iu_{x_1}+MI^{\gamma_g}u_{x_1}^{-\gamma_g}+\frac{1}{8\pi}\left(\frac{L_2+L_1u_{y_1}}{u_{x_1}}\right)^2\right]$$
$$-G_c^2\left[Ku_{x_1}^{2+\gamma_g}-\frac{I}{2}\left(u_{x_1}^{4+\gamma_g}+u_{y_1}^2u_{x_1}^{2+\gamma_g}\right)-G_gMI^{\gamma_g}u_{x_1}^3-\frac{L_2^2}{4\pi}u_{x_1}^{1+\gamma_g}-\frac{L_1L_2}{4\pi}u_{x_1}^{1+\gamma_g}u_{y_1}\right]$$
$$\times\left[J_1u_{x_1}^{2+\gamma_g}-Iu_{x_1}^{3+\gamma_g}-MI^{\gamma_g}u_{x_1}^2-\frac{L_2^2}{8\pi}u_{x_1}^{\gamma_g}-\frac{L_1L_2}{4\pi}u_{x_1}^{\gamma_g}u_{y_1}-\frac{L_1^2}{8\pi}u_{x_1}^{\gamma_g}u_{y_1}^2\right]^2$$
$$\times\left[\frac{1}{2}Iu_{x_1}^2+\frac{1}{2}Iu_{y_1}^2+\frac{1}{2}Iu_{y_2}^2+G_gMI^{\gamma_g}u_{x_1}^{1-\gamma_g}+\frac{L_2}{4\pi}\left(\frac{L_2+u_{y_1}L_1}{u_{x_1}}\right)\right]$$
$$+G_c^3\left[J_1u_{x_1}^{2+\gamma_g}-Iu_{x_1}^{3+\gamma_g}-MI^{\gamma_g}u_{x_1}^2-\frac{L_2^2}{8\pi}u_{x_1}^{\gamma_g}-\frac{L_1L_2}{4\pi}u_{x_1}^{\gamma_g}u_{y_1}-\frac{L_1^2}{8\pi}u_{x_1}^{\gamma_g}u_{y_1}^2\right]^3$$

$$\times \left[\frac{L_2 + L_1 L_2 u_{y_2}}{4\pi} - \frac{G_g}{8\pi}\left(L_2 + L_1 u_{y_2}\right)^2\right] = 0 \tag{2.24}$$

where

$$u_{y_1} = \frac{L_1 L_2 + 4\pi J_2 u_{x_1}}{4\pi I u_{x_1} - L_1^2} \tag{2.25}$$

$$u_{y_2} = \frac{L_1 L_2 + 4\pi J_2 u_{x_2}}{4\pi I u_{x_2} - L_1^2} \tag{2.26}$$

$$u_{x_2} = \frac{\left[K u_{x_1}^{2+\gamma_g} - \frac{I}{2}\left(u_{x_1}^{4+\gamma_g} + u_{y_1}^2 u_{x_1}^{2+\gamma_g}\right) - G_g M I^{\gamma_g} u_{x_1}^3 - \frac{L_2^2}{4\pi} u_{x_1}^{1+\gamma_g} - \frac{L_1 L_2}{4\pi} u_{x_1}^{1+\gamma_g} u_{y_1}\right]}{G_c\left[J_1 u_{x_1}^{2+\gamma_g} - I u_{x_1}^{3+\gamma_g} - M I^{\gamma_g} u_{x_1}^2 - \frac{L_2^2}{8\pi} u_{x_1}^{\gamma_g} - \frac{L_1 L_2}{4\pi} u_{x_1}^{\gamma_g} u_{y_1} - \frac{L_1^2}{8\pi} u_{x_1}^{\gamma_g} u_{y_1}^2\right]} \tag{2.27}$$

Equation (2.24) can be solved numerically (*e.g.* Newton-Raphson methods) for u_{x_1}. Knowing u_{x_1} is sufficient to find the values of the remaining 1-state variables from which the 2-state variables follow. In general, equation (2.24) permits *many* roots and thus a variety of values for u_{x_1}. However all but one of those solutions will be eliminated on physical grounds (*i.e.,* the root allows a negative density or pressure, causes a pressure *drop* across the transition rather than the requisite *rise, etc.*)

For a perpendicular shock ($B_x = 0$), set

$$u_{y_2} = u_{y_1} = 0$$

$$u_{x_2} \rightarrow u_2$$

$$u_{x_1} \rightarrow u_1$$

$$L_2 \rightarrow L$$

$$J_1 \rightarrow J$$

When these are substituted back into eq. (2.24), it reduces to eq. (2.28) in Jun et al (1994)

Drury and Völk (1981) show the structure of a cosmic ray-mediated hydrodynamical shock can include one discontinuity or none. Following Drury and Völk, the criterion for the existence of a shock can be found by differentiating eq. (2.9) with respect to u_x and substituting ρ from eq. (2.8), P_g from eq. (2.13) and B_y from eq. (2.12) and equating $\partial P_c / \partial u_x$ to 0. This yields:

$$\gamma_g P_g = \rho u_x^2 - \frac{B_y^2 \rho u_x^2}{4\pi \rho u_x^2 - B_x^2} \tag{2.28}$$

Therefore, if $P_g > \frac{1}{\gamma_g}(\rho u_x^2 - \frac{B_y^2 \rho u_x^2}{4\pi \rho u_x^2 - B_x^2})$ then P_g has to follow a discontinuous jump to go to the downstream state while if $P_g < \frac{1}{\gamma_g}(\rho u_x^2 - \frac{B_y^2 \rho u_x^2}{4\pi \rho u_x^2 - B_x^2})$, there is no discontinuity and the transition is smooth.

Since the original equations apply to both the shock and non-shock condition, equation 2.24 is completely general in the sense that it can be used to find the final 1- and 2-states whether a subshock exists or not. The absence of a shock will be revealed if the 1- and 2-states turn out to be equivalent.

2.2 Numerical Results

The goal was to find the steady-state solutions for the standard shock-tube problem with oblique magnetic fields. We define the upstream magneto-acoustical Mach number to be

$$M = \left(\frac{\rho u^2}{\gamma_g P_g + \gamma_c P_c + 2P_b} \right)^{\frac{1}{2}} \tag{2.29}$$

where P_b is the magnetic pressure defined as

$$P_b = \frac{B^2}{8\pi}$$

Figures 2 through 5 show the time dependent **ZEUS-3D** solutions, (solid lines), overlaid with the analytical steady-state, (dashed line), for the Mach 10, equipartition ($P_g = P_c = P_b$) runs for various angles (θ) between the magnetic field and the shock normal (obliquity, *e.g.* obliquity $= 0 \Rightarrow$ perpendicular shock, obliquity $= 90 \Rightarrow$ parallel field). All units (for distance, density, pressure, *etc.*) are arbitrary. In all plots, the initial values for density and thermal pressure are 1.0, giving a value of $\sqrt{5/3}$ (arbitrary units) for the sound speed when $\gamma_g = 5/3$. Each frame has 3 epochs plotted, namely $t = 0.1,\ 0.2,\ 0.3$, where $t = \sqrt{3/5}$ is the time for a sound wave to propagate a distance of 1 arbitrary distance unit. It is interesting to note how much more the CR pressure, P_c, is increased than the thermal pressure, P_g. This is because the dependence of P_c on velocity is rather strong, namely $P_c \sim u_x^3$. In contrast,

$P_g \sim u_x^2$.

Figures 6 through 9 show the same plots except for a lower B-field ($P_g/P_b = P_c/P_b = 10$). They look very similar to Figures 2 through 5 because in both cases, the dynamics are dominated by the bulk kinetic energy. Thus, whether the magnetic pressure is comparable to or lower than the thermal pressure has little effect on the dynamics. It should be noted that the percent difference between the values of the steady-states from trial to trial was greater than the percent difference between the values of the steady-states predicted by the **ZEUS-3D** code and the analytical solution finder. This indicates that the variations predicted by both the numerical, time-dependent computer program (**ZEUS-3D**) and the analytical steady-state solution are real. The maximum percent discrepancy between the numerical and analytical solutions is 3% while the minimum is less than 0.1% with a typical value of about 1%. The variables that exhibited a 3% discrepancy varied by at least 10% from one angle of obliquity to another.

Figures 10 through 13 show the same plots for the Mach 3, equipartition shocks, except that the time (defined above) required to reach steady state is 3.0 with plots at $t = 1.0$, 2.0 and 3.0. In this case, a subshock has formed between the original upwind and steady-state downwind states. Note that a subshock did <u>not</u> form in the Mach 10 calculations. This possibly counter-intuitive result can be explained as follows: Diffusion acts to smooth shocks out by allowing particles to be pushed ahead of the shock front. This is manifested by the formation of a “CR-foot” leading the shock front itself (Fig. 1). The transition results in a strong gradient in P_c which results in diffusion. The stronger the transition (*i.e.* higher the Mach number), the greater the diffusion, so much so that high Mach number shocks are entirely smeared out by the effect of the CR's.

Figures 14 through 17 show runs for the Mach 3, low B-field case ($P_g/P_b = P_c/P_b = 10$). The effect of obliquity of the magnetic field are more obvious than in Figures 2 through 9 because the equations are not as dominated by the bulk kinetic energy.

Chapter 3

The Modified “Brio and Wu Problem”

3.1 The Problem

The solutions of the Brio and Wu (1988) have become a standard test for MHD computer programs. It is, of course, limited to MHD and does not include the effects of CR’s. In this chapter, the Brio and Wu problem is generalized to CR-MHD with the aid of the **ZEUS-3D** code. Three runs are shown, the first of which is an MHD calculation with the CR-fluid turned off to reproduce the results Brio and Wu. The second run divides the initial pressures equally between the thermal gas and the CR-fluid. The third run considers the CR-fluid *only*. Some new features are observed and discussed.

3.2 Numerical Results

Figure 18 reproduces the results of Brio and Wu well and will be taken, in part, as evidence that the MHD code is working properly. The features discussed by Brio and Wu (Fig. 18) are clearly visible, namely the fast rarefaction waves (FR), compound wave (CW, slow shock plus a slow rarefaction wave), contact discontinuity (CD) and a slow shock (SS).

In Figure 19, with the CR's included, there is a new feature at $x1 \approx 250$ in the thermal pressure and the relativistic pressure. Recall in the solutions presented in Chapter 2 how P_c was ramped up much higher than P_g across the transition. This is the same effect. The shock at $x1 \approx 300$ causes a greater jump in the cosmic ray pressure than in the thermal pressure. Because both pressures must assume a common value at the base of the complex wave (*i.e.*, they were initialized at the same value), the thermal pressure rises and the CR pressure falls across the contact discontinuity. Without CR's, the thermal pressure was constant across the contact discontinuity. Another noteworthy feature is the formation of a “CR-foot” at the base of the discontinuity at $x1 \sim 310$. It is not very pronounced, but upon comparison with the MHD case (Fig. 18) it is clear that a “CR-foot” is present. The smearing action induced by the CR's is made apparent by simply counting the number of zones the discontinuity at $x1 \sim 310$ occupies. In the MHD case (Fig. 18) the transition occupies only 3 zones while in the CR-MHD case (Fig. 19) the transition occupies approximately 8 zones.

In Figure 20, with only the cosmic ray fluid present, it looks very much like the original no-CR run of Figure 18 except that the features are smoothed out by the diffusive properties of the CR's. The jump in density at $x1 = 310$ (the slow shock of Brio and Wu) is smoothed out so much that there is no longer a true discontinuity.

Chapter 4

Discussion and Conclusions

This thesis has compared the time-dependent solutions to the 1-D jump-shock conditions given by **ZEUS-3D** with the analytical steady-state solutions found by the methods described in Chapter 2. Typically, the discrepancy between the time-dependent numerical solutions and the steady-state analytical calculation are approximately 1% and never exceeded 3%.

Equation 2.24 provides a means of obtaining a steady-state solution to the 1-D jump-shock conditions that is completely general. The only assumptions are 1-D and no time-dependence and can be used to study the effects of the initial pre-shock conditions on the final post-shock conditions. It can also be adapted to a variety of 1-D configurations to test numerical computer programs.

The Brio and Wu test problem was modified to include CR's. With the addition of CR's, the main difference occurred in the modification of the structure at the contact discontinuity. Also new to the CR-MHD version of the Brio and Wu test is the appearance of a "CR-foot" at the base of the slow shock (SS). Another point of note is the smoothing of the features found in the MHD case by the diffusive properties of the CR's (Fig. 19) and the complete smoothing of the slow shock into a continuous transition in the case of only the CR fluid (Fig. 20).

At present, Skilling's equation is being implemented and should be in place and working by the summer of 1995. When complete, it can be tested using equation 2.24 since Frank, Jones and Ryu (1995) have showed that the steady-states are the same, regardless of whether the two-fluid or Skilling approach is used.

The tests performed in this thesis have two main purposes. The first is to help provide an understanding of physics of shock fronts when CR's and oblique B-fields interact. They aid in the understanding of such phenomena as diffusive shock acceleration which can produce energetic particles. The production and re-energizing of charged particles in supernova shocks has been widely accepted as an explanation for the origin of galactic CR's (*e.g.,* Blandford and Eichler, 1987). If this theory is to be validated, it is important to verify the processes occurring in CR-MHD shocks such as supernova remnants (SNR) can adequately accelerate the CR's to give the momentum distributions that are observed in the interstellar medium. Kang and Jones (1991) verified this result by simulating SNR and calculating the acceleration efficiency for the process. Following a similar idea, and using the equation of acceleration efficiency for CR's given in Jun *et al* (1994) as

$$\eta_s = \frac{\gamma_c}{\gamma_c - 1} \frac{(u_2 P_{c_2} - u_0 P_{c_0})}{\frac{1}{2}(\rho_0 u_0^3 - \rho_2 u_2^3)}$$

the acceleration efficiency for a small 1-D section of the SNR could be calculated using the equations in Chapter 2 to give a rough estimate for the overall acceleration efficiency after steady-state has been reached.

The second application is as a test for time-dependent, multi-dimensional computer programs that are much better equipped to simulate realistically observed astrophysical phenomena. Comparisons between the analytical solutions presented in Chapter 2 and solutions generated by **ZEUS-3D** will allow the CR-MHD algorithms to be calibrated and checked for accuracy. Once **ZEUS-3D** is able to reproduce the known analytical solutions, it can be used to solve more complex problems where no analytical solutions exist. Example include the study of astrophysical jets (Clarke *et al* 1992, Clarke 1993) and the stability of astrophysical flows (Hardee, Cooper and Clarke 1994, Hardee, Clarke and Howell 1995), supernova remnants, particle reacceleration, *etc.*. The accurate modeling of astrophysical phenomena greatly increases our confidence in the explanation of these phenomena.

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Figure Captions

Fig. 1. A schematic of the CR-MHD shock front propagating to the right (discontinuous transition) showing various features discussed in the text.

Fig. 2 $\rightarrow$ 9. Time-dependent simulations of the 1-D shock problem (solid lines) with the analytical steady-state solution overlaid (dashed line). Figures 2 through 9 were taken at $t = 0.1, 0.2$, and 0.3 (units discussed in the text) with a numerical resolution of 300 zones. θ is the obliquity of the shock, as defined in the text.

Fig. 2. Mach 10, $P_b = 1.0$, $\theta = 90^\circ$

Fig. 3. Mach 10, $P_b = 1.0$, $\theta = 60^\circ$

Fig. 4. Mach 10, $P_b = 1.0$, $\theta = 30^\circ$

Fig. 5. Mach 10, $P_b = 1.0$, $\theta = 0^\circ$

Fig. 6. Mach 10, $P_b = 0.1$, $\theta = 90^\circ$

Fig. 7. Mach 10, $P_b = 0.1$, $\theta = 60^\circ$

Fig. 8. Mach 10, $P_b = 0.1$, $\theta = 30^\circ$

Fig. 9. Mach 10, $P_b = 0.1$, $\theta = 0^\circ$

Fig. 10 $\rightarrow$ 17. Time-dependent simulations of the 1-D shock problem (solid lines) with the analytical steady-state solution overlaid (dashed line). Figures 10 through 17 were taken at $t = 1.0, 2.0$, and 3.0 with a numerical resolution of 300 zones.

Fig. 10. Mach 3, $P_b = 1.0$, $\theta = 90^\circ$

Fig. 11. Mach 3, $P_b = 1.0$, $\theta = 60^\circ$

Fig. 12. Mach 3, $P_b = 1.0$, $\theta = 30^\circ$

Fig. 13. Mach 3, $P_b = 1.0$, $\theta = 0^\circ$

Fig. 14. Mach 3, $P_b = 0.1$, $\theta = 90^\circ$

Fig. 15. Mach 3, $P_b = 0.1$, $\theta = 60^\circ$

Fig. 16. Mach 3, $P_b = 0.1$, $\theta = 30^\circ$

Fig. 17. Mach 3, $P_b = 0.1$, $\theta = 0^\circ$

Fig. 18. Reproduction of the Brio and Wu test with a numerical resolution of 550 zones, $\gamma_c = \gamma_g = 2.0$.

Fig. 19. Similar test to Fig. 18 except the pressure is divided equally between the thermal fluid and the CR-fluid.

Fig. 20. Similar test to Fig. 18 except all of the pressure is in the CR-fluid.