Ultracold Fermi Gases: Towards BCS

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Abstract

I present a brief overview of ongoing studies with ultracold trapped Fermi gases. Attention is focused on the progress in achieving quantum degeneracy and on ideas for reaching a superfluid BCS transition.

1 Introduction

Since the discovery of Bose-Einstein condensation (BEC) in trapped clouds of alkali atoms in 1995 [1–3], the field of quantum gases has become a mature field with many avenues for fundamental research. The first generation of BEC studies has revealed profound collective effects in these extremely dilute systems (collective oscillations and their damping, asymmetry of free expansion, etc; see [4] for a theory review) and outlined directions for future investigations. At present, large attention is focused on finding novel macroscopic quantum phenomena in ultracold trapped Fermi gases.

Quantum degenerate Fermi gases are fundamentally different from Bose gases. Due to the Pauli exclusion principle, in the case of identical fermions not more than one particle can occupy a given quantum state of motion. Therefore, well below the temperature of quantum degeneracy the distribution of particles is characterized by the presence of a Fermi sea. For identical fermions, occupation numbers of quantum states are equal to unity up to energies close to the Fermi energy T_F, and are zero for larger energies. The Fermi energy represents the ratio of the number of particles to the (energy) density of states and in the uniform case is given by the relation $T_F = \hbar^2(6\pi^2 n)^{2/3}/2m$, where n is the gas density, and m the atom mass [5]. The presence of a finite temperature smooths the step-wise transition from unity to zero occupation numbers in a narrow vicinity of the Fermi surface, i.e. the surface $\varepsilon = T_F$ in the energy space. For such a distribution of fermions, only particles with energies near the Fermi surface provide a response of the system to external perturbations. In this respect, the rest of the Fermi gas becomes a "dead body". This is quite different from Bose-condensed systems where all particles participate in the response.

Macroscopic quantum phenomena in degenerate Fermi gases are mostly expected in relation to the Cooper pairing and superfluid Bardeen-Cooper-Schrieffer (BCS) phase transition. This requires attractive interaction between particles. Then, at sufficiently low temperatures, particles with opposite momenta on the Fermi surface form correlated pairs in the momentum space. This leads to the appearance of a gap in the excitation spectrum which in the uniform case reads (see, e.g., [6] and Fig.1a)

$$\varepsilon_k = \sqrt{[\hbar^2(k^2 - k_F^2)/2m]^2 + \Delta^2}, \tag{1}$$

where ε_k and k are the excitation energy and momentum, $k_F = \sqrt{2mT_F/\hbar^2}$ is the Fermi momentum, and m the particle mass. The gap Δ depends on the interaction parameters, density, and temperature.

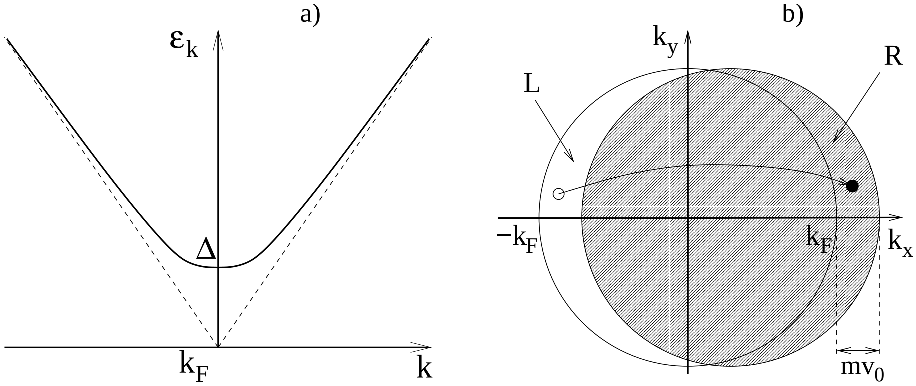

Figure 1: a) Excitation spectrum of a superfluid uniform Fermi gas; b) Momentum-space picture of a moving Fermi gas. Dark space shows the Fermi sphere of this gas.

The presence of the gap is fundamental for understanding the appearance of superfluidity in Fermi gases. Consider a Fermi gas moving with velocity v_0 which is much smaller than the Fermi velocity $v_F = \hbar k_F/m$. In the momentum space, this is equivalent to displacing the entire Fermi sphere (see Fig.1b) and can be viewed as a transfer of particles from the area L to the area R in Fig.1b, with related creation of holes in the area L. The friction is provided by an inverse process, i.e. by transferring particles from the area R to L and creating holes in R. In the superfluid phase characterized by the dispersion law (1), this process is forbidden at $T \to 0$ as the creation of a particle-hole pair costs energy 2Δ. Therefore, there is no friction. This picture requires v_0 to be smaller than the critical velocity v_c. According to the Landau criterion of superfluidity, $v_c = \min\{\varepsilon_k/\hbar k\} \approx \Delta/\hbar k_F$.

We thus see that in some sense superfluid Fermi systems are drastically different from superfluid Bose systems. Only particles near the Fermi surface really form Cooper pairs and, therefore, the "condensate of Cooper pairs" is small. Nevertheless, this tiny fraction of particles drags the rest of the system to the superfluid state, and at zero temperature the whole Fermi system becomes superfluid.

In a dilute Fermi gas, most efficient is the formation of Cooper pairs due to the attractive interparticle interaction in the s-wave channel (binary s-wave scattering). For identical fermions this channel is absent. It requires the presence of at least two fermionic components in the system, with almost equal concentrations and a negative s-wave scattering length a for the intercomponent interaction. Then the transition from the normal to superfluid state occurs at a temperature which in the uniform case is given by the relation [7]:

$$T_c = (2/e)^{7/3} T_F \exp\left(-\pi/2k_F|a|\right). \quad (2)$$

In the dilute limit one has the inequality $k_F|a| \ll 1$. Accordingly, the BCS transition temperature is much smaller than the Fermi energy (temperature of quantum degeneracy). This is one of the main difficulties in achieving the BCS transition in ongoing experiments with Fermi gases.

2 Achieving quantum degeneracy

The first experiments with cold Fermi gases date back to 1980's. They were dealing with atomic deuterium magnetically trapped in a cell with walls covered with liquid helium [8, 9]. These experiments have presented a rather detailed investigation of collisional properties. However, the achieved temperatures $T \sim 0.3$ K and densities $n \sim 10^{14}$ cm^{-3} ($T/T_F \sim 10^4$) were far from the regime of quantum degeneracy.

Experimental studies of ultracold trapped Fermi gases of alkali atoms have been started at JILA [10,11] with ^{40}K, and are now actively being carried out in a number of research centers [12–23]. These experiments have encountered most of the difficulties that one can expect for cooling fermionic atoms to well below the temperature of quantum degeneracy. The key problem is the reduction of elastic collisional rates and, hence, the reduction of the rate of evaporative cooling [24]. The absence of the s-wave scattering for collisions between identical fermions and the decrease of the p-wave collisional cross section with temperature, is not important as one usually cools a mixture of two fermionic isotopes or a mixture of fermions and bosons. However, in the regime of quantum degeneracy ($T < T_F$) elastic collisions are suppressed due to Pauli blocking. At low T/T_F, energy states below the Fermi surface are highly occupied and any collisional event resulting in a state with such an energy is suppressed by the Pauli exclusion principle. In other words, there is a reduction of the phase space available for final states of fermions in a collisional event. The "effective" cross section of elastic collisions strongly decreases with temperature as has been measured at JILA [12]. This, in turn, should lead to the suppression of evaporative cooling (see [24]). The effect of Pauli blocking is smaller for cooling of fermions due to their collisions with bosons, since the blocking then works only for one of the collisional partners.

Another problem is related to the presence of inelastic collisional processes, in particular 3-body recombination. In the regime of quantum degeneracy, aside from reducing the number of particles, the recombination produces holes in the energy distribution of particles below the Fermi surface [25]. Relaxation of the distribution then leads to heating of the sample and, according to the estimates of ref. [25], can strongly reduce possibilities for achieving the superfluid BCS transition.

On the other hand, achieving high densities and ultralow temperatures in Fermi gases is supported by the reduction of inelastic rates with decreasing temperature. This phenomenon has been first found in early deuterium studies [26] for spin dipolar relaxation in pair collisions of identical fermions, and it has the same nature as the reduction of elastic collisions. The reduction is the same for relaxation collisions of fermions in different internal states if they become identical in the outgoing channel, since in this case the orbital angular momentum can only be odd and, hence, the s-wave is absent in the incoming channel. The relaxation rate constant is proportional to T in a classical Fermi gas, and to T_F in a quantum degenerate gas. A similar behavior one finds for the rate of 3-body recombination involving two identical fermions [27,28]. The recombination of three identical fermions is suppressed more strongly: the rate constant behaves as T^2 in a classical Fermi gas, and as T_F^2 in the regime of quantum degeneracy [27].

The onset of quantum degeneracy in trapped Fermi gases has been first observed in the JILA experiment with two-component cloud of ^{40}K [11]. At present, the Fermi degeneracy has been reached in a mixture of ^{6}Li with ^{7}Li [17,18], in a two-component gas of ^{6}Li [19], in a mixture of ^{6}Li with Na [21], and a mixture of ^{40}K with ^{87}Rb [23]. The Fermi degeneracy has been identified through the measurement of the density profile, and densities $n \sim 10^{13}$ cm^{-3} have been reached. The ratio $T/T_F \approx 0.2$ achieved so far, is still not sufficiently small for reaching the superfluid BCS transition.

Theoretical work on degenerate (non-superfluid) trapped Fermi gases was mostly related to the influence of Pauli exclusion principle on their optical and collisional properties [29–33], and to collective oscillations [34–36]. In particular, the inhibition of spontaneous emission [29–31] is promising for optical manipulations of trapped samples. Experimental and theoretical studies of in-phase oscillations of two components in trapped ^{40}K in the hydrodynamic regime have revealed a significant reduction of the damping time compared to classical expectations [13].

3 Mechanisms of superfluid pairing

Superfluid pairing in Fermi gases requires attractive interaction between particles. As mentioned in the Introduction, in the ultracold limit most efficient should be pairing due to the s-wave scattering of particles. The s-wave pairing is possible only between two different fermionic species in the gas, for example fermionic atoms in different hyperfine states [37]. However, it is important to have almost equal concentrations of the two hyperfine species, since the difference in their Fermi energies by an amount exceeding the gap Δ will essentially destroy the pairing.

In sufficiently large trapped fermionic samples, the BCS transition temperature T_c is close to that for a uniform gas of density n equal to the maximum density in the trap [38–40], and one can thus rely on Eq.(2). This requires the condition $T_c \gg \hbar\omega$, where ω is a characteristic trap frequency. Note that temperatures achieved by evaporative cooling in current experiments are usually much larger than $\hbar\omega$. The BCS transition in trapped Fermi gases does not lead to a significant modification of the density profile. This originates from the fact that for a weakly interacting gas the gap Δ and the BCS temperature T_c are much smaller than the Fermi energy and the "condensate of Cooper pairs" is small. Thus, under the condition $\hbar\omega \ll T_c \ll T_F$ the superfluid trapped Fermi gas at temperatures well below T_c can be described in the common Thomas-Fermi approach.

The s-wave superfluid pairing was mostly discussed for ^{6}Li, where the scattering length for the interspecies interaction can be large and negative [41], and one can have BCS temperatures approaching 100 nK at densities $n \sim 10^{13}$ cm^{-3}. In the present stage, experiments with non-degenerate two-component gases of ^{40}K [15] and ^{6}Li [20, 22] have used Feshbach resonances for the intercomponent interaction and tuned the corresponding scattering length in a wide range. In the lithium case [20,22], the decay rates in the vicinity of the Feshbach resonance were found to be much smaller than for bosonic gases. This is consistent with the suppression of inelastic processes in Fermi gases (see previous Section) and is promising for achieving the superfluid transition at a large and negative scattering length. An interesting possibility implies an adiabatic transfer of a small fraction of atoms from a magnetic trap to a superimposed tighter optical trap [42]. This yields a strong increase of the density and Fermi energy for the transferred fraction, without a significant change of the temperature, and brings the system much closer to the BCS transition.

The BCS pairing of identical fermions can occur in the p-wave channel, but the corresponding transition temperature is generally extremely low. On the other hand, this temperature should be strongly enhanced by polarization of the medium in the presence of one more fermionic component [43]. For a large "effective" p-wave interaction the resulting T_c can be of the same order of magnitude as in the case of s-wave pairing and there are no severe restrictions on the concentrations of the components. One may speculate on possibilities of the p-wave pairing in view of the recent JILA experiment with a non-degenerate gas of ^{40}K [16], where the p-wave interaction has been tuned by a Feshbach resonance and the elastic p-wave cross section increased by about 3 orders of magnitude.

Possibilities of a tight optical confinement of a moderate number of atoms stimulate investigations of superfluid pairing in the so called intrashell regime, where the transition temperature T_c and the order parameter Δ are smaller than the trap frequencies [44, 45]. In this regime, the Fermi gas can be in many aspects regarded as a large nucleus. In a spherical trap, the Cooper pairs are formed between particles which belong to the same harmonic oscillator shell and have angular quantum numbers (l, m) and $(l, -m)$. Although the transition temperature is smaller than the trap frequency, it can still be much higher than the prediction of Eq.(2) [45]. For frequencies of the order of several kHz one can have T_c of several tens of nanokelvins.

In the case of the s-wave pairing, a comparatively high BCS temperature T_c is reached at a large negative scattering length a for the intercomponent interaction. If the scattering length becomes very large and the parameter $k_F|a| > 1$, the gas leaves the weakly interacting regime and Eq.(2) is no longer valid. The gas then enters a peculiar strongly interacting regime. It is still dilute and the mean interparticle separation is much larger than the characteristic radius of interaction between particles. However, the amplitude of binary interactions (scattering length) exceeds the interparticle separation. Then the scattering length looses its meaning and one has an intriguing goal to analyze how close the BCS transition temperature could be to the Fermi energy T_F. The effects of increasing two-particle interactions due to the exchange of density fluctuations in Fermi gases and Fermi-Bose mixtures and possible upper bound for the ratio T_c/T_F have been discussed in refs. [46–49].

The use of Feshbach resonances can allow one to achieve very large positive or negative values of an effective scattering length and thus reach the strongly interacting regime in an ultracold gas. This will be the case in the vicinity of a Feshbach resonance, where for the two-particle problem one has a strong coupling between (zero-energy) continuum states of colliding atoms and a weakly bound molecular state of another hyperfine domain. The effective coupling strength depends on the detuning from resonance (the energy difference between the bound molecular state and the continuum states), which can be varied by changing the magnetic field. In the strongly interacting regime, the effective scattering length for the Feshbach resonance is no longer a relevant quantity to describe the system and one has to explicitly incorporate resonance coupling into the many-body physics.

For a large detuning from resonance the gas is still in the weakly interacting regime. In a two-component Fermi gas, for a large negative detuning (negative effective scattering length) one expects the BCS pairing between distinguishable fermions, and for a large positive detuning encounters the problem of Bose-Einstein condensation of diatomic molecules of these fermions. A subtle question is then to describe a crossover from the BCS to BEC behavior and find the superfluid transition temperature in the crossover regime. This type of crossover has been discussed in literature in the context of superconductivity [50–53] and in relation to superfluidity in two-dimensional films of ^{3}He [54, 55].

The described idea of resonance coupling through a Feshbach resonance for achieving a superfluid phase transition in ultracold two-component Fermi gases has been proposed in refs. [56, 57] and then developed in several contributions [58–60]. The superfluid transition in the crossover regime is strongly influenced by fluctuations and, according to refs. [59, 60], the maximum transition temperature for a spatially homogeneous gas is $T_c \approx 0.26 T_F$ [59]. Such value of T_c is achievable in current experimental studies and, as discussed above, there are now many experiments investigating the behavior of ultracold Fermi gases in the vicinity of Feshbach resonances.

Fermi gases of polar molecules present a different physical picture [61] and are now attracting a great deal of interest in view of the recent progress in cooling and trapping these molecules [62, 63]. Being electrically or magnetically polarized, polar molecules interact with each other via long-range anisotropic dipole-dipole forces. In the ultracold limit, the main contribution to the dipole-dipole scattering amplitude comes from distances between particles of the order of their de Broglie wavelength. Accordingly, the amplitude is energy independent for any angular momenta in the incoming and outgoing channels (see [61] and refs. therein). This opens prospects to achieve superfluid pairing in single-component Fermi gases of polar molecules, where only scattering with odd orbital momenta is present. These prospects are especially interesting, since inelastic collisions occur at short interparticle distances and will be suppressed in the same way as in single-component atomic Fermi gases.

The amplitude of dipole-dipole scattering with odd orbital momenta is independent of short-range physics. The BCS pairing originates from the fact that the dipole-dipole interaction is partially attractive. In a weakly interacting gas the Fermi energy T_F greatly exceeds the mean dipole-dipole interaction nd^2, where d is the dipole moment of a particle. Then, in the uniform case the BCS transition temperature is determined by the relation [61]:

$$T_c = 1.44 T_F \exp\left(-\pi T_F / 12 n d^2\right). \tag{3}$$

For most polar molecules the dipole moment ranges from 0.1 to 1 Debye. For example, fermionic molecules $^{15}ND_3$ trapped in the Rijnhuizen experiment [63] have $d = 1.5$ D. From Eq.(3) one then finds that the BCS transition temperature for a single-component ND_3 dipolar gas approaches 100 nK at densities exceeding $\sim 10^{12}$ cm^{-3}.

4 How to detect a BCS state?

A large part of the theoretical work on superfluid trapped Fermi gases has been focused on the problem of identifying the presence of a BCS state. The existing ideas rely on the detection of Cooper pairing through the observation of effects related to the excitation spectrum [34,44,45,64–66,73–75] and optical [67–73] characteristics. Below the critical temperature T_c, superfluid pairing and the presence of the gap modify the spectrum of single-particle excitations and lead to the appearance of collective Anderson-Bogoliubov excitations. These can be found from hydrodynamic equations. Well below T_c, collective oscillations are not damped and can manifest themselves as eigenmodes of the density oscillations [65]. Then the lowest eigenfrequencies of collective modes can be measured by modulating the trap frequencies, i.e. in the same way as in Bose condensates. These eigenfrequencies are very different from those of a non-superfluid gas in the collisionless regime [34, 44, 45, 65], and one thus can conclude the presence of superfluid pairing. Interestingly, the eigenfrequencies of surface and scissors modes of a superfluid gas are independent of the density profile [74, 75]. Accordingly, for the BCS-paired state they will be the same as in the case of Bose condensates. Single-particle excitations can also be addressed, in particular by light. In a trapped gas below T_c, their spectrum is strongly influenced by the spatial inhomogeneity of the gap [64]. The existence of low-energy in-gap excitations proves to be important for laser probing of superfluid pairing [71].

There are several proposals for spectroscopic measurements of the BCS pairing and the order parameter (gap). It has been predicted that pairing influences the spectral and spatial distribution of scattered off-resonant light [67,69], and increases the optical line width and shift [68]. The idea of using on-resonant light for transferring atoms from the internal state which is Cooper-paired to another (hyperfine) state which is not paired, suggests that the absorption peak is shifted and becomes asymmetric because of the existence of the gap [70, 71]. A random-phase approximation for finding the dynamical structure factor which determines the response of the system to incident light or probe particles, has been developed in ref. [73].

We now discuss how to identify a superfluid phase transition from the free expansion of a trapped gas, after abruptly switching off the trap. The free expansion of trapped Bose gases gave a spectacular evidence of BEC in the pioneering experiments at JILA [1] and MIT [2]. The expansion of a trapped condensate is asymmetric, whereas a collisionless thermal cloud expands symmetrically and (if the temperature exceeds the chemical potential) much faster than the condensate.

What is the situation with free expansion in trapped Fermi gases [76]? A non-superfluid degenerate gas in the collisionless regime at $T \ll T_F$ expands symmetrically with velocities $\sim v_F = \sqrt{2T_F/m}$. Interparticle collisions play a minor role due to Pauli blocking [76]. The behavior of a BCS-paired gas is different. In the Thomas-Fermi regime well below T_c, a major part of the gas is in the superfluid phase and its expansion can be described in the hydrodynamic approach. The continuity and Euler equations read:

$$\dot{n} + \nabla(n\mathbf{v}) = 0; \quad m\dot{\mathbf{v}} + \nabla[\mu(n) + V(\mathbf{r}) + mv^2/2] = 0, \tag{4}$$

where $\mathbf{v}$ is the velocity field, $V(\mathbf{r})$ is the trapping potential which is switched off at a time $t = 0$, $n(\mathbf{r}, t)$ is the density at the point $\mathbf{r}$, and $\mu(n)$ is the chemical potential of a uniform gas at density equal to $n(\mathbf{r}, t)$. The same equations are valid in the case of a Bose-Einstein condensate and for a hydrodynamic thermal cloud. The only difference is related to the dependence $\mu(n)$.

In the case of harmonic trapping and power law dependence $\mu(n)$, the hydrodynamic equations (4) allow a scaling solution (see [76] and refs. therein). For a condensate one has $\mu \sim n$ and the scaling solution [77–79] describes existing experimental data. A degenerate Fermi gas, as well as a hydrodynamic thermal cloud, is characterized by the chemical potential $\mu \propto n^{2/3}$. The scaling solution of Eqs. (4) for this case has been obtained in ref. [79]. The asymmetry of the expansion is almost the same as in the case of condensates. For example, an initially cigar-shaped cylindrical sample ultimately becomes a pancake. The time dependence of the aspect ratio of a cylindrical cloud for several initial aspect ratios is calculated in ref. [76], and the influence of interparticle interaction on the dependence $\mu(n)$ is found to be not important.

One thus sees that the free expansion of a quantum degenerate trapped Fermi gas will give clear signatures of the presence of BCS pairing. However, the absolute velocity of the expansion is of the order of the Fermi velocity v_F, as well as in the case of a non-superfluid gas. Therefore, at temperatures comparable with T_c, where the non-superfluid fraction is significant, the asymmetry of the expansion can be smaller than that following from Eqs. (4). The reason is that the symmetrically expanding non-superfluid fraction does not "fly away" from the superfluid one. One should also make sure that the expanding gas does not leave the Thomas-Fermi regime. This is likely to require that initially the sample is deeply in this regime.

Another "underwater stone" is related to the fact that the nonsuperfluid degenerate Fermi gas in the hydrodynamic regime expands in the same asymmetric way as the superfluid gas, except for a small difference originating from the interparticle interaction. At temperatures well below T_F, irrespective of the presence/absence of superfluidity, the chemical potential is $\mu \propto n^{2/3}$ and Eqs. (4) describing the nonsuperfluid hydrodynamic expansion take the same form as in the superfluid case. Therefore, in order to conclude on achieving superfluidity from the asymmetry of expansion one should make sure that above T_c the gas is in the collisionless regime. This is especially important as the attempts to reach the superfluid transition rely on tuning the scattering length to a large negative value. Accordingly, the collisional frequency increases and the mean free path of particles reduces, which drives the gas towards the hydrodynamic regime.

A large asymmetry of free expansion has been observed in the recent Duke experiment [80] with a strongly degenerate ($T/T_F \sim 0.2$) two-component gas of ^{6}Li. The aymmetry can be due to the presence of superfluidity or, alternatively, due to the intermediate hydrodynamic/collisionless type of nonsuperfluid expansion. Importantly, the gas is in the unitarity limit for atomic collisions: the scattering length has been tuned to $a \approx -5000$ Å and, before switching off the trap, the parameter $k_F|a|$ is close to 7. In these conditions the cross section of elastic collisions is $\sigma \propto (1/k_F^2)F(T/T_F)$, where the factor $F(T/T_F)$ takes into account the reduction of the collisional rate due to Pauli blocking. The mean free path of particles is $\lambda \sim (1/n\sigma)$ and initially the ratio of λ to the radial size R of the cigar-shaped cloud is slightly smaller than unity. So, the gas is at most in the intermediate regime. However, in the course of expansion it is getting deeper to the hydrodynamic regime. This is due to the unitarity limit for atomic collisions. As the ratio T/T_F remains unchanged in the expanding cloud, the mean free path scales with density as $\lambda \propto k_F^2/n \propto n^{-1/3}$. On the other hand, even at the largest times of flight the axial size of the cloud in the Duke experiment is not significantly changing. Hence, the radial size is changing as $R \propto n^{-1/2}$, which gives the ratio λ/R decreasing with density as $n^{1/6}$. This should be the case until the parameter $k_F|a| \propto n^{1/3}$ becomes of order unity and the gas leaves the unitarity limit, which in the Duke experiment occurs at about the largest expansion times.

5 Concluding remarks

A brief overview of research on ultracold Fermi gases shows that they present remarkable physics, and one expects more interesting developments in the near future. For example, recent LENS experiments [81] have found a collapse of a trapped fermionic cloud of ^{40}K mixed with a BEC of ^{87}Rb. This phenomenon finds its origin in a strong attraction of fermions by condensed bosons located in the trap center: if the number of bosons is increased such that the attraction energy "beats" the Fermi energy, the fermionic cloud collapses [82].

The field of cold fermionic atoms is continuously being filled with interesting new ideas. The recent idea of ref. [83] predicts superfluid pairing between two fermionic components characterized by different Fermi energies. The pairing occurs near the Fermi surface of the smaller Fermi sphere, and at $T = 0$ one then gets a system which contains a superfluid and a normal Fermi gas simultaneously. The superfluid transition temperature is expected to be of the order of the "ordinary" BCS transition temperature in Fermi gases. Recently, it has also been predicted that a novel system of fermionic atoms in an optical lattice can have a high transition temperature to a superfluid state [84]. Experimental studies in this direction can provide a new insight into the origin of high-temperature superconductivity in cuprates.

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References

[1] M.H. Anderson *et al.*, Science **269**, 198 (1995).

[2] K.B. Davis *et al.*, Phys. Rev. Lett. **75**, 3969 (1995).

[3] C.C. Bradley *et al.*, Phys. Rev. Lett. **75**, 1687 (1995).

[4] F. Dalfovo *et al.*, Rev. Mod. Phys. **71**, 463 (1999).

[5] In the presence of N fermionic components with equal masses and concentrations, one has to replace n by n/N in the expression for T_F.

[6] E.M. Lifshitz and L.P. Pitaevskii, *Statistical Physics* (Pergamon Press, Oxford, 1980), Part 2.

[7] L.P. Gor'kov and T.K. Melik-Barkhudarov, Zh. Eksp. Teor. Fiz. **40**, 1452 (1961) [Sov. Phys. JETP **13**, 1018 (1961)].

[8] I.F. Silvera and J.T.M. Walraven, Phys. Rev. Lett. **45**, 1268 (1980).

[9] I. Shinkoda *et al.*, Phys. Rev. Lett. **57**, 1243 (1986).

[10] B. DeMarco *et al.*, Phys. Rev. Lett. **82**, 4208 (1999).

[11] B. DeMarco and D.S. Jin, Science **285**, 1703 (1999).

[12] B. DeMarco *et al.*, Phys. Rev. Lett. **86**, 5409 (2001)

[13] B. DeMarco and D.S. Jin, Phys. Rev. Lett. **88**, 040405 (2002).

[14] S.D. Gensemer and D.S. Jin, Phys. Rev. Lett. **87**, 173201 (2001).

[15] T. Loftus *et al.*, Phys. Rev. Lett. **88**, 173201 (2002).

[16] C.A. Regal *et al.*, cond-mat/0209071.

[17] A.G. Truscott *et al.*, Science **291**, 2570 (2001).

[18] F. Schreck *et al.*, Phys. Rev. Lett. **87**, 080403 (2001).

[19] S.R. Granade *et al.*, Phys. Rev. Lett. **88**, 120405 (2002).

[20] K.M. O'Hara *et al.*, cond-mat/0207717.

[21] Z. Hadzibabic *et al.*, Phys. Rev. Lett. **88**, 160401 (2002).

[22] K. Dieckmann *et al.*, cond-mat/0207046.

[23] G. Roati *et al.*, Phys. Rev. Lett. **89**, 150403 (2002).

[24] M.J. Holland *et al.*, Phys. Rev. A **61**, 053610 (2000).

[25] E. Timmermans, Phys. Rev. Lett. **87**, 240403 (2001)

[26] J.M.V.A. Koelman *et al.*, Phys. Rev. Lett. **59**, 676 (1987); Phys. Rev. B, **38**, 9319 (1988).

[27] B.D. Esry *et al.*, Phys. Rev. A **65**, 010705(R) (2001).

[28] D.S. Petrov, cond-mat/0209246.

[29] Th. Busch *et al.*, Europhys. Lett. **44**, 1 (1998).

[30] B. DeMarco and D.S. Jin, Phys. Rev. A, **58**, R4267 (1998).

[31] J. Ruostekoski and J. Javanainen, Phys. Rev. Lett. **82**, 4741 (1999).

[32] G. Ferrari, Phys. Rev. A, **59**, R4125 (1999).

[33] B. DeMarco *et al.*, Phys. Rev. Lett. **82**, 4208 (1999).

[34] G.M. Bruun and C.W. Clark, Phys. Rev. Lett. **83**, 5415 (1999).

[35] L. Vichi and S. Stringari, Phys. Rev. A, **60**, 4734 (1999).

[36] G.M. Bruun, Phys. Rev. A, Phys. Rev. A, **63**, 043408 (2001).

[37] H.T.C. Stoof *et al.*, Phys. Rev. Lett. **76**, 10 (1996).

[38] H.T.C. Stoof and M. Houbiers, in *Bose-Einstein Condensation in Atomic Gases*, Varenna School, 1999, course CXL.

[39] M.A. Baranov and D.S. Petrov, Phys. Rev. A **58**, R801 (1998).

[40] G.M. Bruun *et al.*, Eur. Phys. J. D **7**, 433 (1999).

[41] E.P.I. Abraham *et al.*, Phys. Rev. A **55**, R3299 (1997).

[42] L. Viverit *et al.*, Phys. Rev. A, **63**, 033603 (2001).

[43] M.A. Baranov *et al.*, JETP Lett. **64**, 301 (1996) and refs. therein.

[44] G.M. Bruun and B.R. Mottelson, Phys. Rev. Lett. **87**, 270403 (2001); G.M. Bruun and H. Heiselberg, Phys. Rev. A, **65**, 053407 (2002); H. Heiselberg and B.R. Mottelson, Phys. Rev. Lett. **88**, 190401 (2002).

[45] G.M. Bruun, cond-mat/0207213.

[46] R. Combescot, Phys.Rev. Lett. **83**, 3766 (1999).

[47] M.J. Bijlsma, B.A. Heringa, and H.T.C. Stoof, Phys. Rev. A, Phys. Rev. A, **61**, 053601 (2000).

[48] H. Heiselberg *et al.*, Phys. Rev. Lett. **85**, 2418 (2000).

[49] L. Viverit, Phys. Rev. A, **66**, 023605 (2002).

[50] D.M. Eagles, Phys. Rev. **186**, 456 (1969).

[51] A.J. Leggett, in *Modern Trends in the Theory of Condensed Matter*, edited by A. Pekalski and J. Przystawa (Springer, Berlin,1980).

[52] P. Nozieres and S. Schmitt-Rink, J. Low Temp. Phys. **59**, 195 (1985).

[53] See for review M. Randeria, in *Bose-Einstein Condensation*, edited by A. Griffin, D.W. Snoke, and S. Stringari (Cambridge University Press, Cambridge,1995).

[54] K. Miyake, Progr. Theor. Phys. **69**, 1794 (1983).

[55] See for review M.Yu. Kagan, Sov. Physics Uspekhi **37**, 69 (1994).

[56] M. Holland *et al.*, Phys. Rev. Lett. **87**, 120406 (2001).

[57] E. Timmermans *et al.*, Phys. Lett. A **285**, 228 (2001).

[58] M.L. Chiofalo *et al.*, Phys. Rev. Lett. **88**, 090402 (2002); S.J.J.M.F. Kokkelmans *et al.*, Phys. Rev. A, **65**, 053617 (2002).

[59] J.N. Milstein *et al.*, Phys. Rev. A **66**, 043604 (2002).

[60] Y. Ohashi and A. Griffin, Phys. Rev. Lett. **89**, 130402 (2002).

[61] M.A. Baranov *et al.*, Phys. Rev. A **66**, 013606 (2002).

[62] J.D. Weinstein *et al.*, Nature **395**, 148 (1999); J.M. Doyle and B. Friedrich, Nature **401**, 749 (1999).

[63] H.L. Bethlem *et al.*, Phys. Rev. Lett. **83**, 1558 (1999); H.L. Bethlem *et al.*, Nature **406**, 491 (2000); H.L. Bethlem *et al.*, Phys. Rev. A **65**, 053416 (2002) and refs. therein.

[64] M.A. Baranov, JETP Lett. **70**, 396 (1999).

[65] M.A. Baranov and D.S. Petrov, Phys. Rev. A, **62**, 041601 (2000).

[66] G.M. Bruun and C.W. Clark, J. Phys. B **33**, 3953 (2000).

[67] W. Zhang *et al.*, Phys. Rev. A **60**, 504 (1999).

[68] J. Ruostekoski, Phys. Rev. A **60**, R1775 (1999).

[69] F. Weig and W. Zwerger, Europhys. Lett. **49**, 282 (2000).

[70] P. Torma and P. Zoller, Phys. Rev. Lett. **85**, 487 (2000).

[71] G.M. Bruun *et al.*, Phys. Rev. A **64**, 033609 (2001).

[72] M. Rodriguez and P. Torma, Phys. Rev. A **66**, 033601 (2002).

[73] A. Minguzzi *et al.*, Eur. Phys. J. D **17**, 49 (2001).

[74] F. Zambelli and S. Stringari, Phys. Rev. A **63**, 033602 (2001).

[75] A. Minguzzi and M.P. Tosi, Phys. Rev. A **63**, 023609 (2001).

[76] C. Menotti *et al.*, cond-mat/0208150.

[77] Yu. Kagan *et al.*, Phys. Rev. A **54**, R1753 (1996).

[78] Y. Castin and R. Dum, Phys. Rev. Lett. **77**, 5315 (1996).

[79] Yu. Kagan *et al.*, Phys. Rev. A **55**, R18 (1997).

[80] K.M. O'Hara *et al.*, Science, 07/11/2002, p. 1079107.

[81] G. Modugno *et al.*, Science **297**, 2240 (2002).

[82] R. Roth and H. Feldmeier, Phys. Rev. A **65**, 021603 (2002); R. Roth, Phys. Rev. A **66**, 013614 (2002).

[83] W. Vincent Liu and F. Wilczek, cond-mat/0208052.

[84] W. Hofstetter *et al.*, cond-mat/0204237.