

Homework 1

Astronomy 202a

Fall 2009

Solutions

Problems:

1. A galaxy has an integrated blue magnitude of 6.5, a rotation velocity (essentially flat) of 107 km s^{-1} , and is elliptical in overall shape with major and minor axes of $73' \times 43'$. (a) Using Opik's method of estimating distances, what is the approximate distance to this galaxy?

Another galaxy has a B magnitude of 9.27, a rotation velocity of 280 km s^{-1} , an apparent diameter of $8.7'$ and is essentially edge on. It is at a known distance of 16.7 Mpc. (b) What is its blue mass-to-light ratio in solar units?

(Hints: You will need to correct the rotation velocity for inclination effects. You will also need to know the conversion between blue magnitudes and flux. See the course webpage for that information. *Explain any assumptions.*)

Solution (a) Opik's equation is derived from:

$$\frac{GM}{r^2} = \frac{V_{rot}^2}{r}; \quad L = 4\pi D^2 l; \quad r = \theta D; \quad Y = M/L$$

$$D = \frac{V_{rot}^2 \theta}{4\pi G l Y}$$

where l is the galaxy flux and Y is the mass to light ratio in the same band and $\theta = 73'/2 = 10^{-2} \text{ rad}$ is the angular radius of the disk. **NOTE THE ERRORS IN THE LECTURE NOTES!** First one must correct the rotation velocity for the inclination of the galaxy. There are various formulae for such corrections based on assumptions about the intrinsic thickness of the disks (realistically the disk + the bulge). One of the simplest is the Danvers formula using the major and minor axes, a and b :

$$\cos^2(i) = \frac{(b/a)^2 - \alpha^2}{1 - \alpha^2}$$

where α , the intrinsic thickness of an average spiral, is approximately 0.13, and

$$V_{rotation} = \frac{V_{observed}}{\sin(i)}.$$

For $a = 73'/2$, $b = 43'/2$, we have: $V_{rotation} = 132$ km/s. The B-band apparent magnitude of the Sun is -26 , so the B-band flux of the galaxy is

$$l = 10^{(6.5 - (-26)) / (-2.5)} = 10^{-13} l_{B,\odot}$$

where $l_{B,\odot}$ is the B-band flux of the Sun. Assuming a B-band M/L for the galaxy of $Y \sim 3 M_{\odot} / L_{B,\odot}$, where $L_{B,\odot}$ is the B-band magnitude of the Sun, we have:

$$lY = (10^{-13} l_{B,\odot}) \left(3 \frac{M_{\odot}}{L_{B,\odot}} \right) = \frac{3 \times 10^{-13}}{4\pi (1AU)^2} M_{\odot}.$$

Putting it all together, we find:

$$D = 3.2 \text{ Mpc} \left(\frac{Y}{3 M_{\odot} / L_{B,\odot}} \right)^{-1}.$$

(b) As before:

$$\frac{Y}{M_{\odot} / L_{B,\odot}} = \frac{V_{rot}^2 \theta}{4\pi G l D} \frac{L_{B,\odot}}{M_{\odot}}$$

with $V_{rot} = 280$ km/s, $D = 16.7$ Mpc and $\theta = 8.7'/2 = 1.3 \times 10^{-3}$ rad. The B flux of the galaxy is

$$l = 10^{(9.27 - (-26)) / (-2.5)} = 8 \times 10^{-15} l_{B,\odot} = 8 \times 10^{-15} \frac{L_{B,\odot}}{4\pi (1AU)^2}.$$

Putting it together yields:

$$Y = 4 M_{\odot} / L_{B,\odot}.$$

2. Write a program to do coordinate conversions between equatorial (celestial) coordinates, galactic coordinates and supergalactic coordinates. What are the galactic and supergalactic coordinates that correspond to $13.00^h +28^\circ$? $5.50^h -10^\circ$? $12.45^h +12.9^\circ$? (attach your program in the solution)

Solution The celestial points

$$\{(13.00^h, +28^\circ), (5.50^h, +10^\circ), (12.45^h, +12.9^\circ)\}$$

correspond to the galactic points

$$\{(58.1^\circ, +87.9^\circ), (213^\circ, -22.6^\circ), (279.9^\circ, +74.7^\circ)\}$$

and the supergalactic points

$$\{(89.6^\circ, +8.37^\circ), (309^\circ, -68.5^\circ), (102.1^\circ, -3.09^\circ)\}.$$

3. The NFW Profile (Navarro, Frenk and White 1996, ApJ 462, 563) is the most commonly used modern density profile for galaxies and is based on the results of CDM simulations:

$$\frac{\rho(r)}{\rho_{crit}} = \frac{\delta_c}{(r/r_s)(1 + r/r_s)^2}$$

where $r_s = r_{200}/c$ is a characteristic radius and $\rho_{crit} = 3H^2/8\pi G$ is the cosmological critical density, and δ_c and c are two dimensionless parameters. This profile is linked to the critical density. The mass of the so described halo is

$$M_{200} = 200\rho_{crit}(4\pi/3)r_{200}^3$$

and the definition of r_{200} is that the mean density within r_{200} is $200\rho_{crit}$, with

$$\delta_c = \frac{200}{3} \frac{c^3}{[\ln(1+c) - c/(1+c)]}$$

δ_c can be considered the characteristic overdensity of the halo, r_s is its scale radius, and c is its “concentration.”

What is the surface brightness profile predicted from this density profile if you assume $M/L = \text{constant}$? (Remember you have to integrate the projected radial density along the line of sight). How does the shape of this profile differ from a deVaucouleurs $r^{1/4}$ law profile? (Plot your results).

Solution The goal of this problem was to get you to think about how observations of the shapes (profiles) of galaxies are compared (1) with each other and (2) with theory, especially since the theory predicts $\rho(r)$, the run of density, and the observations are $I(r)$, the run of surface brightness.

For the purpose of comparing the profile shapes, all you need is the parametric form for the shape as a function of projected radius. For the NFW profile, we have the 3-D spherical density as

$$\frac{\rho(r)}{\rho_{crit}} = \frac{\delta_c}{(r/r_s)(1 + r/r_s)^2}$$

or, setting the simple constants to unity,

$$\rho(r) = \frac{1}{(r/r_s)(1 + r/r_s)^2}$$

Its useful to keep the r_s for now just to demonstrate later how the scaling affects the profile. This profile needs to be integrated along the line of sight (versus projected radius, R) to get the projected surface brightness profile.

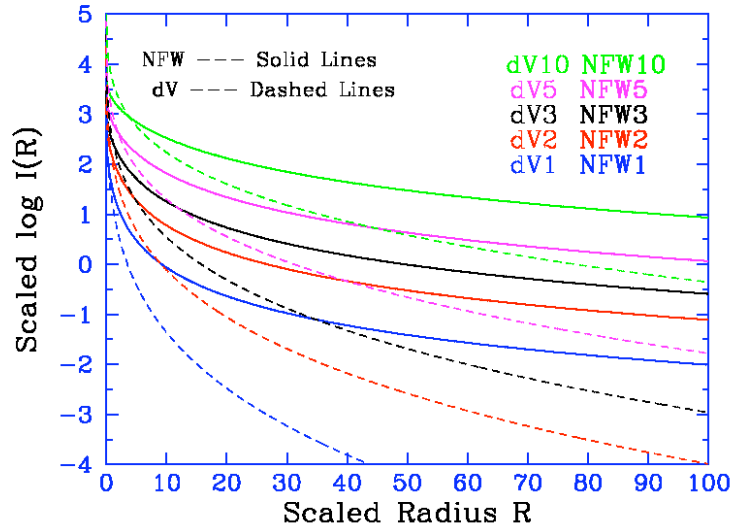


Figure 1: Large radii comparison between the surface brightness profiles predicted by the NFW density profile and deVaucouleurs' profiles with similar characteristics.

The deVaucouleurs profile, using similar notation, is

$$I(R) = I(0) \exp[-7.67((R/R_e)^{1/4} - 1)] ,$$

and again is easy to compute with a short program.

What about the comparison? First, just look at the equations. The NFW profile has a singularity at $r=0$ and this is not removed by the line-of-sight integration at small R . The DV profile, on the other hand does not.

4. The bulge-to-disk ratio for a spiral galaxy is the ratio of the luminosity in the bulge to that in the disk. Typically bulge-to-disk ratios decrease with increasing (later) morphological type. Calculate the bulge-to-disk ratio for a typical spiral galaxy with a de Vaucouleur's Law bulge and an exponential disk as a function of the variables r_s , r_e , I_s and I_e .

Solution Integrate the two surface brightness relations. Remember

For Ellipticals:

$$I(R) = I_e e^{-7.67((r/r_e)^{1/4} - 1)}$$

For Spirals:

$$I(R) = I_s e^{r/r_s}$$

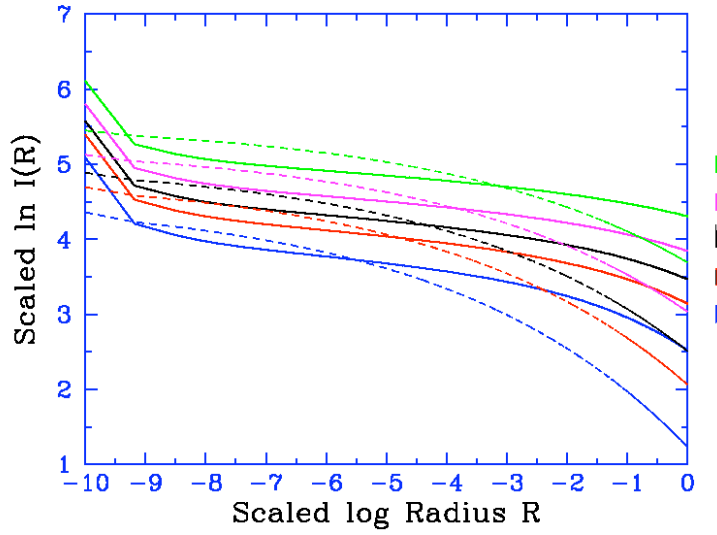


Figure 2: Comparison of the inner profiles.

So the bulge luminosity is

$$\begin{aligned}
 L_{Bulge} &= 2 \int_0^{r_e} I(r) 2\pi r dr \\
 &= 7.22\pi r_e^2 I_e
 \end{aligned}$$

(the trick is to remember that r_e is the “half-light” radius), and the disk luminosity is

$$L_{Disk} = 2\pi \int_0^\infty I_S e^{-r/r_s} f dr$$

thus

$$\frac{B}{D} = 3.57 \left(\frac{r_e}{r_s} \right)^2 \frac{I_e}{I_S}$$