Homework 2

Astronomy 202a

Fall 2009

Due October 7th, 2009

Problems:

Problem 1.Getting a feel for distribution functions:

1. For a Maxwellian distribution of velocities with one dimensional velocity dispersion σ, show that: (a) the mean speed is $\bar{v} = \sqrt{8/\pi\sigma}$; (b) the mean square speed is $\bar{v^2} = 3\sigma^2$; (c) the mean square of one component of the velocity is $\bar{v_x^2} = \sigma^2$; (d) the mean square relative speed of any two particles is $\bar{v_r^2} = 6\sigma^2$; (d) the fraction of particles with $v^2 = 4\bar{v^2}$ is 0.00738 (BT problem 4-18) what is the significance of this last number?

2. Consider a Hernquist model (ApJ **356** 359, 1990), $\rho(r) = Ma/2\pi r(r+a)^3$. Find the formulae for the enclosed mass and the gravitational potential as a function of radius, r. Using the distribution function obtained in that paper (equation 17) plot the velocity distribution for the z component of the velocity at $r/a = 0.01, 1, 100$ (you will need to integrate numerically over v_x and v_y but the dependence is only through $v_x^2 + v_y^2$ so you it will be easier to use polar coordinates). Compare the results with a Gaussian distribution with the same variance.

Problem 2. Applying the Virial Theorem

The measurement of the masses of astronomical objects is one of the most fundamental jobs of an astrophyscist. There are several techniques for doing so

On the course website (http://cfa-www.harvard.edu/∼huchra/ay202/homework) is a file containing coordinates and apparent radial velocities in km/s for galaxies in a 3 degree circle around the center of the Coma Cluster. The file format is basically object name, coordinates in right ascenscion and declination, apparent magnitude, heliocentric radial velocity (in km/s) and velocity error.

1. Write a program to look at the velocity histogram for the cluster and then to calculate (after eliminating background and foreground objects!) the mass of the cluster core

using the virial theorem estimator and projected mass estimators of Heisler, Tremaine and Bahcall (1985, ApJ 298, 8). For Coma, calculate the mean heliocentric velocity, correct for galactic rotation ($300 km/s sin(l) cos(b)$) (small! – use your galactic coordinate conversion program to find l and b) and assume a Hubble Constant of 70 km/s/Mpc to estimate the distance (and thus all the projected separations). What do you need to do to eliminate the background and foreground galaxies? Justify your assumptions.

$$M_{VT} = \frac{3\pi}{2G} N \frac{\sum V_i^2}{\sum_{i<j}(\frac{1}{R_{ij}})}$$

where $\sum V_i^2$ is $N \times$the velocity dispersion of the system,

$$\sigma^2 = \frac{1}{N} \sum V_i^2 = \frac{1}{N} \sum (v_i - v_{mean})^2$$

and

$$M_{PM} = \frac{f_p}{GN} \sum_{i=1}^{N} V_i^2 R_{i,C}$$

where $R_{i,C}$ is the projected separation in physical units between the ith object and the center of the system, and f_p is the projection factor, $64/\pi$ for radial orbits, $48/\pi$ for isotropic orbits and $32/\pi$ for circular orbits. (Note: these numbers are updated to the second paper in the PM series).

2. What might you expect the average orbital eccentricities to be for or test particles in the cluster?

   You can also define the system size in terms of the mean projected radius

$$R_P = \frac{1}{N} \sum R_{i,C}$$

   or the mean harmonic radius,

$$R_H = \frac{\pi}{2} \frac{N^2}{\sum_{i<j}(\frac{1}{R_{ij}})}$$

   How differerent are the two size estimators for our systems?

   Please give the mass estimates in solar units. After you've done this, consider why, for this cluster, these estimates of the mass may be incorrect. Give me your two best reasons.

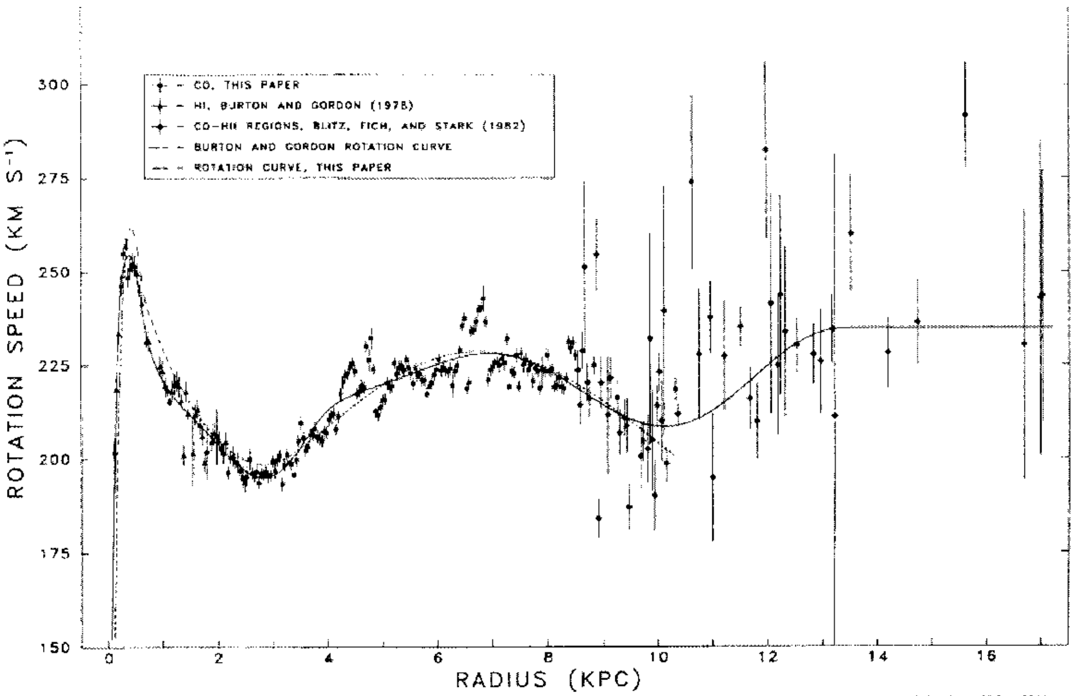

FIG. 3.—Plots of the rotation speed versus galactocentric radius. The solid lines correspond to the polynomials, and the dashed lines are the BG rotation curve. *(upper panel)* $(R_0, \theta_0) = (10$ kpc, 220 km s$^{-1})$; *(lower panel)* (8.5 kpc, 220 km s^{-1}).

Figure 1: Milky Way rotation curve derived by D. Clemens (ApJ 1985).

3. Using the magnitudes in the file for the Coma Cluster, calculate the mass-to-light ratio of the cluster in solar units. These are B-band (blue) magnitudes, so you will need to convert the absolute magnitude of each galaxy into a luminosity in solar units using the absolute B magnitude of the Sun, 5.42, and sum the luminosities of all the galaxies to get the Cluster luminosity.

Problem 3. Milky Way Mass Profile

Given Clemen's rotation curve for the Milky Way (Fig. 1), calculate its mass as a function of radius. You can read off the v(r) values from the graph.