Homework 3 Solutions

Astronomy 202a

Fall 2009

Due October 21st, 2009

Problems:

Problem 1.Luminosity Functions

A useful parametrization of the $\phi(L)$, derived by Schechter, is

$$\phi(L) = \phi_0 L^{-1} (L/L^*)^\alpha exp(-L/L^*).$$

L^* is the characteristic luminosity near the knee, ϕ_0 is the normalization and α is the slope at the faint end. The Schechter function has the interesting property that the integral luminosity density, useful in cosmology, is just given by

$$L_{int} \;=\; \phi_0 L^* \Gamma(\alpha \;+\; 2)$$

where Γ is the incomplete gamma function.

The Schechter function is also useful for estimating the amount of luminosity you fail to measure when you work with flux or magnitude limited samples. For example, in a nearby galaxy cluster you might be able to sample the LF 4 magnitudes fainter than M*, while in one 5 times further away, to the same apparent magnitude limit, you will only sample 0.5 magnitudes below M*.

A second issue, still hotly debated, is how much luminosity is actually hiding in low luminosity / low surface brightness objects. Modern redshift surveys find flat faint end slopes but only for relatively luminous galaxies and high surface brightness. An alternative view is that the real faint end slope rises much more steeply (see the works of Mike Disney, Greg Bothun and Chris Impey).

1. For a faint end slope of -1.1, how much of the integrated luminosity of the above more distant cluster will be lost relative to the nearby one?

   Solution There are two ways to interpret this questions: (a) what is the drop off due in observed luminosity due to the extra distance or (b) compare the amount of luminosity lost in each case. For case (a), the fraction is:

$$\frac{\int_{L_{\rm min,far}}^{\infty} \phi(L)\, L\, dL}{\int_{L_{\rm min,near}}^{\infty} \phi(L)\, L\, dL} \approx 50\%,$$

while for case (b) it's:

$$\frac{\int_{0}^{L_{\rm min,far}} \phi(L)\, L\, dL}{\int_{0}^{L_{\rm min,near}} \phi(L)\, L\, dL} \approx 10.$$

2. For Schechter functions of the same M*, what fraction of the integrated luminosity density lies below M* for α's of -1.0, -1.25, -1.5 and -1.85?

   Solution The fraction is:

   $$f = \frac{\int_{L^*}^{\infty} \phi(L)\, L\, dL}{\int_{0}^{\infty} \phi(L)\, L\, dL} = \frac{\int_{1}^{\infty} e^{-x}\, x^{\alpha+1}\, dx}{\Gamma(\alpha+2)}.$$

   For $\alpha = (-1.0, -1.25, -1.5, -1.85)$ the fraction of the total luminosity observed (i.e. below M^*, i.e. above L^*) is $f = (0.378, 0.260, 0.157, 0.038)$.

Problem 2. Mass Functions

The route to the luminosity and mass functions usually starts with a galaxy sample and proceeds through the determination of the masses either dynamically or via other properties such as morphological type or color. An interesting old galaxy sample to play with is that from the 1st CfA survey, which is complete to a blue magnitude of 14.5 and also has complete morphological type information. The data file can be found on the website at

http://www.cfa.harvard.edu/~huchra/zcat/cfa1.dat

and the format is moderately self explanatory. Write a program to first calculate the luminosity function, and then using an estimate of the M/L versus morphologival type, turn this into the mass function. The way to accomplish this second step is to use something like the Bell (2003) et al. calibration of g-r or B-V color versus M/L (their Figure 6 or Table 7) and a calibration of B-V versus type (can you find one!). Be sure to specify/use the right band — remember the CfA1 magnitudes are B.

Do you need to worry about K-corrections? What about dust? Galactic extinction? How do you estimate distances and thus luminosities? And what should you do about the galaxies with negative or near zero velocities?

Solution Use the Hubble relation to find the distance to each object (tossing out the ones that are too close to be moving with the Hubble flow) and the get the luminosity from the apparent magnitude. One simple correction worth making to the velocity is that to the centroid of the local group, it is, according to the IAU correcting for galactic rotation plus additional terms, simply

$$V_{GC} = 300 sin(l) cos(b)$$

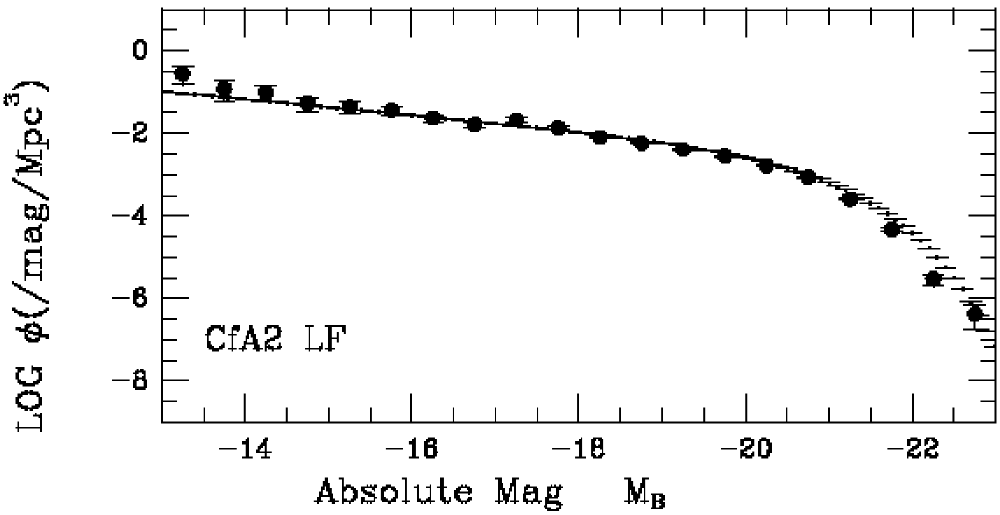

Figure 1: Luminosity function for the CfA1 sample with error bars and a Schechter function fit between -14 and -22.5. $\phi^* = 0.45\times10^{-2}$, M* = -20.32, L* = 2.1×10^{10}, and the faint end slope α = -1.48. This LF was actually derived using a full up flow field model and measured distances to nearby galaxies so it will be slightly (only slightly!) different from yours.

or if you want to be more strictly correct, according to Sandage, Tammann & Yahil

$$V_{LG} \;=\; 296 sin(l) \;-\; 79 cos(l) cos(b) \;-\; 36 sin(b)$$

which has a small component vertical to the plane of the MW. Distances should be estimated with the Hubble relation: $d = v/H_0$ where we generally adopt H_0 = 70 km/s/Mpc. Objects with abnormally low velocities will contaiminate the very faint end of the LF which really won't bug you too much for this exercise so we just eliminated galaxies with corrected velocities $\leq \sim$50 km/s. Galaxies with negative or zero velocity are not moving with the Hubble flow, and their distances cannot, thus, be estimated with the Hubble relation. In the absence of another distance indicator for these objects, they should be removed from the sample.

Sort the objects into luminosity bins. For each bin, calculate the volume out to which an objects of that luminosity can be seen. Divide the number of galaxies in each bin by the corresponding volume to get the luminosity function. That gets you the LF in magnitudes.

Repeat using the M/L for each morphology to get the mass and sorting the galaxies into mass bins. Use (1) the type-color relation (T vs B-V) from JPH (in class) and (2) the equation relating B-V to M/L from Bell et al. (2003, ApJS 149, 289)

$$log(M/L)_B \;=\; -0.942 \;+\; 1.737(B-V)$$

Remember that the mass function is the *number density* of galaxies of a given mass.

TABLE 4

MEAN COLORS OF GALAXIES

Type	Number	$B-V$	$U-B$	Q	BSB
Field Galaxies					
E..............	64	0.96	+0.49	−0.20	21.0
E^+.............	11	0.95	+0.46	−0.23	21.0
L^-.............	17	0.97	+0.46	−0.24	21.0
L..............	20	0.92	+0.44	−0.22	21.1
L^+.............	16	0.91	+0.42	−0.23	21.5
S0/a............	8	0.90	+0.40	−0.25	21.6
Sa.............	11	0.87	+0.31	−0.32	21.6
Sab.............	14	0.83	+0.26	−0.34	21.6
Sb..............	36	0.77	+0.18	−0.38	21.8
Sbc............	32	0.71	+0.08	−0.43	21.9
Sc..............	28	0.62	0.00	−0.44	21.9
Scd.............	8	0.54	−0.14	−0.52	21.9
Sd..............	8	0.58	−0.09	−0.51	22.3
Sdm............	5	0.54	−0.14	−0.53	21.8
Sm.............	5	0.42	−0.28	−0.58	21.7
Im.............	4	0.46	−0.23	−0.55	21.4
Irr II...........	9	0.83	+0.25	−0.35	21.2
E,L,S0 P........	17	0.74	+0.18	−0.35	21.2
Sa–Im P........	19	0.57	−0.05	−0.46	21.8
P..............	9	0.73	+0.07	−0.46	21.3
All Types.....	341	0.79	+0.23	−0.34	21.5

Since the galaxies are at low redshift, you don't have to worry about K-corrections. Dust could make a significant contribution for spiral galaxies and galactic extinction for galaxies too near the galactic plane, but this sample is all above |30| degrees galactic latitiude.