Homework 4

Astronomy 202a

Fall 2009

Due November 4th, 2009

Problems:

Problem 1.Malmquist Bias The H_0 Key project has measured distances to a number of nearby galaxies:

http://www.cfa.harvard.edu/ huchra/ay202/homework/key.dist.dat

By definition, one could calculate a Hubble Constant by fitting these distances to a relation versus redshift. This isn't correct. Why? Describe how you would correct the redshift distance plot for these galaxies for the distance-error "flavor" of the Malmquist bias. Estimate H_0 for the data. What other effects are important? (You will need to look up redshifts for the galaxies. There are a number of sources. Justify your choices).

Solution Naively fitting the given distances to the a relation versus recession velocity isn't correct for a variety of reasons including non-Hubble motions (e.g. Virgo infall, the gravitationally bound Local Group), errors in distance or velocity that are not symmetric, and selection biases that can limit regions of the available data space. Also, several of the galaxies are in the Virgo Cluster where we know that the velocity dispersion can significantly spread things out in velocity, so it might be more appropriate to use the cluster mean velocity instead. (N.B. These galaxies were chosen by the KP precisely because they were in Virgo so a distance to the cluster could be measured.)

Malmquist bias appears in this problem is several ways. To begin with, a symmetric error in the distance modulus does not correspond to a symmetric error in the distance itself; the relationship between the two is nonlinear. Additionally, any effect that makes the probable true distance to a galaxy different than the given measured distance contributes. For example, the volume element spanned by the far side of the distance probability distribution is larger than that for the near end because the survey volume is slightly conical and not a perfect parallelepiped. Any evolution in the luminosity function with distance (redshift) will have a similar effect, increasing the probability that the measured galaxy is farther (or nearer depending on the direction of the evolution) than measured.

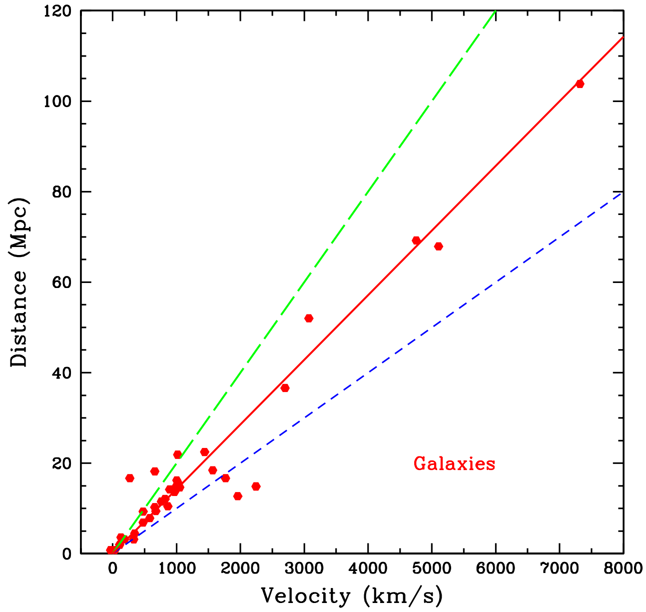

Figure 1: Hubble Constant from the Key Project distances and flow field velocities. Red line is H_0=70 km/s/Mpc, green = 50 and blue = 100.

A reasonable place to find the redshifts is NED. I used the redshifts corrected to the centroid of the Local Group and then corrected for the infall model used by the Key Project team. Here is a plot of the results.

You should get a number in the low 70's depending on the details of your fitting technique (chi sq vs max likelihood, w or w/o errors, etc.).

Problem 2. Gravitational Lensing A new, bright, two image gravitational lens has been found (separation=2.1", $z_{\rm lens} = 0.6$, $z_{\rm source} = 2.2$, component A is 0.7" from the center of the lensing galaxy and component B is 1.4"), and old observations from the Harvard plate stacks indicate that there is a time delay of 1 year between component A and B. Estimate the distance of the lens? What simplifying assumption do you need to make?

Solution In this problem, the distances can be calculated using the Hubble law once the Hubble parameter has been estimated from the data given and not assumed. Here, the redshifts are not small, so the recession velocities are given by the relativistic Doppler formula: $v/c = [(1+z)^2 - 1]/[(1+z)^2 + 1]$. This yields recession velocities of 0.44c and 0.82c for the lens and the source, respectively.

If the lens is a point pass, then the angle β between the lens and the true position of the source is given by the difference between the positions of the two images: $\beta = \theta_1 - \theta_2$. The time delay can be related to the geometry of the problem as $\Delta t = D_S\, D_L\, \beta\, \theta_E / D_L S\, c$, where the Einstein radius is approximately distance between the two images: $\theta_E \approx \theta_1 + \theta_2$. The distances D_L and D_S can be related to the redshift and Hubble parameter (which we will not assume we know). We will also assume space is flat and distances add. Remember, too, that all angles must be in radians.

Putting everything together, the distances are:

$$D_L = \frac{(v_S - v_L)\, c\, \Delta t}{v_S\, (\theta_1 - \theta_2)\, (\theta_1 + \theta_2)} \approx 4\,\mathrm{Gpc}$$

and

$$D_S = \frac{(v_S - v_L)\, c\, \Delta t}{v_L\, (\theta_1 - \theta_2)\, (\theta_1 + \theta_2)} \approx 7\,\mathrm{Gpc}$$

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Problem 3. Galaxy Formation From just the Jean's model, what is the Jeans mass at a redshift of 100 if the universe has its current matter density ($\Omega \sim 0.25$)? At a redshift of 10? Assume that the Universe has not reionized and that the IGM has cooled adiabatically since it was last in thermal equilibrium with the CMB at $z = 1000$.

Solution The Jeans Mass is the mass enclosed in a sphere of radius $R_J = c_s/2\sqrt{G\,\rho}$ containing matter at the mean density: $M_J = 4/3\,\pi\,\bar{\rho}\,R_J^3$. The speed of sound is given by $c_s = \sqrt{5/3\,k_B\,T/m_p}$ and the mean density is $\bar{\rho} = 3\,H_0\,\Omega(z)\,(1+z)^3/8\,\pi\,G$ (note that $\Omega(z) = 1$ for the relevant redshifts in this problem despite being 0.25 today). The temperature CMB at $z = 1000$ was $\sim 3000\,\mathrm{K}$, and the temperature of the IGM subsequently cooled as $T \sim (1+z)^2$ (in reality, the IGM maintains the CMB temperature until about $z = 200$ as we'll discuss next semester). Putting this all together, we have $M_J(z = 100) = 3000\,M_\odot$ and $M_J(z = 10) = 100\,M_\odot$.

Problem 4. Quasars The quasar SDSS J1148+5251 at $z = 6.42$ has a bolometric luminosity of $10^{14}\,L_\odot$. At this redshift, the age of the universe is approximately $0.85\,\mathrm{Gyr}$. If the quasar is powered by accretion onto a supermassive black hole with an efficiency of 10%, what are the minimum and maximum masses that the black hole can have and what creates these limits? If the black hole mass has its minimum value, for what fraction ϵ_{DC} of the age of the universe must the quasar have been active? Given that there is a relation between the velocity dispersion of the host galaxy and the black hole mass, with what other observational constraint must the black hole mass and ϵ_{DC} be consistent? Explain.

Solution The quasar's mass is lowest for the given luminosity if this luminosity is at the Eddington value, in which case $M_{bh} = (L/3.3e4\,L_\odot)\,M_\odot = 3 \times 10^9\,M_\odot$. The quasar's mass is highest if it is shining below Eddington but for the age of the Universe. In this case, the mass is $M_{bh} = \dot{M}_{bh}\,t = L/(0.1\,c^2)\,t_H(z = 6.42) = 6 \times 10^{10}\,M_\odot$. If the quasar shines at the Eddington luminosity, then it only needs to be accreting for $t = M_{bh}/\dot{M}_{bh} = (0.1\,M_{bh}\,c^2)/L =$ 0.042Gyr or 5% of the age of the Universe.

If you assume there are many galactic nuclei with the same duty cycle and that the accretion is stochastic, the duty cycle corresponds to the fraction of all objects that appear as quasars at any moment; we've just seen how the mass of the black hole determines this fraction. At the same time, the black hole mass is related to the velocity dispersion and mass of the host halo, the particular abundance of which is given by Press-Schechter (and sims). This abundance of halos and the fraction we expect to be accreting gives a theoretical estimate for the abundance of quasars, which had better match up with observations. The consistency between the theoretical and observed numbers is still debated.