# Homework 5

Astronomy 202a

Fall 2009

**Solutions**

## Problems:

### Problem 1.Timing Arguments

1. Finish out the Kahn & Woltjer timing argument for the Local Group discussed in class. The measured velocity difference between M31 and the center of the MW is -118 km/s (they are coming together) an the current separation is 770 kpc. Assume an age for the Universe of 13.7 Gyr. Derive the LG mass assuming they are on their first pass. What is the "phase angle," $\theta$ of the orbit? How much does this change if they are in fact on their second pass?

   **Solution** So far in the argument we have:

$$r = \frac{R_{\rm max}}{2}\,(1 - cos\,\theta)$$

$$t = \sqrt{\frac{R_{\rm max}^3}{8\,G\,M_{\rm tot}}}\,(\theta - sin\,\theta)$$

$$\frac{dr}{dt} = \sqrt{\frac{2\,G\,M_{\rm tot}}{R_{\rm max}}}\,\frac{sin\,\theta}{(1 - cos\,\theta)}$$

.

$$\frac{v\,t}{r} = \frac{sin\,\theta\,(\theta - sin\,\theta)}{(1 - cos\,\theta)^2}.$$

   Putting the present-day values into the last equations and solving numerically, we get $\theta = 4.24$ today if the two are on their first pass. This gives $R_{\rm max} = 1058\,\text{kpc}$ and $M_{\rm tot} = 4 \times 10^{12}\,M_\odot$.

   If they are on their second pass, $\theta = 10.1$, $R_{\rm max} = 870\,\text{kpc}$, and $M_{\rm tot} = 10^{13}\,M_\odot$.

2. The same argument can be applied to to the MW-LMC system to derive an estimate of the mass of the combined system (mostly the MW). The relative galactocentric velocity is 84 km/s and the separation is now 51 kpc. What is the MW mass? (N.B. Here it is more likely that the LMC has gone around more than once.)

   **Solution** As before, we solve for $\theta$ now using 84 km/s, 51 kpc, and 13.7 Gyr. We find no solution between 0 and $\pi$ but one just beyond at $\theta = 7.35$ (i.e. after just having interacted for the first time). There is also a solution at $\theta = 13.9$ (i.e. after just having interacted twice). In each case, $R_{\rm max} = 200\,{\rm kpc}$ and $R_{\rm max} = 130\,{\rm kpc}$, respectively, while the MW mass is $M_{\rm tot} = 5 \times 10^{10}\,M_\odot$ and $M_{\rm tot} = 6.5 \times 10^{10}\,M_\odot$, respectively. Note that this is a worse approximation than for the LG since the relative velocity of the MW and LMC is less close to being purely radial and the two objects are close enough to each other that the fact that they're not point masses matters more.

**Problem 2. Cluster Structure** The Coma Cluster is the nearest large galaxy cluster and as such has been fairly extensively studied spectroscopically. On the course website you can find a large data file containing positions and velocities for galaxies in the general direction of the cluster.

http://www.cfa.harvard.edu/ huchra/ay202/comasurvey.dat

1. Find the cluster centers derived from a luminosity weighted mean and the number weighted mean. Explain your assumptions and your methodology.

2. Derive the projeced radial number and luminosity density profiles and determine the 3-D radial luminosity profile using the techniques discussed in class.

3. Extra credit: Plot the cluster isopleths scaled to the mean galaxy density in the surveyed area.

N.B. The data file for this problem is much larger than for the virial problem of HMW2 and covers a much larger area.

**Solution:** The way to first approach this problem is to take the large dataset and select out the redshift range of the cluster. Coma has a heliocentric redshift of z $\sim$ 0.0231 or 6930 km/s which you can get from NED. Its velocity dispersion is 1000 km/s so to be safe, select out galaxies in the sample with $v_{Coma} \pm 3000$ km/s, or between say 4430 and 9430 km/s, about $2.5\sigma$. (Other reasonable choices are ok, this is my best cut at a compromise between adding too much noise and missing the cluster core). You can plot the redshift space diagram for the cluster to see that this makes some sense. The to find the center, we essentially top hat smooth the survey and look for the peaks in the number and luminosity density. I did this by first constructing a grid of centers around the approximate cluster center then finding the peak number and luminosity densities inside 2 degree radius circles. This also requires iteration to hone down to a more accurate center, so one starts with a coarse grid, say on 1 degree centers, then zeroes on with grids of finer and finer spacing.

Here is an example of the number density (left) and magnitude (right) output on a 5x5 grid centered on the catalog center for Coma (essentially very close to the center of the X-ray emission).

| RA/DEC | 2 degree circles 1 degree centers | | | | | Magnitudes | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 196.96 | 195.96 | 194.96 | 193.96 | 192.96 | 196.96 | 195.96 | 194.96 | 193.96 | 192.96 |
| 29.98 | 2 7 | 44 | 50 | 36 | 21 | 7.19 | 6.77 | 6.60 | 6.95 | 7.58 |
| 28.98 | 62 | 95 | 111 | 112 | 69 | 6.19 | 5.82 | 5.64 | 5.63 | 6.21 |
| 27.98 | 80 | 113 | 128 | 131 | 104 | 5.97 | 5.64 | 5.51 | 5.48 | 5.73 |
| 26.98 | 48 | 97 | 112 | 114 | 87 | 6.52 | 5.83 | 5.66 | 5.62 | 5.97 |
| 25.98 | 10 | 30 | 60 | 51 | 47 | 8.48 | 7.10 | 6.16 | 6.60 | 6.62 |

You can see that both the number count and luminosity (expressed as magnitudes) data will perhaps be a little off the x-ray center, but one needs a finer grid and perhaps a different smoothing scale to pin it down further. We then hone in it using 1/2 degree centers, remember that we are looking for the maximum count rate and minimum (brightest) integrated magnitude:

| RA/DEC | 2 degree circles 0.5 degree centers | | | | | | Magnitudes | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 195.5 | 195.0 | 194.5 | 194.0 | 193.5 | 193.0 | 195.5 | 195.0 | 194.5 | 194.0 | 193.5 | 193.0 |
| 29.0 | 103 | 112 | 114 | 115 | 100 | 76 | 5.73 | 5.63 | 5.64 | 5.60 | 5.74 | 6.13 |
| 28.5 | 117 | 123 | 126 | 120 | 112 | 95 | 5.61 | 5.53 | 5.51 | 5.58 | 5.62 | 5.81 |
| 28.0 | 123 | 128 | 130 | 130 | 125 | 108 | 5.54 | 5.51 | 5.48 | 5.49 | 5.54 | 5.68 |
| 27.5 | 124 | 125 | 128 | 131 | 119 | 101 | 5.53 | 5.53 | 5.51 | 5.51 | 5.57 | 5.75 |
| 27.0 | 107 | 112 | 118 | 114 | 106 | 89 | 5.68 | 5.66 | 5.60 | 5.62 | 5.66 | 5.96 |

and eventually 0.5 degree circles on 0.1 degree centers somewhere near RA=194.75 and DEC=27.8 ...

| RA/DEC | 0.5 degree circles 0.1 degree centers Magnitude | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 195.3 | 195.2 | 195.1 | 195.0 | 194.9 | 194.8 | 194.7 | 194.6 | 194.5 |
| 28.1 | 6.52 | 6.53 | 6.54 | 6.54 | 6.55 | 6.67 | 6.64 | 6.80 | 6.88 |
| 28.0 | 6.52 | 6.48 | 6.54 | 6.53 | 6.53 | 6.67 | 6.70 | 6.83 | 6.73 |
| 27.9 | 6.56 | 6.53 | 6.51 | 6.54 | 6.57 | 6.57 | 6.66 | 6.75 | 6.75 |
| 27.8 | 6.69 | 6.61 | 6.54 | 6.51 | 6.57 | 6.60 | 6.71 | 6.70 | 6.97 |
| 27.7 | 6.81 | 6.71 | 6.67 | 6.65 | 6.67 | 6.72 | 6.79 | 6.77 | 7.04 |
| 27.6 | 7.13 | 6.81 | 6.77 | 6.81 | 6.69 | 6.84 | 6.94 | 7.29 | 7.49 |
| 27.5 | 8.00 | 7.32 | 6.98 | 7.04 | 6.95 | 7.29 | 7.42 | 7.73 | 7.64 |

which gives a very broad peak in the luminosity weighted center — i.e. Coma is not cuspy, at approximately

$$\alpha \;=\; 13^h\ 00^m\ 48^s \text{ and } \delta \;=\; +28^c irc\ 00'\ 00''$$

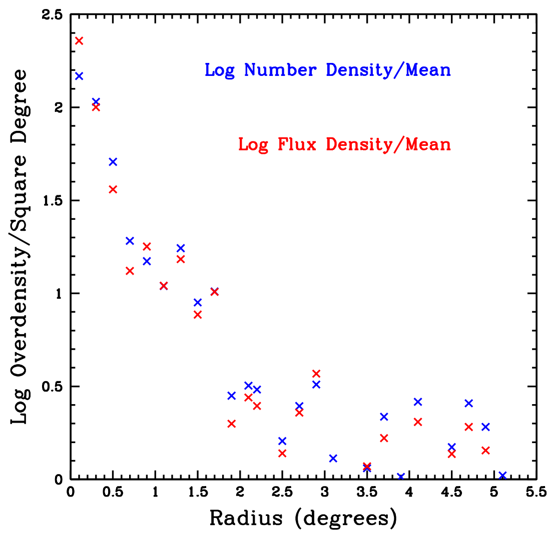


Figure 1: Surface Density profile for the Coma Cluster in both galaxy number density and integrated "flux."

with the number weighted peak near but very indeterminate. What you get depends on the smoothing legnth chosen and whether you luminosity weight or number weight. This luminosity weighted center is very close, but not exactly the same as the X-ray peak that we started from.

Now to the profiles. The surface density profiles are comparatively easy once the center is determined. Count galaxies or flux in annuli around the center relative to the background. More properly one should subtract the background, but this can produce messy plots. Here's what we get in terms of the surface densities (both galaxy number and integrated flux overdensities): again pre-cutting the catalog between 4430 and 9430 km/s to restrict foreground and background to objects in the same redshift range as the cluster. We estimated the background in an annulus between 7 and 8 degrees. What you get is also going to be sensitive to where and how you estimate the background.

**Problem 3. Correlation Functions** The simplest correlation function to calculate is just the angular two-point CF, $\omega(\theta)$:

$$\delta P \;=\; N[1+\omega(\theta)]d\Omega$$

where $\delta P$ is the excess probability of finding a galaxy at angular separation $\theta$ from any galaxy, $N$ is the mean surface density of galaxies, and $d\Omega$ is the solid angle element.

Using the same data file as above, calculate and plot the angular two-point CF over scales from 0.1 to to 10 degrees.

Remember to take into account edge effects! That is to say, for galaxies in the sample near the edge of the survey boundary when calculating the expected number of galaxies at any given radius you must take into account area missed due to the survey's spatial limits. The spatial limits of this sample are given in the file header. This is actually the hard part of the problem! Explain your approach.

**Problem 4. Cluster Gas Metallicity** As discussed, one way of bringing the metallicity of the cluster gas up to 1/4 Solar is by winds from the galaxies of enriched material from stars driven by stellar mass loss. The Sun's mass loss rate is $2\times10^{-14}$ $M_\odot$ per year, way too small. What would the *average* mass loss rate for the stars in a cluster need to be to bring a primordial gas halo of typical mass up to 1/4 solar? Assume the winds are solar metallicity.

You will need to estimate the number of stars in the galaxy cluster. You should also make some other assumptions about the stars and about the total gas mass and primordial gas mass. For simplicity, assume all stars are the same.

Why does this number either make or not make sense?

**Solution** The metallicity of the cluster gas is given by:

$$\text{cluster metallicity} = \frac{\text{total rate of injected solar metallicity x age of universe}}{\text{total gas mass in cluster}}.$$

This assumes that the gas in the cluster has initially zero metallicity and that none of the metal-rich gas is reprocessed (i.e. it doesn't get blown out of a star twice, and new stars are only made with pristine gas) and that supernovae don't contribute. If the cluster has a halo mass of $10^{14}$ and 10% of this is in gas, than the total injected gas mass at solar metallicity is $180\,M_\odot/\text{yr}$. If there are 1000 galaxies in the cluster and each galaxy has 100 billion stars, this works out to an average mass loss rate per star of $1.8\times10^{-12}\,M_\odot/\text{yr}$.