Homework 6

Astronomy 202a

Fall 2009

Due December 4th, 2009

Problems:

Problem 1.Mass-to-Light Ratios What is the mass to light ratio of a stellar population with a Salpeter Initial Mass Function

$$N(M) \; = \; M^{-\alpha} dM$$

with $\alpha = 2.35$, of zero age (i.e. assume all stars are alive and well and on the main sequence and that L $\propto$ M^3)? What else do you need to assume to do this problem?

Solution To do this problem, you need to assume high- and low-mass cut-offs for the IMF; typical values are $200\, M_\odot$ and $0.1\, M_\odot$, respectively. The mass to light ratio can be found by integrating the IMF over mass and luminosity (assuming $L_\mathrm{B}/L_{\odot,\mathrm{B}} = (M/M_\odot)^3$).

$$\frac{M}{L} = \frac{\int_{0.1}^{200} x^{-1.35}\, dx}{\int_{0.1}^{200} x^{0.65}\, dx}\, Y_\odot = 1.5 \times 10^{-3}\, Y_\odot.$$

It make sense that this value is small since the large luminosity of super-solar stars dominates the mass of the sub-solar ones thanks to the luminosity's M^3 dependence.

(b) What is the M/L of this population when it is $5{\times}10^9$ years old? (Hint: the mass stays the same, but only stars that live longer than 5 billion years still emit light). Again, assume only main sequence stars (otherwise the problem takes three months).

N.B. In reality, because the red giant stage for low mass stars produces significant luminosity, this simple M/L change isn't quire right.

Hint: Use the Basic Stellar Data link on the website.

http://www.cfa.harvard.edu/ huchra/ay145/stars.pdf

and extrapolate and interpolate as necessary using what you know about the stellar mass-luminosity relation — as needed.

Solution Interpolating through the table given on the website, we find that, after 5 billion years, stars larger than $\sim 1.25\, M_\odot$ have left the main sequence. Thus, we change the maximum mass in the denominator from part (a) to $1.25\, M_\odot$ (from $200\, M_\odot$). We expect the mass-to-light ratio to now be greater than unity because most of the luminosity now comes from sub-solar mass stars with low M/L. Redoing the integral, we find the mass-to-light ratio of the evolved system to be about $6.9\,Y_\odot$. You could have made a case for this problem that for the older population, the mass of dead stars doesn't really "count" since they're not really part of the population anymore.

Problem 2. IMF How do the above M/L's vary with (a) the slope, α of the IMF (say between 0, high mass star enriched, and 4, low mass star dominated)? (b) the upper mass limit of integration of the IMF (say between 20 and 200 $M_\odot$)? and (c) the lower masss limit of integration of the IMF (say between 0.005 and 0.1 $M_\odot$)?

Solution Doing the integral with variable α, $M_{\rm max}$, and $M_{\rm min}$ gives:

$$\frac{M}{L} = \frac{(4-\alpha)\,\left[(M_{\rm max,init})^{2-\alpha} - (M_{\rm min})^{2-\alpha}\right]}{(2-\alpha)\,\left[(M_{\rm max,lum})^{4-\alpha} - (M_{\rm min})^{4-\alpha}\right]}\,Y_\odot,$$

where $M_{\rm max,init}$ and $M_{\rm max,lum}$ are the maximum masses of the initial population at the luminous population at time t, respectively. Figure 1 shows the plot of mass-to-light ratio vs. α for $(M_{\rm min}, M_{\rm max,init}, M_{\rm max,lum}) = \{(0.1, 200, 200), (0.1, 200, 1.25)\}\ M_\odot$. Setting $\alpha = 2.35$ and assuming $M_{\rm min} << M_{\rm max}$, the dependence on $M_{\rm min}$ and $M_{\rm max,init}$ is approximately $Y = 4.71\,M_{\rm min}^{-0.35}\,M_{\rm max,init}^{-1.65}\,Y_\odot$, for a zero-age population where $M_{\rm max,init} = M_{\rm max,lum}$, or $Y = 4.71\,M_{\rm min}^{-0.35}\,M_{\rm max,lum}^{-1.65}\,Y_\odot$ (i.e. constant in $M_{\rm max,init}$) for a 5 Gyr population.

Problem 3. Motions Supposed we know of two moderately well determined "flows" in the local Universe, the infall into Virgo (250 km/s towards l=284 b=75) and the motion towards the Great Attractor (450 km/s towards l=310 b=9). What extra motion is needed produce the Local Group motion towards the CMB (direction & velocity)?

Solution The brute-force way of adding vectors in spherical coordinates, $(r = v, \theta = 90^\circ - b, \phi = l)$, is to first convert them into Cartesian coordinates. The two flows are $(15.7, -62.8, 241.5)$ km/s and $(285.7, -340.5, 70.4)$ km/s. The Local Group motion measured from the CMB dipole is 612 km/s towards l=269.8°, b=28.6° or $(-1.8, -537.2, 293.2)$ km/s in Cartesian coordinates. The necessary additional velocity unaccounted for by the two flows is, thus, $(-303.2, -133.9, -18.7)$ km/s or, in galactic coordinates, 332 km/s towards l=204°, b=-1.75°.

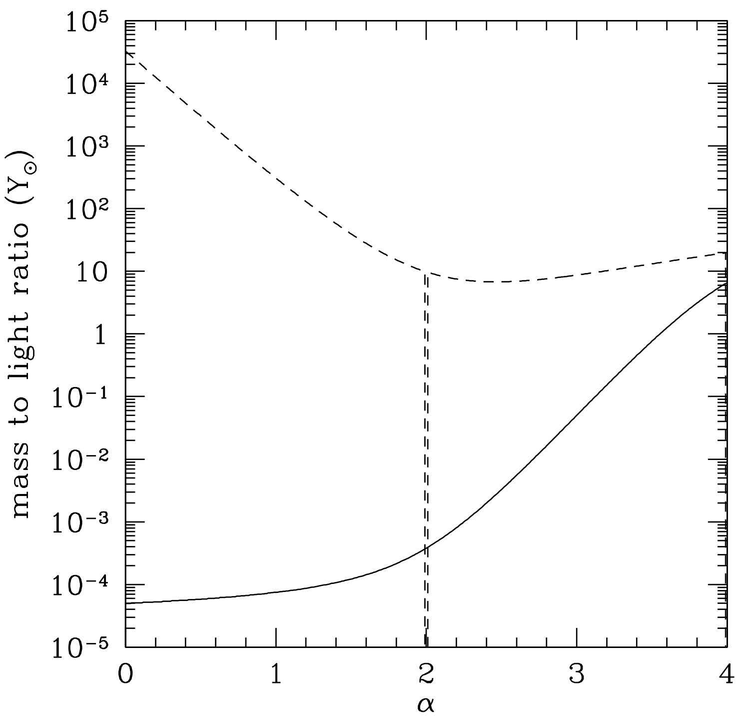

Figure 1: The dependence of the mass-to-light ratio on the IMF power-law index α. The solid and dashed lines show results for zero-age and 5 billion year old populations with $(M_{\rm min}, M_{\rm max,init}, M_{\rm max,lum}) = (0.1, 200, 200)\, M_\odot$ and $(0.1, 200, 1.25)\, M_\odot$, respectively. Ignore the divergence at $\alpha = 2$.