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PROPAGATION EFFECTS IN LOW-DENSITY TRANSRELATIVISTIC PLASMAS

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ABSTRACT

The transfer of polarized radiation in magnetized and non-magnetized relativistic plasmas is an area of research with numerous flaws and gaps. The present paper is aimed at filling some gaps and eliminating the flaws. Trubnikov's linear response tensor for thermal plasma is a starting point. The analytic expressions are derived of temperature dependence for the tensor components in a high-frequency limit. The Faraday rotation coefficient and the Faraday conversion coefficient are computed in the first orders in the ratio of the cyclotron frequency to the observed frequency. The computed coefficients bridge the known non-relativistic and ultra-relativistic limiting formulas. The fitting low-density expressions are found for high temperatures, where the high-frequency limit fails. The plasma eigenmodes found to become linearly polarized at temperatures much higher than thought before. The results are applied to the propagation delays through hot ISM, diagnostics of hot accretion flows and jets.

Subject headings: radiative transfer — polarization — magnetic fields

1. INTRODUCTION

We learn most of information about astrophysical objects by observing the light they emit. Polarization properties of this light are its important characteristics. Observations of polarization can tell us the geometry of the emitter, strength of the magnetic field, density of plasma and temperature. The proper and correct theory of optical activity is essential for making accurate predictions. While the low-temperature propagation characteristics of plasma are well-established (Landau & Lifshits 1980), the theory of relativistic effects should be explored more. In this paper I discuss the propagation effects through homogeneous magnetized relativistic plasma. The non-magnetized case emerges as an essential limit of a magnetized case. The discussion can be divided into three separate topics.

Two linear plasma propagation effects are Faraday rotation and Faraday conversion (Azzam & Bashara 1987). Traditionally, these effects are considered in their lowest orders in the ratio of the cyclotron frequency Ω_0 to the circular frequency of light ω, id est in a high-frequency approximation. The distribution of particles is taken to be thermal. The dimensionless temperature is T in the units of particle rest mass temperature $m_p c^2/k_B$. The Faraday rotation measure RM and conversion measure are known in non-relativistic $T \ll 1$ and ultra-relativistic $T \gg 1$ limits (Melrose 1997c). I derive an analytic expression in for the arbitrary temperature T. It appears to be as compact as the limiting expressions.

Smallness of the ratio $\beta = \Omega_0/\omega$, $\beta \ll 1$ for the real systems made some authors (Melrose 1997a) consider the described high-frequency approximation to always work. However, there is a clear indication that it breaks at high temperatures $T \gg 1$. It was claimed that the eigenmodes of plasma become linearly polarized for high temperatures $T \gg 1$ (Melrose 1997c). The claim is based on the result that the second order term $\sim \beta^2$ becomes larger than the first order term $\sim \beta$ because of the T-dependent factor. The arbitrarily large T-factor may stand in front of higher order expansion terms in β of the relevant expressions. I find the generalized rotation measure as a function of β and T without expanding in β and compare the results with the known high-frequency expressions. Indeed the high-T behavior is found to be significantly different.

Plasma physics involves complicated calculations. This led to a number of errors in literature (Melrose 1997c). Some of them are still not fixed. In the article I check all the limiting cases numerically and analytically and expound all the steps of derivations. Thus I correct the relevant errors and misinterpretations done by previous authors, hopefully not making new mistakes. The analytical and numerical results are obtained in Mathematica 6 system. It has an enormous potential in these problems (Marichev 2008).

The paper is organized as follows. The formalism of plasma response and calculations are described in Section 2. The plasma dispersion relation and the refinement of the method are discussed in Section 3. Section 4 contains numerous applications to observations. Section 5 concludes with a short summary and future prospects.

2. CALCULATIONS

2.1. *Geometry of the problem*

I assume the traditional geometry:

- the Euclidean basis ($\tilde{\mathbf{e}}^1$, $\tilde{\mathbf{e}}^2$, $\tilde{\mathbf{e}}^3$),
- magnetic field along the 3-d axis $\tilde{\mathbf{B}} = (0,0,B)^T$,
- the wave vector of the wave $\tilde{\mathbf{k}} = k(\sin\theta, 0, \cos\theta)^T$ with an angle θ between $\mathbf{k}$ and $\mathbf{B}$.

The basis is rotated from ($\tilde{\mathbf{e}}^1$, $\tilde{\mathbf{e}}^2$, $\tilde{\mathbf{e}}^3$) to ($\mathbf{e}^1$, $\mathbf{e}^2$, $\mathbf{e}^3$), so that the wave propagates along $\mathbf{k} = (0,0,k)^T$ in a new basis. The transformation has a form

$$\mathbf{e}^1 = \tilde{\mathbf{e}}^1 \cos\theta - \tilde{\mathbf{e}}^3 \sin\theta, \quad \mathbf{e}^2 = \tilde{\mathbf{e}}^2, \quad \mathbf{e}^3 = \tilde{\mathbf{e}}^1 \sin\theta + \tilde{\mathbf{e}}^3 \cos\theta, \tag{1}$$

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that can be conveniently written as

$$\mathbf{e}^\mu = \tilde{\mathbf{e}}^\nu S^{\nu\mu}, \quad S^{\nu\mu} = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}. \tag{2}$$

Vectors and tensors then rotate according to

$$A^\mu = (S^T)^{\mu\nu}\tilde{A}^\nu, \quad \alpha^{\mu\nu} = (S^T)^{\mu\sigma}\tilde{\alpha}^{\sigma\delta}S^{\delta\nu}. \tag{3}$$

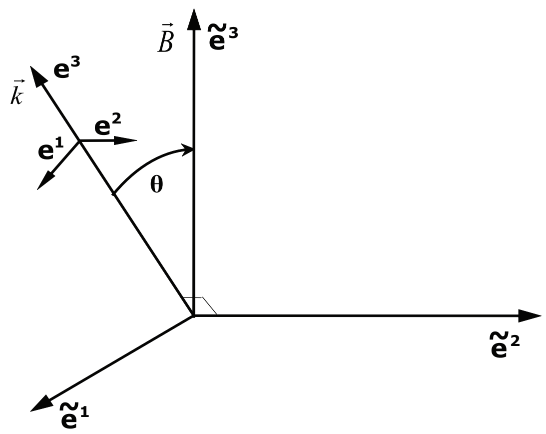

FIG. 1.— Geometry of the problem.

2.2. *Linear plasma response*

The propagation of weak EM waves in a homogeneous magnetized plasma can be fully described by the response tensor $\alpha^{\mu\nu}$. It defines the linear proportionality between the induced current density and the wave vector potential $j^\mu(\omega) = \alpha^\mu{}_\nu A^\nu(\omega)$. The spatial projection of such defined 4-D tensor $\alpha^\mu{}_\nu$ is equal to 3-D tensor α_{ij} in $\vec{j} = \alpha_{ij}\vec{A}$.

I consider the Trubnikov's from of the response tensor (Trubnikov 1958; Melrose 1997a). I work in a low-density regime, when $|\mathbf{k}| = \omega/c$ as in vacuum. I take the tensor $\tilde{\alpha}^{\mu\nu}$ from the first-hand derivations (Trubnikov 1958; Melrose 1997a), make a transformation (3) and take the 1-st and 2-nd components in both indices. Thus the projection onto $(\mathbf{e}^1, \mathbf{e}^2)$ plane in CGS units is

$$\alpha^\mu{}_\nu(k) = \frac{iq^2 n\omega\rho^2}{cmK_2(\rho)} \int_0^\infty d\xi \left[t^\mu{}_\nu \frac{K_2(r)}{r^2} - R^\mu \tilde{R}_\nu \frac{K_3(r)}{r^3} \right], \tag{4}$$

$$t^\mu{}_\nu = \begin{pmatrix} \cos^2\theta\cos\Omega_0\xi + \sin^2\theta & \eta\cos\theta\sin\Omega_0\xi \\ -\eta\cos\theta\sin\Omega_0\xi & \cos\Omega_0\xi \end{pmatrix}, \tag{5}$$

$$R^\mu = \frac{\omega\sin\theta}{\Omega_0}\left(\cos\theta(\sin\Omega_0\xi - \Omega_0\xi), -\eta(1-\cos\Omega_0\xi)\right), \tag{6}$$

and

$$\tilde{R}_\nu = \frac{\omega\sin\theta}{\Omega_0}\left(\cos\theta(\sin\Omega_0\xi - \Omega_0\xi), \eta(1-\cos\Omega_0\xi)\right), \tag{7}$$

$$r = \left[\rho^2 - 2i\omega\xi\rho + \frac{\omega^2\sin\theta^2}{\Omega_0^2}\left(2 - \Omega_0^2\xi^2 - 2\cos\Omega_0\xi\right)\right]^{1/2}, \tag{8}$$

where η is the sign of the charge[1]. The inverse temperature is

$$\rho = T^{-1} = \frac{m_p c^2}{k_B T_p} \tag{9}$$

for the actual temperature of particles T_p. The response of plasma is usually characterized by the dielectric tensor. Its projection onto $(\mathbf{e}^1, \mathbf{e}^2)$ plane is

$$\varepsilon^\mu{}_\nu = \delta^\mu{}_\nu + \frac{4\pi c}{\omega^2}\alpha^\mu{}_\nu. \tag{10}$$

The wave equation for the transverse waves in terms of $\varepsilon^\mu{}_\nu$ is

$$(n^2\delta^\mu{}_\nu - \varepsilon^\mu{}_\nu)\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = 0 \tag{11}$$

where E_1 and E_2 are the components of the electric field along $\mathbf{e}^1$ and $\mathbf{e}^2$ and $n^2 = k^2c^2/\omega^2$ (Swanson 2003).

2.3. *High frequency limit*

Let me first calculate the limiting expression for $\alpha^\mu{}_\nu$ in a high-frequency limit $\Omega_0 \ll \omega$. I denote

$$\alpha = \omega\xi, \qquad \beta = \frac{\Omega_0}{\omega}, \tag{12}$$

substitute (12) into the expression (4) and expand the response tensor $\alpha^\mu{}_\nu$ in β. I retain only up to the 2-nd order of the expansion what gives the conventional generalized Faraday rotation (Melrose 1997c). The first terms of the series of r, $t^\mu{}_\nu$ and $R^\mu\tilde{R}_\nu$ read

$$r^2 = r_0^2 + \delta r^2, \quad r_0^2 = \rho^2 - 2i\alpha\rho, \quad \delta r^2 = -\frac{\sin^2\theta}{12}\beta^2\alpha^4, \tag{13}$$

$$t^\mu{}_\nu = \begin{pmatrix} 1 - \cos^2\theta\cdot\alpha^2\beta^2/2 & \alpha\beta\eta\cos\theta \\ -\alpha\beta\eta\cos\theta & 1 - \alpha^2\beta^2/2 \end{pmatrix}, \tag{14}$$

$$R^\mu\tilde{R}_\nu = -\frac{\alpha^2\beta^2}{4}\sin^2\theta\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{15}$$

Melrose (1997c) used the approximation $r_0^2 = -2i\alpha\rho$ instead of the expansion (13) and obtained the approximate high-T expressions as his final answers.

However, one can take the emergent integrals if considers the exact expansions (13,14,15). Three terms appear in the expanded expression for $\alpha^\mu{}_\nu$:

$$\int_0^\infty d\alpha \left[t^\mu{}_\nu \frac{K_2(r_0)}{r_0^2} \right], \tag{16}$$

$$\int_0^\infty d\alpha \left[t^\mu{}_\nu \frac{K_3(r_0)\delta r^2}{r_0^3} \right], \tag{17}$$

$$\int_0^\infty d\alpha \left[R^\mu\tilde{R}_\nu \frac{K_3(r_0)}{r_0^3} \right]. \tag{18}$$

The 2-nd term (17) originates from the expansion of $K_2(r)/r^2$ in r^2 to the first order

$$\frac{K_2(r)}{r^2} - \frac{K_2(r_0)}{r_0^2} = -\frac{\delta r^2}{2}\frac{K_3(r_0)}{r_0^3}. \tag{19}$$

[1] Note that the analogous expression in Melrose (1997c) has an extra factor $\Omega_0\xi$ in the component t^{11} and the opposite sign of $R^\mu\tilde{R}^\nu$ term by an error. The author corrects his formulas in a subsequent book (Melrose 2008).

Integrals (16,17,18) can be evaluated knowing that

$$\int_0^\infty d\alpha \left[\alpha^n \frac{K_2(\sqrt{\rho^2-2i\rho\alpha})}{\rho^2-2i\rho\alpha}\right] = n! i^{n+1}\frac{K_{n-1}(\rho)}{\rho^2}, \quad (20)$$

$$\int_0^\infty d\alpha \left[\alpha^n \frac{K_3(\sqrt{\rho^2-2i\rho\alpha})}{(\rho^2-2i\rho\alpha)^{3/2}}\right] = n! i^{n+1}\frac{K_{n-2}(\rho)}{\rho^3}. \quad (21)$$

2.4. *Components in high-frequency limit*

I substitute the high-frequency expansions (13,14,15) into the expression for the projection of the dielectric tensor $\varepsilon^\mu{}_\nu$ (10) with the projection of the response tensor $\alpha^\mu{}_\nu$ (4) and take the integrals (16,17,18) analytically. The components of the dielectric tensor (10) in the lowest orders in Ω_0/ω are

$$\varepsilon^1{}_1 = 1 - \frac{\omega_p^2}{\omega^2}\left(\frac{K_1(\rho)}{K_2(\rho)}\left(1+\frac{\Omega_0^2}{\omega^2}\cos^2\theta\right) + \frac{\Omega_0^2\sin^2\theta}{\omega^2\rho}\right), \quad (22)$$

$$\varepsilon^2{}_2 = 1 - \frac{\omega_p^2}{\omega^2}\left(\frac{K_1(\rho)}{K_2(\rho)}\left(1+\frac{\Omega_0^2}{\omega^2}\right) + \frac{7\Omega_0^2\sin^2\theta}{\omega^2\rho}\right), \quad (23)$$

$$\varepsilon^1{}_2 = -\varepsilon^2{}_1 = -i\eta\frac{\omega_p^2\Omega_0}{\omega^3}\frac{K_0(\rho)}{K_2(\rho)}\cos\theta. \quad (24)$$

The plasma frequency in CGS units is defined as

$$\omega_p^2 = \frac{4\pi n q^2}{m_p}. \quad (25)$$

The results reproduce the non-relativistic limits for $\rho \to +\infty$

$$\varepsilon^1{}_1 = 1 - \frac{\omega_p^2}{\omega^2}\left(1+\frac{\Omega_0^2}{\omega^2}\cos^2\theta\right), \quad (26)$$

$$\varepsilon^2{}_2 = 1 - \frac{\omega_p^2}{\omega^2}\left(1+\frac{\Omega_0^2}{\omega^2}\right), \quad (27)$$

$$\varepsilon^1{}_2 = -\varepsilon^2{}_1 = -i\eta\frac{\omega_p^2\Omega_0}{\omega^3}\cos\theta, \quad (28)$$

when all Bessel functions of ρ approach unity[2] (Landau & Lifshits 1980; Trubnikov 1996; Swanson 2003; Bellan 2006). The correspondent relativistic limits $\rho \to 0$ of the same components are

$$\varepsilon^1{}_1 = 1 - \frac{\omega_p^2}{\omega^2}\left(\frac{1}{2T}\left(1+\frac{\Omega_0^2}{\omega^2}\cos^2\theta\right) + T\frac{\Omega_0^2\sin^2\theta}{\omega^2}\right), \quad (29)$$

$$\varepsilon^2{}_2 = 1 - \frac{\omega_p^2}{\omega^2}\left(\frac{1}{2T}\left(1+\frac{\Omega_0^2}{\omega^2}\right) + T\frac{7\Omega_0^2\sin^2\theta}{\omega^2}\right), \quad (30)$$

$$\varepsilon^1{}_2 = -\varepsilon^2{}_1 = -i\eta\frac{\omega_p^2\Omega_0}{\omega^3}\frac{\ln(T)}{2T^2}\cos\theta \quad (31)$$

consistently with (Melrose 1997c; Quataert & Gruzinov 2000)[3]. The ultra-relativistic non-magnetized dispersion relation then reads

$$\omega^2 = \frac{\omega_p^2}{2T} + c^2k^2 = \frac{2\pi n q^2}{mT} + c^2k^2 \quad (32)$$

[2] The non-diagonal term has a wrong sign in (Melrose 1997c).
[3] The diagonal plasma response is 2 times larger in (Melrose 1997c) by an error.

according to the relation (11). The expression (32) is consistent with Landau & Lifshits (1980), chapter 32.

The plasma propagation effects can usually be parameterized by only the difference of the diagonal components and the non-diagonal component of $\varepsilon^\mu{}_\nu$. I define $\mathbf{X}$ to be a vector of T, θ, Ω_0/ω. I write the difference between the diagonal components with the gaunt factor $f(\mathbf{X})$ as

$$\varepsilon^1{}_1 - \varepsilon^2{}_2 = f(\mathbf{X})\frac{\omega_p^2\Omega_0^2}{\omega^4}\left(\frac{K_1(T^{-1})}{K_2(T^{-1})} + 6T\right)\sin^2\theta \quad (33)$$

and the non-diagonal component with the gaunt factor $g(\mathbf{X})$ as

$$\varepsilon^1{}_2 = -i\eta g(\mathbf{X})\frac{\omega_p^2\Omega_0}{\omega^3}\frac{K_0(T^{-1})}{K_2(T^{-1})}\cos\theta. \quad (34)$$

Both gaunt factors equal unity in the considered above high-frequency limit $f(\mathbf{X}) = g(\mathbf{X}) = 1$. Now we can turn to a more general case.

2.5. *General low density regime*

The ultra-relativistic expressions (29,30,31) allow me to trace the T-factors in front of the first 3 expansion coefficients of the dielectric tensor in β. The coefficient at β^2 is $\sim T^3/\ln(T)$ times larger than at β. Thus at temperature $T \gtrsim 10$ the second order becomes larger than the 1-st order for the ratio $\Omega_0/\omega \sim 10^{-3}$. This indicates that the expansion in β may become invalid at these plasma parameters[4]. The gaunt factors $f(\mathbf{X})$ and $g(\mathbf{X})$ are likely to be far from 1. I consider only the real parts of these gaunt factor, since the imaginary parts correspond to absorption. The contour plots of the numerically calculated $f(\mathbf{X})$ and $g(\mathbf{X})$ for $\theta = \pi/4$ are shown on Figure 2 and Figure 3 respectively.

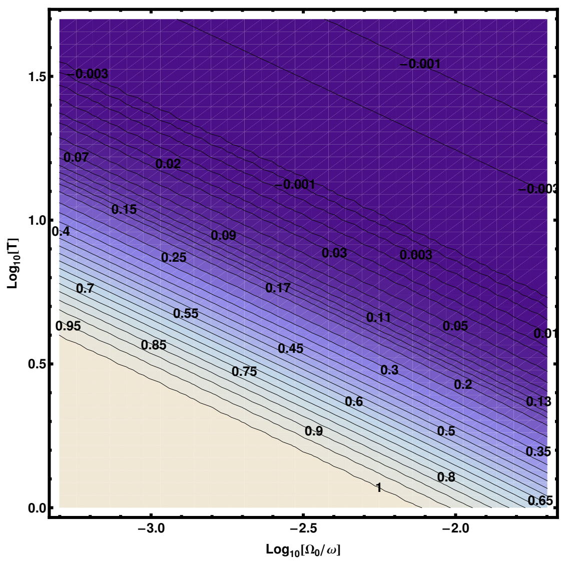

FIG. 2.— Gaunt factor $f(\mathbf{X})$ for the difference of diagonal components $\varepsilon^1{}_1 - \varepsilon^2{}_2$.

Let me define X to be the following combination of the parameters

$$X = T\sqrt{\sqrt{2}\sin\theta\left(10^3\frac{\Omega_0}{\omega}\right)}. \quad (35)$$

[4] One cannot claim that the diagonal magnetized terms become larger then the non-diagonal (Melrose 1997c).

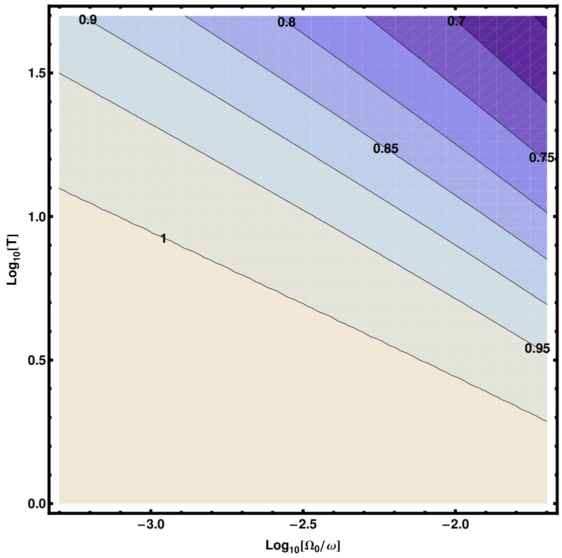

FIG. 3.— Gaunt factor $g(\mathbf{X})$ for the non-diagonal component $\varepsilon^1{}_2$.

For the fiducial $\Omega_0/\omega = 10^{-3}$, $\theta = \pi/4$ the parameter X is just temperature $X = T$.

I first identify the boundaries, where the high-frequency limit is valid and then find a fit for the gaunt factors at higher X. The expression for the difference $\varepsilon^1{}_1 - \varepsilon^2{}_2$ (33) is accurate within 10% for $X < 0.1$ if we set $f(X) = 1$. The expression for $\varepsilon^1{}_2$ (34) is accurate within 10% for $X < 30$ if we set $g(X) = 1$. The accuracy depends on the parameter X rather than on the individual parameters T, Ω_0/ω, θ. The expression

$$f(X) = 2.011 \exp\left(-\frac{X^{1.035}}{4.7}\right) - \\ - \cos\left(\frac{X}{2}\right) \exp\left(-\frac{X^{1.2}}{2.73}\right) - 0.011 \exp\left(-\frac{X}{47.2}\right) \quad (36)$$

extends the applicability domain of the formula (33) up to $X = 200$. Figure 4 shows the fit for $f(X)$ in comparison with the numerical results. The expression

$$g(X) = 1 - 0.11 \ln(1 + 0.035X) \quad (37)$$

extend up $X = 200$ the domain of the formula (34). Figure 5 shows the fit for $g(X)$ in comparison with the numerical results.

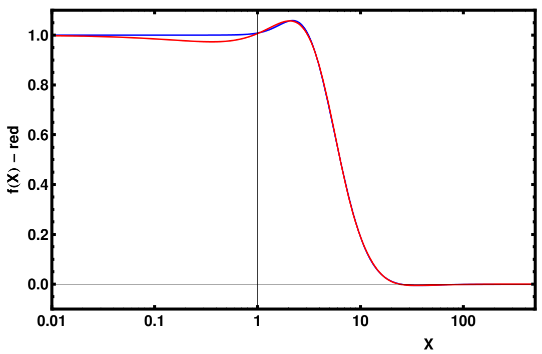

FIG. 4.— Gaunt factor $f(\mathbf{X})$ for the difference of diagonal components $\varepsilon^1{}_1 - \varepsilon^2{}_2$.

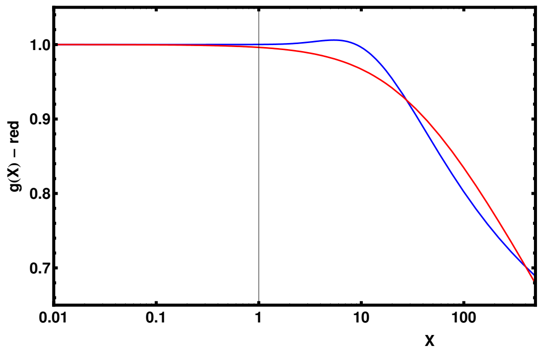

FIG. 5.— Gaunt factor $g(\mathbf{X})$ for the non-diagonal component $\varepsilon^1{}_2$.

2.6. *Exact plasma response*

The expression for the response tensor (4) is written for a vacuum wave with $|\mathbf{k}|c = \omega$. The actual plasma wave is being modified by the plasma response. More general self-consistent response tensor should be used (Trubnikov 1958; Melrose 1997c). One needs to solve the dispersion relation similar to (11) to obtain the eigenmodes. These eigenmodes should be self-consistently modified by the plasma response. One should not forget about the anti-hermitian and longitudinal components of the dielectric tensor $\varepsilon^\mu{}_\nu$ that modify the dispersion relation.

3. EIGENMODES

The used calculation method is applicable to a non-magnetized plasma as well as a magnetized. Dispersion relation of EM waves in a non-magnetized plasma reads

$$\omega^2 = k^2c^2 + \omega_p^2 \frac{K_1(T^{-1})}{K_2(T^{-1})} \quad (38)$$

in a high-frequency approximation $\omega \gg \omega_p$. The opposite limit of $kc \ll \omega$ was considered by Bergman (2001).

Now we turn to a magnetized case. Melrose (1997c) only considered the first terms of in the expansion of $\alpha^\mu{}_\nu$ in β to get the eigenmodes. I do the next step: consider the full expression in β in the low-density regime $kc = \omega$, but take only the hermitian part $(\alpha^\mu{}_\nu)^H$. The ellipticity $\Upsilon = (\varepsilon^1{}_1 - \varepsilon^2{}_2) : |\varepsilon^1{}_2|$ determines the type of eigenmodes. If $\Upsilon \gg 1$, then the eigenmodes are linearly polarized unless θ is close to 0. If $\Upsilon \ll 1$, then the eigenmodes are circularly polarized for θ far from $\pi/2$. Let me consider the fiducial model with $\Omega_0/\omega = 10^{-3}$ and $\theta = \pi/4$. Figure 6 shows the ratio Υ calculated in a high-frequency approximation (see Section 2.3) (thin line) and in a general low-density approximation (see Section 2.5) (thick line). The high-frequency approximation produces the linear eigenmodes already at $T \gtrsim 10$ consistently with (Melrose 1997c). However, the general low-density limit produces the eigenmodes with $\Upsilon \sim 1$ up to very high temperatures $T \sim 50$. Unexpectedly, the sign of the diagonal difference $(\varepsilon^1{}_1 - \varepsilon^2{}_2)$ changes at about $T \approx 25$.

4. APPLICATIONS

4.1. *Dispersion measure*

The calculated transrelativistic propagation effects have far-reaching consequences in many topics of astronomy. Let me concentrate on four applications: propagation delay, Faraday rotation measure of light from the

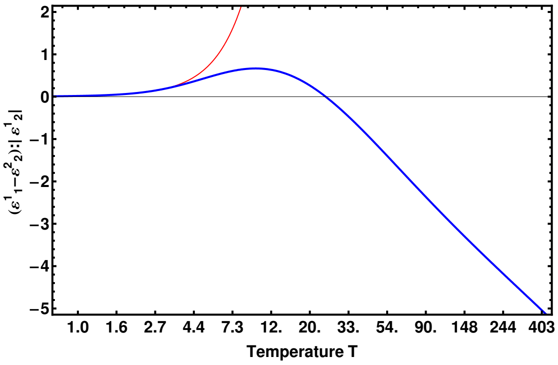

FIG. 6.— Ellipticity $\Upsilon = (\varepsilon^1{}_1 - \varepsilon^2{}_2) : |\varepsilon^1{}_2|$ of eigenmodes. The ratio Υ much above unity — linear eigenmodes, much below unity — circular eigenmodes. Thick line — this paper, thin line — previous calculations.

Galactic Center (GC), circularly polarized light from the GC, diagnostics of jets.

Propagation delay is an important effect in pulsar dispersion (Phillips, & Wolszczan 1992). The relativistic part of this delay can be obtained from the dispersion relation (38). I retain only the first-order correction in T, since $T \ll 1$ in cosmic medium (Cox,& Reynolds 1987). Since $K_1(T^{-1})/K_2(T^{-1}) \approx 1 - 3T/2$ at low T, the non-relativistic Dispersion Measure (DM) should be modified as

$$DM_{\rm rel} = DM_{\rm nonrel}\left(1 - \frac{3}{2}T\right). \quad (39)$$

The correspondent gas density appears underestimated, if the non-relativistic expression is used[5]. The effects in magnetized plasma plasma are also relevant for pulsars.

4.2. *Magnetized radiative transfer*

Relativistic plasmas exhibit the generalized Faraday rotation for the general orientation of the magnetic field (Azzam & Bashara 1987). One can decompose it into two effects: Faraday rotation and Faraday conversion. The former operates alone at $\theta = 0, \pi$, the latter operates alone at $\theta = \pi/2$ and both should be considered together for the intermediate angles. The transfer equations (Mueller calculus) for the Stokes parameters I, Q, U, V were devised to treat together the propagation effects, emission and absorption (Azzam & Bashara 1987; Melrose, & McPhedran 1991). Good approximation for emission and absorption were long known (Trubnikov 1958; Rybicki, & Lightman 1967; Melrose, & McPhedran 1991). Now it is possible to combine them with given approximations of the propagation effects given by

$$\frac{d}{ds}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\rho_V & \rho_U \\ 0 & \rho_V & 0 & -\rho_Q \\ 0 & -\rho_U & \rho_Q & 0 \end{pmatrix}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}, \quad (40)$$

$$\rho_V = -\frac{\omega}{c} i\varepsilon^1{}_2, \quad \rho_Q = -\frac{\omega}{2c}(\varepsilon^1{}_1 - \varepsilon^2{}_2), \quad \rho_U = 0 \quad (41)$$

and do the radiative transfer calculations. One of the most interesting objects of such calculations is our Galactic Center Sgr A*.

Transfer equations were recently solved for the simple time-independent model of the GC (Huang et al. 2008). The authors treat the ordinary and extraordinary modes as linearly polarized by assuming these eigenmodes constitute a basis where either U or Q components of emissivity and propagation coefficients vanish. Actually, U components vanish $\rho_U = 0$ already in the basis $(\mathbf{e}^1, \mathbf{e}^2)$, since the projection of the magnetic field onto $(\mathbf{e}^1, \mathbf{e}^2)$ is parallel to $\mathbf{e}^1$ (see Melrose, & McPhedran (1991) p.184). As I shown in the Section 3, plasma modes are far from being linearly polarized at temperatures $T \lesssim 10$ estimated for the GC (Sharma et al. 2007). Thus, the propagation coefficients should be taken from equations (33) and (34). The Faraday conversion coefficient ρ_Q cannot be defined via emissivities and Faraday rotation coefficient ρ_V as in (Huang et al. 2008). The Faraday rotation measure was calculated from a simulated accretion profile in (Sharma, Quataert, & Stone 2007). However, the paper considered only the propagation effects, and did not do the self-consistent treatment of propagation. It is impossible to disentangle the effects of Faraday rotation and Faraday conversion in a relativistic plasma.

The crucial part of any radiative transfer is the proper transfer coefficients. It allows one to estimate the electron density near the accreting object (Quataert & Gruzinov 2000; Shcherbakov 2008). Several formulas were suggested for the temperature dependence of the component $\varepsilon^1{}_2$, responsible for Faraday rotation. These formulas were yet given for the high-frequency approximation (see Section 2.3). Let me compare them with the exact temperature dependence (24) $J = K_0(T^{-1})/K_2(T^{-1})$ and its limits. The limits are $J \to 1$ as $T \to 0$ and $J \to \ln(T)/(2T^2)$ as $T \to +\infty$. The results of this comparison are shown on Figure 7. Ballantyne,

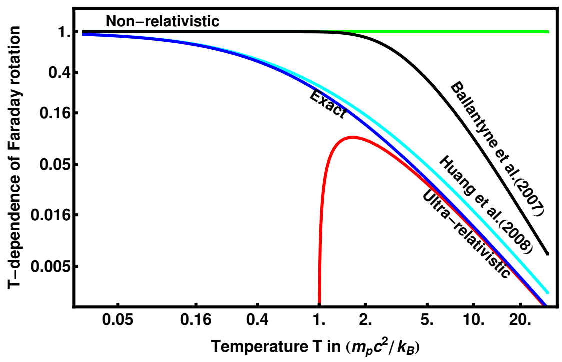

FIG. 7.— Temperature dependence of the Faraday rotation measure.

Ozel, & Psaltis (2007)[6] divided the thermal distribution into ultra-relativistic and non-relativistic parts as marked by the electron energy $\gamma_{\rm crit} = 10$. They sum the contributions of both species with calculated densities. To make a plot, I take their effective temperature Θ of plasma above $\gamma_{\rm crit}$ to be just temperature $\Theta = T$ and not the average kinetic energy as (Ballantyne, Ozel, & Psaltis 2007) suggests. This brings Θ to lower values and decreases the rotation measure. Even with this decrease the rotation measure is severely overestimated at $T \sim 1$. The convergence to the relativistic limit is not achieved

[5] The formula in (Phillips, & Wolszczan 1992) has no references/checks and is not correct

[6] The paper (Ballantyne, Ozel, & Psaltis 2007) has likely confused 3-D projection of 4-D response tensor in $j^\mu = \alpha^{\mu\nu} A_\nu$ (Melrose 1997c) with 3-D response tensor $\vec{j} = \alpha_{ab}\vec{A}$ that has the opposite sign.

even at $T \sim 30$. The paper (Huang et al. 2008) found the simpler fitting formula that reproduces the limits. Their expression is quite accurate.[7]

The rise of circular polarization at frequency 1THz is predicted (Huang et al. 2008) for the GC. The phase of Faraday conversion approaches unity and the destructive interference does not occur at this frequency. The result seems to be qualitatively correct regardless of the expression for the conversion measure, but the proper expressions (33) and (34) should be used for quantitative predictions.

The better treatment of propagation effects may also play a role in observations of jets. As we saw in Section 2.5, the propagation effects in thermal plasma cannot be described in the lowest orders in Ω_0/ω, if the temperature T is sufficiently high. Power-law distribution of electrons has a quite high effective temperature. Thus the high-frequency limit (Sazonov 1969; Jones & O'Dell 1977; Melrose 1997b) may not approximate well the hermitian part of the response tensor. Careful analysis of jet observations (Beckert & Falcke 2002; Wardle et al. 1998) may be needed. It should be based on the expressions for $\varepsilon^{\mu}{}_{\nu}$ in a general low-density regime.

5. DISCUSSION & CONCLUSION

The paper presents several new calculations and amends the previous calculations of propagation effects in uniform thermal magnetized plasma. The expression (4) for the correct response tensor is given in a high-frequency approximation. The exact temperature dependence (22) and (24) is found in first orders in Ω_0/ω in addition to known highly-relativistic and non-relativistic results. The importance on higher order terms is demonstrated for relativistic plasmas in jets and hot accretion flows. The fitting expressions (33) and (34) are found for the dielectric tensor at relatively high temperatures.

The results of numerical computations are given only when the correspondent analytical formulas are found. One can always compute the needed coefficients numerically for every particular frequency ω, plasma frequency ω_p, cyclotron frequency Ω_0 and distribution of electrons. However, the analytic formulas offer the simpler and faster way of dealing with the radiative transfer for a non-specialist. The eigenmodes were not considered in much detail. Radiative transfer problems do not require the knowledge of eigenmodes. However the knowledge of eigenmodes is needed to compute the self-consistent response tensor.

The response tensor in a form (4) can be expanded in Ω_0/ω and ω_p/ω. This expansion is of mathematical interest and will be presented in a subsequent paper as well as the expressions for the power-law electron distribution. The propagation through dense plasmas will also be considered separately.

As we saw, the bad approximations of polarized radiative transfer are sometimes made. The full system of transfer equations should be examined carefully before applying it to the real objects. The expressions should be given the boundaries of applicability (see (Wolfe, & Melia 2006) for the discussion of emissivity). The author expresses his wish for the proper treatment of polarized radiative transfer by the other authors.

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[7] "Temperature" γ_c in (Huang et al. 2008) should be redefined as $\gamma_c = 1 + T$, otherwise the lower limit is not reproduced.

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