The Solar Rotation and its Evolution During Cycle 23

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Introduction

- We present the most exhaustive and accurate inferences of the internal solar rotation rate and its evolution during solar cycle 23.

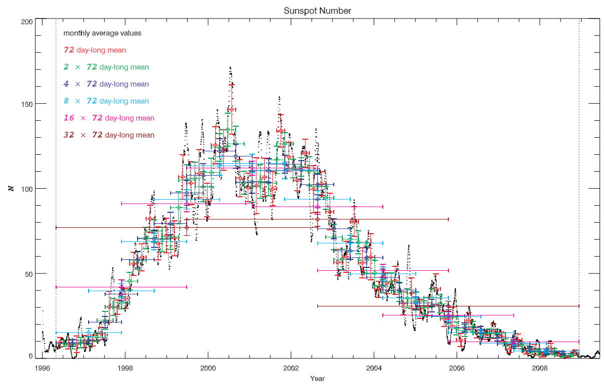


Fig. 1: *A full solar cycle of MDI observations has been analyzed using our state of the art fitting methodology.*

- We computed and fitted power spectra derived from time series of varying lengths: from a single 4608-day long epoch (64×72 day or 12.6 year) down to 64 segments each 72-day long.
- We carried out rotation inversions for all the available fitted mode sets and all available segments, including the MDI and GONG standard ”pipe-line” sets.

The Fitting Method

This method fits *individual* modes, using an asymmetric profile, and an *optimal* multi-tapered spectral estimator.

It fits simultaneous all m for a given n, ℓ multiplet, uses an iterative scheme to include mode contamination, and a “sanity” rejection.

- Key elements of this method:
  - ▹ include leakage matrix, fit an asymmetric profile, use *optimal* multi-tapered spectral estimator, simultaneous fit of individual modes (all m) sanity rejection, and mode contamination (iterative), fit time-series of varying lengths.
- Use *improved* SHC time-series: `1996.05.01 -- 2008.12.12`
  - ▹ spatial decomposition includes our best estimate of the image plate scale and of the MDI instrumental image distortion.
- Use *improved* leakage matrix:
  - ▹ includes effect of distortion by differential rotation (effective leakage matrix)
- Use varying time series lengths:
  - ▹ $64\times, 32\times, 16\times, 8\times, 4\times$ & 2×72-day long, overlapping, time-series, as well as 72-day long non-overlapping epochs.

Comparison with Standard “Pipe-Lines”

Frequencies Comparison

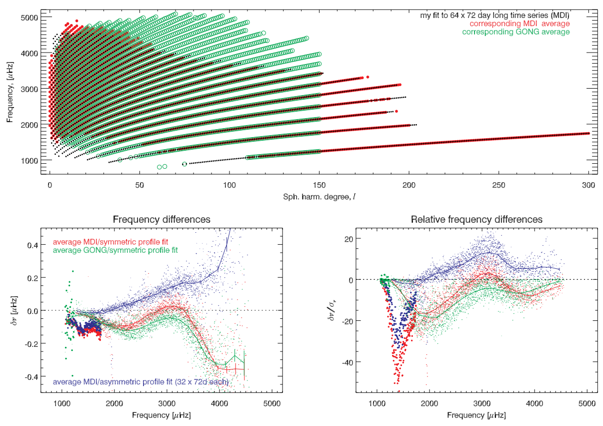


Fig. 2: *Top: mode coverage comparison. Bottom: frequencies and relative frequencies differences, as a function of mode frequency.*

- Systematic differences between methodologies, even when asymmetry is included in pipe-line processing,
- Specific f-mode systematics.

Splittings Comparison: a_1

	δa_1	$\delta a_1/\sigma_{a_1}$
GONG/sym 64e vs SGK/asym 64e	-0.277 ± 0.984	-0.917 ± 1.279
TPL/sym 64e vs SGK/asym 64e	0.051 ± 0.635	0.534 ± 2.888
TPL/asym 32e vs SGK/asym 32e	0.096 ± 0.769	1.398 ± 2.384

The Attrition Issue

Observed Mode Attrition

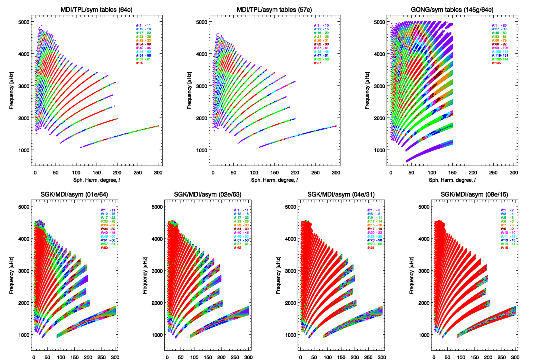


Fig. 3: *Mode attrition for different mode fitting methods. Top: MDI and GONG pipe-lines; bottom: my method for different time series lengths. The colors represent how often a mode was fitted.*

The Problem with Mode Attrition

- Inverse Theory

$$y_i = \int K_i\, x(p) dp$$

  - Inverse problems are singular, require regularization,
  - produce an *estimate* of the solution

$$\hat{x}(p_k) = \int R(p, p_k) x(p) dp$$

  - R resolution kernels – depend on the input set
- Solar Rotation

$$\delta\nu_{n,\ell,m} = \iint K_{n,\ell,m}(r,\theta)\, \Omega(r,\theta)\, dr\, d\theta$$

  - input set is defined by $\{n, \ell, m\}$ or $\{n, \ell, a_i\}$
  - temporal changes in the input set affect R, hence $\hat{x}$

⇒ We chose to invert a constant input set to avoid injecting changes of the input sets into inverted rotation profile changes.

Other Issues

- Leakage matrix
  - closest leaks – $\Delta\nu_{\delta m=2,\delta\ell=0}$ – are *rarely* resolved
    - ∗ $\Delta\nu \gg \Gamma$, $\Delta\nu \simeq 2 \times \frac{\Omega}{2\pi} \simeq 0.8\,\mu$Hz
  - plate scale, image & eigen values distortions, orientation (B_0)
    - ∗ new MDI Sph. Harm. Coefs
      accounts for plate scale and image distortion
    - ∗ distortion by differential rotation — 1 – 6% effect
    - ∗ $B_0 = B_0(t)$ — 3 – 15% effect
    - ∗ other geometric variations negligible
    - ⇒ very long time-series indicate remaining mismatch for f-mode
  - Independent leakage computation
    - ∗ small differences

Leakage Matrix – f modes

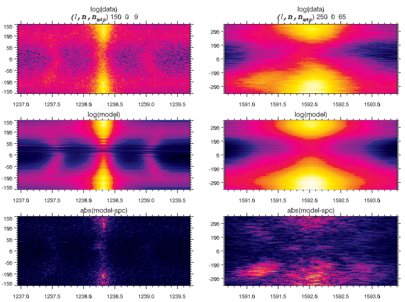
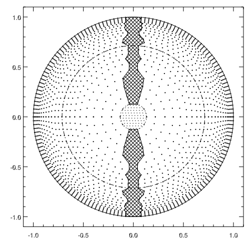


Fig. 4: *Data and model (top and middle respectively) of the f-mode at $\ell = 150$ & 250 (left & right resp.). Bottom: difference. Note the mismatch of the leaks at $l = 250$.*

The Inversion Method

- The inversion methodology is an iterative methodology based on a least-squares regularization (Eff-Darwich & Korzennik, 2007).
- Implements a model grid optimization derived from the actual information in the input set. This optimized model grid is itself irregular, namely with a variable number of latitudes at different depths.



- ▹ Iterative approach
- ▹ *optimal* and *non-uniform* grid

Fig. 5: *Model grid derived iteratively from the actual resolution potential of the input set.*

Mean Solar Rotation Rate for Cycle 23

The 12.6 year-long MDI Data Set

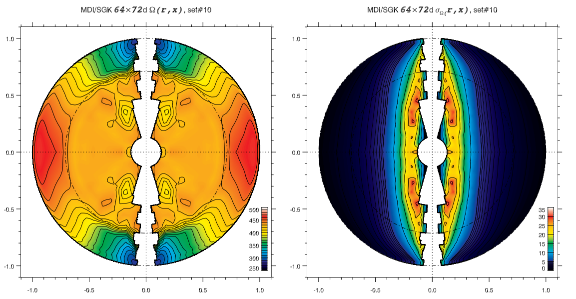


Fig. 6: *Rotation rate (left) as a function of depth and latitude derived from a solar cycle worth of data. Right: The formal uncertainty of that inversion.*

Rotation Rate Changes

GONG $r/R = 1.00 - 0.71$

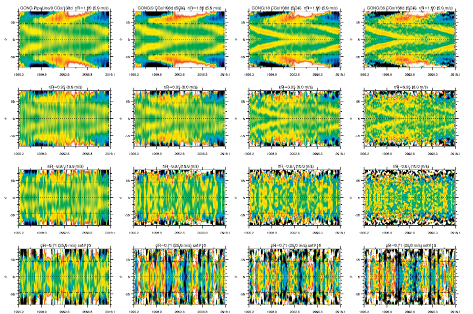


Fig. 7: *Change of the rotation rate, as a function of time and latitude, derived from inversions using Clebsch-Gordan coefficients, at four depths ($r/R = 1, 0.95, 0.87, 0.71$ top to bottom) and using either the GONG pipeline 9 term expansion, or my fit to GONG frequency tables using 9, 18 or 36 terms (left to right).*

MDI/JS/TPL $r/R = 1.00 - 0.71$

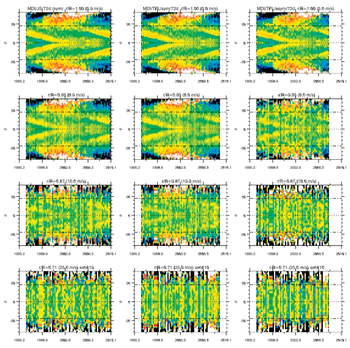


Fig. 8: *Change of the rotation rate, as a function of time and latitude, derived from inversions using Clebsch-Gordan coefficients, same four depths as in Fig. 7, but using MDI's tables (standard & improved sets, using symmetric or asymmetric profile – left to right).*

MDI/SGK $r/R = 1.00 - 0.71$

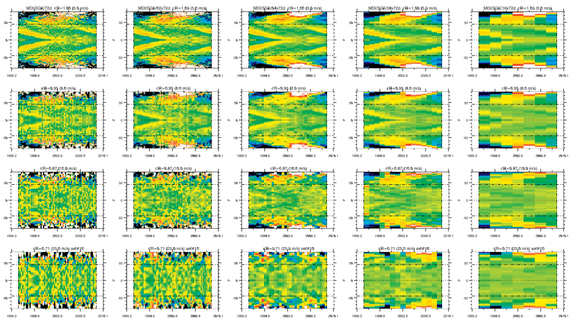


Fig. 9: *Change of the rotation rate, as a function of time and latitude, derived from inversions using individual frequency tables, same four depths, using results of my fit to MDI's 72d, and 2, 4, 8 & 16 × 72d long epochs (left to right).*

Conclusions

- Fitting: *Issues* still remain to be solved.
- Mean rotation
  - Very long time-series improved precision, resolution & extent
  - Dip at $(0.4, 63^o)$ – a 1σ, rising branch of cycle
- Evolution
  - *Easy* at the surface, and low latitudes
  - Remains challenging down to base of CZ & difficult below CZ
  - A more consistent picture emerges when using longer time series