Modes of Star Formation: A Retrospective

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A Brief History

4 Stages of Star Formation

(Shu, Adams, Lizano, 1987)
\[
\frac{\partial \Sigma}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (ru\Sigma) = 0
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{a^2}{\Sigma} \frac{\partial}{\partial r} (\Theta \Sigma) = g + l
\]

\[
g + l = \frac{1}{r^2 \Sigma} \int_{0}^{\infty} 2\pi r' dr' K_0(r'/r) \left[ -G \Sigma(r) \Sigma(r') + \frac{B_z(r)B_z(r')}{(2\pi)^2} \right]
\]

\[
\left[ \frac{\partial B_z}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (urB_z) \right] = \frac{1}{r \Sigma} \left[ \frac{(2\zeta_0)^{1/2}}{2\pi \gamma C} \frac{r B_z B_r^+}{\sqrt{\Sigma}} \right]
\]

\[
B_r^+ = \frac{1}{r^2} \int_{0}^{\infty} K_0(r'/r) B_z(r') r' dr'
\]

\[
K_0(q) = \frac{1}{2\pi} \int_{0}^{2\pi} d\varphi \frac{1 - q \cos \varphi}{(1 + q^2 - 2q \cos \varphi)^{3/2}}
\]
\( v_\infty = 0.495 \ a_s \)
Fred: Your measurements of only `half the sound speed’ must be wrong: you should get 0.495 times a_s. You need to get more accurate measurements...
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Phil: … you and Frank never do anything 0.495-hearted or 0.495-assed…
Physical Diffusion Solutions

Graphs showing the variation of density $n$ (cm$^{-3}$) and velocity $-v$ (km/s) with radius $r$ (pc) for different times $t$ (Myr). The graphs illustrate how the density and velocity change with time, with distinct curves for each time interval, showing the diffusion process in a medium.
Mass Infall Rates with Nonzero Head Start Velocity
Most Stars Form in Clusters:

[1] What is the distribution of cluster environments?
[2] How does the cluster environment affect the formation of stars/planets?
Conjecture:

Cluster environments actively affect the formation of planets more than the formation of stars themselves.
Cumulative Distribution: Fraction of stars that form in stellar aggregates with $N < N$ as function of $N$

Median point: $N=300$

Dynamical Studies

I. Evolution of clusters as astrophysical objects

II. Effects of clusters on forming solar systems

• Distribution of closest approaches
• Radial position probability distribution
Simulations of Embedded Clusters

- Modified NBODY2 (and 6) Codes (*S. Aarseth*)
- Simulate evolution from embedded stage to age 10 Myr
- Cluster evolution depends on the following:
  - cluster size
  - initial stellar and gas profiles
  - gas disruption history
  - star formation history
  - primordial mass segregation
  - initial dynamical assumptions
- 100 realizations are needed to provide robust statistics for output measures

*(E. Proszkow thesis - work with Phil!)*
Virial Ratio  $Q = |K/W|$  

- Virial $Q = 0.5$; cold $Q = 0.04$

Mass Segregation: largest star at center of cluster

**Simulation Parameters**

- **Cluster Membership**
  $N = 100, 300, 1000$

- **Radius**
  $R(N) = 1\text{pc}\left(\frac{N}{300}\right)^{1/2}$

- **Initial Stellar Density**
  $\rho_* \propto r^{-1}$

- **Gas Distribution**
  $\rho \propto r^{-1}$

- **SF Efficiency** $= 0.33$

- **Embedded Epoch** $t = 0-5$ Myr

- **SF time span** $t = 0-1$ Myr
Simulation | $\Gamma_0$ | $\gamma$ | $b_C$ (AU)
---|---|---|---
100 Subvirial | 0.166 | 1.50 | 713
100 Virial | 0.0598 | 1.43 | 1430
300 Subvirial | 0.0957 | 1.71 | 1030
300 Virial | 0.0256 | 1.63 | 2310
1000 Subvirial | 0.0724 | 1.88 | 1190
1000 Virial | 0.0101 | 1.77 | 3650

Typical star experiences one close encounter with impact parameter $b_C$ over $t = 10$ Myr

Subvirial initial conditions matter - thanks Phil...
\[
\langle \sigma \rangle_{ej} = C_0 \left( \frac{a_P}{AU} \right) \left( \frac{M_*}{M_{\odot}} \right)^{-1/2}
\]

where

\[C_0 = 1350 \pm 160 \ (AU)^2\]

\[\langle \sigma \rangle_{\text{disrupt}} \approx (400 \ AU)^2\]
Clusters have relatively moderate effects on their constituent solar systems through dynamical interactions
Photoevaporation of a circumstellar disk
Radiation from the background cluster often dominates radiation from the parent star (Johnstone et al. 1998; Adams & Myers 2001)
FUV radiation (6 eV < E < 13.6 eV) is more important in this process than EUV radiation
FUV flux of $G_0 = 3000$ will truncate a circumstellar disk to $r_d$ over 10 Myr, where $r_d = 36 AU \left[ \frac{M_*}{M_{\text{sun}}} \right]$
FUV Flux depends on:
- Cluster FUV luminosity
- Location of disk within cluster

Assume:
- FUV point source at center of cluster
- Stellar density $\rho \sim 1/r$

$G_0$ Distribution

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>900</td>
</tr>
<tr>
<td>Peak</td>
<td>1800</td>
</tr>
<tr>
<td>Mean</td>
<td>16,500</td>
</tr>
</tbody>
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$G_0 = 1$ corresponds to FUV flux $1.6 \times 10^{-3}$ erg s$^{-1}$ cm$^{-2}$
Photoevaporation Model

(Adams et al. 2004)
Evaporation Time vs FUV Field

(for disks around solar mass stars)
Evaporation is much more effective for disks around low-mass stars: Giant planet formation can be compromised
Evaporation vs Accretion

Disk accretion aids and abets the disk destruction process by draining gas from the inside, while evaporation removes gas from the outside...

Total time scale of 8 Myr, consistent with observations...
Radiation effects Dominate over Dynamical effects in Clusters
Group/Cluster Transition

Constraints on Solar Birth Cluster

Supernova
Neptune
Sedna
Radiation

\[ N = 4300 \pm 2800 \]
Cluster size: $N = 1000 - 7000$

Reasonable \textit{a priori} probability (few percent)

Allows meteoritic enrichment and scattering survival

UV radiation field evaporates disk down to 30 AU

Scattering interactions truncate Kuiper belt at 50 AU
leave Sedna and remaining KBOs with large $(a,e,i)$
Bottom Line:

Clusters in solar neighborhood exert an intermediate level of influence on their constituent solar systems: Neither Dominant Nor Negligible.

What’s next:

Extend analysis to larger $N$
Distribution of cluster sizes $N$
Fundamental Plane for Clusters

Density: 1, 10, 100
Relax time: 1-1000 Myr
In spherical limit, orbits are Spirographs:
Orbits in Spherical Potential

\[ \rho = \frac{\rho_0}{\xi(1 + \xi)^3} \Rightarrow \Psi = \frac{\Psi_0}{1 + \xi} \]

\[ \varepsilon \equiv \left| \frac{E}{\Psi_0} \right| \text{ and } q \equiv \frac{j^2}{2\Psi_0 r_s^2} \]

\[ \varepsilon = \frac{\xi_1 + \xi_2 + \xi_1\xi_2}{(\xi_1 + \xi_2)(1 + \xi_1 + \xi_2 + \xi_1\xi_2)} \]

\[ q = \frac{(\xi_1\xi_2)^2}{(\xi_1 + \xi_2)(1 + \xi_1 + \xi_2 + \xi_1\xi_2)} \]
\[ q_{\text{max}} = \frac{1}{8\varepsilon} \left(1 + \sqrt{1 + 8\varepsilon} - 4\varepsilon\right)^3 \]  
(angular momentum of the circular orbit)

\[ \xi_* = \frac{1 - 4\varepsilon + \sqrt{1 + 8\varepsilon}}{4\varepsilon} \]  
(effective semi-major axis)

\[ \frac{\Delta \theta}{\pi} = \frac{1}{2} + \left[ (1 + 8\varepsilon)^{-1/4} - \frac{1}{2} \right] \left[ 1 + \frac{\log(q/q_{\text{max}})}{6 \log 10} \right]^{3.6} \]

\[ \lim_{q \to q_{\text{max}}} \Delta \theta = \pi (1 + 8\varepsilon)^{-1/4} \]  
(circular orbits do not close)
These results determine the radiation exposure of a star, averaged over its orbit, as a function of energy and angular momentum:

$$\langle F_{\text{fuv}} \rangle \approx \frac{L_{\text{fuv}}}{8r_s^2 \sqrt{q} \cos^{-1} \sqrt{\varepsilon + \sqrt{\varepsilon \sqrt{1 - \varepsilon}}} \cdot \frac{A \varepsilon^{3/2}}{\varepsilon}$$

where \( 1 \leq A(q) \leq \sqrt{2} \)
Triaxial Density Distributions

• Relevant density profiles include NFW and Hernquist

\[ \rho_{nfw} = \frac{1}{m(1 + m)^2} \quad \rho_{\text{Hern}} = \frac{1}{m(1 + m)^3} \]

• Isodensity surfaces in triaxial geometry

\[ m^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \quad a > b > c > 0 \]

• In the inner limit both profiles scale as 1/r

\[ m \ll 1 \quad \longrightarrow \quad \rho \propto \frac{1}{m} \]
Triaxial Potential

\[ \Phi = \int_0^\infty du \frac{\psi(m)}{\sqrt{(u + a^2)(u + b^2)(u + c^2)}} \]

where \( \psi(m) = \int_\infty^m \rho(m) dm^2 \)

• In the inner limit the above integral can be simplified to

\[ \Phi = -I_1 + I_2 \]

where \( I_1 \) is the depth of the potential well and the effective potential is given by

\[ I_2 = 2 \int_0^\infty du \frac{\sqrt{\xi^2 u^2 + \Lambda u + \Gamma}}{(u + a^2)(u + b^2)(u + c^2)} \]

\( \xi, \Lambda, \Gamma \) are polynomial functions of \( x, y, z, a, b, c \)
Triaxial Forces

\[
F_x = \frac{-2 \text{sgn}(x)}{\sqrt{(a^2 - b^2)(a^2 - c^2)}} \ln \left( \frac{2G(a)\sqrt{\Gamma} + 2\Gamma - a^2\Lambda}{2a^2\xi G(a) + \Lambda a^2 - 2a^4\xi^2} \right)
\]

\[
F_y = \frac{-2 \text{sgn}(y)}{\sqrt{(a^2 - b^2)(b^2 - c^2)}} \left[ \sin^{-1} \left( \frac{\Lambda - 2b^2\xi^2}{\sqrt{\Lambda^2 - 4\Gamma\xi^2}} \right) - \sin^{-1} \left( \frac{2\Gamma/b^2 - \Lambda}{\sqrt{\Lambda^2 - 4\xi^2\Gamma}} \right) \right]
\]

\[
F_z = \frac{-2 \text{sgn}(z)}{\sqrt{(a^2 - c^2)(b^2 - c^2)}} \ln \left( \frac{2G(c)\sqrt{\Gamma} + 2\Gamma - c^2\Lambda}{2c^2\xi G(c) + \Lambda c^2 - 2c^4\xi^2} \right)
\]

\[
G(u) = \xi^2 u^4 - \Lambda u^2 + \Gamma
\]

\[
\xi^2 = x^2 + y^2 + z^2
\]

\[
\Lambda = (b^2 + c^2)x^2 + (a^2 + c^2)y^2 + (a^2 + b^2)z^2
\]

\[
\Gamma = b^2c^2x^2 + a^2c^2y^2 + a^2b^2z^2
\]

(Adams, Bloch, Butler, Druce, Ketchum 2007)
Orbit Gallery
New Cluster Result

Kinematic observations of the Orion Nebula Cluster show that the system must have:

- Non-spherical geometry
- Non-virial initial conditions
- Viewing angle not along a principal axis

(with E. Proszkow, J. Tobin, and L. Hartmann, 2009)
Lots of chemistry and many heating/cooling lines determine the temperature as a function of $G, n, A$. 

Results from PDR Code
Solution for the Fluid Fields

outer disk edge

sonic surface

\[ \xi = \frac{r}{r_d} \]