Afterglows Theory
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Gamma-Ray Burst:

4 Stages

1) Compact Source, $E > 10^{51}$ erg

2) Relativistic Kinetic Energy

3) Radiation due to Internal Shocks = GRB

4) Afterglow by External Shocks
Gamma-Ray Burst: 4 Stages

Coasting, IS=GRB

Deceleration= Afterglow

Energy release

Thermal acceleration
4 Stages

- Energy release
- Thermal acceleration
- Coasting, IS=GRB
  - Early Afterglow
    - FS + RS
  - Late Afterglow
    - Newtonian
- Deceleration=

Graph with time on the x-axis and energy release (γ) on the y-axis, showing different stages of the release.
Afterglow was Predicted!

Paczyski & Rhoads 1993
Katz 1994
Vietri 1996
Meszaros & Rees 1997
Sari & Piran 1997

GRB: internal shocks
afterglow: external shocks
GRB proper is NOT the early afterglow


**Simple Theory**

**Dynamics:** deceleration of the relativistic shell by collision with the surrounding medium  
(Blandford & McKee 1976; Meszaros & Rees 1997; Waxman 1997; Sari 1997; Cohen, Piran & Sari 1998; Rhoads 1999; Best & Sari 2000)

**Radiation:** synchrotron & Inverse Compton (IC)  
(Meszaros & Rees; Katz & Piran; Waxman; Sari, Piran & Narayan; Granot Piran & Sari)

Clean, well defined problem.

**Few parameters:**  
\[ E, n, p, \varepsilon_e, \varepsilon_B \]
Theoretical Spectra
(Sari, Piran & Narayan 1998)
Theoretical Spectra
(Sari, Piran & Narayan 1998)

\[ N(\gamma_e) \propto \gamma_e^{-p} + B \]

Cooling

\[ v^2 \sim B \gamma_e^2 \]

"universal"
Synchrotron Radiation: Calculating $\nu_a \; \nu_m \; \nu_c \; F_{max}$

- Synchrotron: $\nu \sim \Gamma B \gamma_e^2 \; \& \; F_{\nu} \sim \Gamma B$

- $B^2/8\pi \sim \varepsilon_B \Gamma^2 n m_p c^2$

- $\nu_m$: $\gamma_e \sim 1840 \varepsilon_e \Gamma$

- $\nu_c$: $P \tau / \Gamma = \gamma_e m_e c^2$

- $F_{\text{max}} \sim \Gamma B * 4\pi R^3 n/3$

- $F_{\nu < \nu_a} \sim 2c^{-2} \nu^2 kT \left(\frac{R}{\Gamma}\right)^2 \Gamma$
Observations vs. Theory

(Galama et. al., 1998 compared to Sari, Piran & Narayan 98)

Good agreement between theory and observations
Theory & Observations

(Harrison et. al.; Yost et. al.)

Good agreement between theory and observations
Inferring Physical Parameters from the Observed Spectra

We observe $\nu_a$, $\nu_m$, $\nu_c$, $F_m$

We infer $R$, $B$, $\gamma_{\text{min}}$, $N$

With synchrotron only

Pure powerlaws

There is always a single solution.
Physical Parameters from the Observed Spectra

- The parameters: $E$, $n_0$, $\varepsilon_e$, $\varepsilon_B$, $p$
- $p$: the slope at high frequencies
- remaining parameter: Solving equations for: $\nu_a$, $\nu_m$, $F_m$, $\nu_c$ (Wijers & Galama 1998)
- For GRB970508 we obtained:
  - $E = 5.3 \times 10^{51}$ ergs
  - $n_0 = 5.3$ cm$^{-3}$
  - $\varepsilon_e = 0.57$
  - $\varepsilon_B = 0.0082$
- This method is sensitive to the details of the model
- Can be applied at several epochs

4 observables & 4 unknowns
Inverse Compton

(a) Slow cooling

(b) Fast cooling

\[ \log_{10} \left( \frac{\nu f_\nu}{\nu_{m} f_\nu(\nu_{m})} \right) \]

\[ \log_{10} \nu \ [\text{Hz}] \]
Inverse Compton

- Another 4 powerlaw segments
- One powerlaw segment $\nu_c < \nu < \nu_m$ extended
- Frequency higher by $\gamma_e^2$
- May be observed in X-ray if $n > 10$
- Energy in IC: $\sigma_T n R \gamma_e^2$ or $\sqrt{\xi_e / \xi_B}$

With synchrotron + IC

No Pure powerlaws

There are two or no solution.

$$C \equiv 0.06 t_d^4 D_{L,28}^{-2} \eta \nu_{a,9}^{10/3} \nu_{m,13}^{13/6} \nu_{c,14}^{3/2} F_{m,mJy}^{-1} < \frac{1}{4}$$
Dynamics - Scaling

Adiabatic:

- ISM: \( M \propto R^3 \) \( \gamma \propto R^{-3/2} \) \( \gamma \propto t^{-3/8} \)
- Wind: \( M \propto R \) \( \gamma \propto R^{-1/2} \) \( \gamma \propto t^{-1/4} \)
- Jet: \( M \propto \theta^2 \) \( \gamma \propto e^{-R/R_j} \) \( \gamma \propto t^{-1/2} \)

\[ \theta = \theta_0 + 1/\gamma \quad R \sim \text{Const.} \]

Newtonina: \( \beta \) instead of \( \gamma \)
Sidewise Expansion

(Rhoads; Meszaros & Rees; Sari, Piran & Halpern)

- Fluid frame expansion ~ c.
- Observer frame: $\Delta R_\perp \sim R/\gamma$.

- Initially: $\theta \sim \theta_0$
- After $1/\gamma \geq \theta_0$: $\theta \sim 1/\gamma$

$t_{\text{jet}}: \gamma(t_{\text{jet}}) = 1/\theta_0$
Dynamics + Radiation = Light Curves

(Sari, Piran & Narayan 1998)

The figure shows two plots with different time regimes:

1. **High Frequency** ($\nu > \nu_0$):
   - Region A: $t^{1/6}$, $[t^{-1/3}]$
   - Region B: $t^{-1/4}$, $[t^{-4/7}]$
   - Region C: $t^{(2-\beta)p/4}$
   - Region D: $t^{(2-\beta)p/7}$
   - Region E: $t^{(2-\beta)p/4}$

2. **Low Frequency** ($\nu < \nu_0$):
   - Region F: $t^{1/2}$
   - Region G: $t^{3(1-p)/4}$
   - Region H: $t^{(2-\beta)p/4}$

The plots illustrate the flux vs. time relationship with markers for different time points: $t_c$, $t_m$, $t_0$. The diagrams highlight the alpha-beta game in the context of light curve analysis.
Adiabatic v.s. Radiative

(Blandford & McKee 1976; Sari 1997; Cohen, Piran & Sari 1998)

- Most models assume adiabatic evolution.

- Strongly radiative: only if $\varepsilon_e \sim 1$ and all electrons cool.

- Partially radiative evolution: lasts for a long time, especially if $p \sim 2$.

$$E = E_0 \left( \frac{t}{t_0} \right)^{-17\varepsilon/12} \rightarrow E_0 / E = 2 - 30$$

Not negligible!

$F_m$ $E_{aft,x}$ $E_{cal}$
More Complications

- **Pre acceleration:**
  - High energy photons fronts
    - (Thompson & Madau; Beloborodov)
  - Decupled Neutrons

- **Change of external density**
  - Termination shocks
    - \[ r_t = 7.5 p c \dot{M}_{-5}^{3/10} \rho_{-24}^{-3/10} v_{w,3}^{1/5} t_{6}^{2/5} \]

- **Additional energy**
  - Slow shells may have most of the energy - reverse shocks
    - (Kumar & Piran; Sari & Meszaros)
Dynamics in more detail

- **Self similar solution: BM76.**
  - Characteristic width $w = R/16\gamma^2$
  - Hydrodynamic profiles $(1+(R-r)/w)^{-17/12}$

- **Extensions**
  - Partial radiative solutions: Cohen, Piran & Sari
  - Exponential density: Perna & Vietri

- **Jets?**
  - No detailed solution. Only rough scalings
  - Panaitescu Kumar.
  - Hydrodynamic simulations: Granot et. al.
The Contribution to the Observed Flux:
Equal arrival time surface & volume
A Prediction: Rings
(Waxman; Sari; Panaitescu & Meszaros; Granot, Piran & Sari abc; Granot & Sari)

- If a nearby GRB occurs it can be resolved. (redshift $z<0.1$)

- High frequency: narrow ring
- Low frequency: disk with brighter edges.
- Scintillation & Microlensing
Temporal Breaks with Jets

\[ F_v \propto t^{-\alpha} \]

Change from spherical ISM

- \( v > v_m \) \( F_v \propto t^{-p} \sim t^{-2.2} \) \( \Delta \alpha \approx -1.1 \) (>p1)
- \( v_a < v < v_m \) \( F_v \propto t^{-1/3} \) \( \Delta \alpha = -5/6 \) (>p1)
- \( v < v_a \) \( F_v \propto t^0 \) \( \Delta \alpha = -1/2 \) (>p1)

The break is substantial at all frequencies.
Observed Jets

$\dot{t} = -0.82$

$\dot{t} = -2.18$

$\dot{t} \text{jet} = 1.2d$

$t^{-0.82}$

$t^{-2.18}$

$\text{time (days)}$
Jets Without Breaks
(Sari, Piran & Halpern 1999)

- $F_\nu \propto t^{-\alpha}$
- Two classes: $\alpha \sim 1.2$ & $\alpha \sim 2$

- Jet candidates without breaks:
  - $980519 \; \alpha = 2.05 \pm 0.04$
  - $980326 \; \alpha = 2.10 \pm 0.13$

- The Large L distribution is due to jets.
  - (Based on 4 bursts, 2 without redshift)
3 Classes of Optical Afterglows

<table>
<thead>
<tr>
<th></th>
<th>Break</th>
<th>No Break</th>
</tr>
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<tbody>
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<td>( t^{-1} )</td>
<td>Jets !!</td>
<td>Spherical</td>
</tr>
<tr>
<td>( t^{-2} )</td>
<td></td>
<td>probably jets</td>
</tr>
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</table>
Wind for fast decays?

- Requires $p=3$ and slow cooling: $\nu_c \geq 10^{18} \text{ Hz}$
- BUT: IC should be accounted for!

$$C \equiv 0.06 t_4 \ d_4 \ D_{L,28}^{-2} \ \eta \ \nu_{a,9}^{10/3} \ \nu_{m,13}^{13/6} \ \nu_{c,14}^{3/2} \ F_m^{\frac{1}{2}} < \frac{1}{4}$$

- 980519: $\nu_c \leq 10^{14} \text{ Hz}$
- Wind interpretation for 980519 is difficult.
Distribution of $\theta_0$
Beaming Luminosity Relation
GRB – Standard Candles!

Luminosity distribution is spread over a factor of 500

After correcting for beaming, spread < 10

Frail et al.
Direction of Polarization

$B \parallel \rightarrow B_{\perp}$

$B \parallel \leftarrow B_{\perp}$
Direction of Polarization $B_{\perp}$

$\Pi = 0, \Pi = \Pi_m / 3, \Pi = \Pi_m$
Polarization Evolution

Offset = 0.3θ₀

Offset = 0.95θ₀
Polarization Lightcurves
Observations

- **Radio upper limits:**
  - 19% GRB 980329 (Taylor et. al.)
  - 8% GRB 980703 (Frail et. al.)

- **Optical upper limit:**
  - <2.3% GRB 990123 (Hjorth et. al.)

- **First detection**
  - 1.7% for GRB 990510
  - (Covino et. al.; Wijerse et. al.)
Polarization - Summary

- Fluctuations occur if coherent B cells are large.

- For Beamed Afterglows:
  - polarization even for small coherence length
  - 1 or 3 peaks of polarization
  - polarization may rotate by $90^\circ$.
  - Direction of polarization is towards the center of the jet or perpendicular to that.
  - Together with proper motion: infer the direction of B

Need observations at multiple times around $t_{\text{jet}}$. 
Confirmation of The Fireball Model? YES! BUT...

- Most afterglow observations detect only late stage with $\gamma<10$.

$100<\gamma_0<10^5$

- No direct evidence to internal or external shocks.

- $M=E/\gamma_0$

Moderately “Clean”

Extremely “Clean”
4 Stages

- Energy release
- Thermal acceleration
- Coasting, IS=GRB
- Early Afterglow (FS + RS)
- Deceleration = Afterglow
- Late Afterglow
- Newtonian

Energy release

time
Initially: More complicated hydrodynamics

(Sari & Piran 1995)

- Four critical radii:
  - $R_s$ where shell may spread and IS may happen.
  - $R_\Delta$ where reverse shock crosses the shell.
  - $R_\gamma$ where the shock had collected mass of $M/\gamma = E/c\gamma^2$.
  - $R_N$ where the reverse shock turns from Newtonian to relativistic.
Two Possible Evolutions

(Sari & Piran 1995)

\[ \xi^2 R_s = \xi^{1/2} R_\Delta = R_\gamma = \xi^{-1} R_N \]

where \( \xi \sim (E/n)^{1/6} \Delta^{-1/2} \gamma_0^{-4/3} \)

Two possible orderings:

- \( \xi > 1 \): shell spreads \((R_s)\), reverse shocks crosses the shell \((R_\Delta)\) and then shell decelerate \((R_\gamma)\)
- \( \xi < 1 \), the reverse shock first becomes relativistic \((R_N)\), \( R_\gamma \) is no important, and the self similar stage begins \((R_\Delta)\), there is no spreading.
The initial external shocks (afterglow) may overlap the internal shocks (GRB) signal.

Confirmation requires very early (10-100s) afterglow observations.
Predicted optical flash

\[ \gamma - \text{rays} \]

\[ \text{Reverse shock} \]

\[ \text{forward shock} \]

\[ \text{X -rays} \]

\[ \text{optical} \]
Reverse Shock in GRB 990123 - radio
Early Afterglow - Summary

■ Optical:
  Gaps - Onset of afterglow \((\gamma_0 + \text{IS vs. ES})\)
  Reverse shock – ejecta emission \(\gamma_0\), loading.
  Transition reverse-forward
  Polarization – Jet geometry \(\longrightarrow\) correct energy.

■ X-ray:
  Gaps - Onset of afterglow.
  GRB – Afterglow mismatch \((\text{IS vs. ES})\)
  Partially Radiative phase – True energy \(E_0\)

■ Radio:
  Radio flares – ejecta emission \(\gamma_0\), loading.
  Winds or ISM (early value of \(n_0\)).

SHAPE, WHERE, HOW, HOW FAST, WITH WHAT
jets \(n\) int – ext \(\gamma_0\) Baryons or B
Reverse, Forward, Jets ...

- Rising spectrum $\nu^{1/3}$
- Polarization $\Pi \leq 10\%$
- $t^{-2}$
- $t^{+1/2}$
- $t^{-1}$
- $t^{-2.2}$
- $t^{-1.5}$
- $30\text{s}$
- $30\text{m}$
- $2\text{h}$
- $1\text{d}$
- $100\text{d}$