Testing gravity: equivalence principle, and gravitational waves

Lam Hui
Columbia University
Outline:

1. Equivalence principle testing.
   Collaborators: Phil Chang, Alberto Nicolis, Chris Stubbs.

2. Gravitational wave detection.
   Collaborators: Sean McWilliams, I-Sheng Yang.
Essentially all attempts to explain cosmic acceleration by modifying gravity involves introducing new long range forces. The simplest option is a scalar force i.e. these theories are some kind of scalar-tensor theory (at least in appropriate limits). This includes theories that at first sight might not look like a scalar-tensor theory e.g. DGP, massive gravity.
Essentially all attempts to explain cosmic acceleration by modifying gravity involves introducing new long range forces. The simplest option is a scalar force i.e. these theories are some kind of scalar-tensor theory (at least in appropriate limits). This includes theories that at first sight might not look like a scalar-tensor theory e.g. DGP, massive gravity.

- By Weinberg/Deser theorem, at low energies, a Lorentz invariant theory of a massless spin 2 particle must be general relativity (GR). This is why essentially all proposed long distance modifications to GR end up introducing a new particle, usually a scalar, mediating an extra long range force.
• Also: if one is willing to consider some form of dark energy other than the cosmological constant e.g. some kind of light scalar (quintessence), it is very natural to think this scalar might couple to matter. In fact, absent symmetries that forbid it, such a scalar-matter coupling is inevitable, i.e. a scalar-tensor theory.
What do these scalar-tensor theories look like?

\[ S = S_{GR} + \int d^4x \left[ -\frac{1}{2} (\partial \varphi)^2 + \mathcal{L}_{\text{int}}(\varphi) \right] + \int d^4x \left[ h_{\mu\nu} T_{\mu\nu}^{m} + \varphi T_{m} \right] \]

- Graviton-matter coupling is universal i.e. graviton couples to all forms of matter with the same strength. This is forced upon us by consistency i.e. Einstein’s equivalence principle (Weinberg’s theorem).
- Scalar-matter coupling is also universal. This is by construction. i.e. Let us consider the minimal models where the scalar is coupled to electrons, protons, dark matter, etc. at the same (grav.) strength, i.e. these theories are constructed to obey the equivalence principle.
Despite the (microscopic) universal scalar-matter coupling, “equivalence principle” is violated anyhow.

- **Microscopic**: scalar does not couple to photons whose \( T_m = T_m^{\mu\mu} = 0 \), i.e. photons bend in a gravitational field but not in a scalar field. This is why comparing lensing versus dynamical masses is useful (see Rachel Mandelbaum’s talk).

\[
S = S_{GR} + \int d^4x \left[ -\frac{1}{2} (\partial \varphi)^2 + \mathcal{L}_{\text{int}}(\varphi) \right] + \int d^4x [h_{\mu\nu} T_m^{\mu\nu} + \varphi T_m]
\]
Despite the (microscopic) universal scalar-matter coupling, equivalence principle is violated anyhow.

- **Macroscopic**: scalar charge gets renormalized in astronomical objects.

  \[
  \text{net force on an object } = -M \nabla \Phi_\text{grav.} - Q \nabla \varphi
  \]

  mass \hspace{1cm} scalar charge

These scalar-tensor theories give \( Q/M = 1 \) for microscopic particles i.e. \( Q = \int d^3x T_m = M \).

Macroscopic objects, however, can have \( Q/M \neq 1 \); in fact \( Q/M \) ratio depends on the internal structure of the objects, thus leading to equivalence principle violations.

- **Chameleon/symmetron theories**: non-derivative interaction e.g. \( \mathcal{L}_{\text{int}}(\varphi) \sim 1/\varphi \).

  \[
  Q \sim \int d^3x [T_m + \partial \varphi \mathcal{L}_{\text{int}}(\varphi)] \rightarrow 0 \quad \text{for objects with sufficiently deep GM/R.}
  \]

- **Galileon theories**: derivative interactions e.g. \( \mathcal{L}_{\text{int}}(\varphi) \sim (\partial \varphi)^2 \Box \varphi \) (DGP).

  The symmetry \( \varphi \rightarrow \varphi + c + b \cdot x \) preserves \( Q/M = 1 \), because scalar equation takes a Gauss law form: \( \partial \cdot J \sim T_m \).

  A more careful derivation using Einstein, Infeld, Hofmann (LH, Nicolis, Stubbs).

\[
S = S_{GR} + \int d^4x \left[ -\frac{1}{2} (\partial \varphi)^2 + \mathcal{L}_{\text{int}}(\varphi) \right] + \int d^4x \left[ h_{\mu\nu} T_m^{\mu\nu} + \varphi T_m \right]
\]
Chameleon screening.

- The scalar force has a range that depends on environment: in an object with a deep GM/R (e.g. the Sun!), the range becomes very small.

\[ GM/R > \varphi_{\text{ext}} \quad \text{screened object } Q/M = 0 \quad \text{unscreened object } Q/M = 1 \]

Observationally, we know objects with \( GM/R > 10^{-6} \) are screened e.g. Sun, Milky way. (Otherwise, we would have known about the scalar force already!)

Screened and unscreened object fall at rates that are \( \mathcal{O}(1) \) different. Examples:

- Diffuse gas (e.g. HI) and stars could yield discrepant dynamical masses from rotation curves of dwarf galaxies. (Beware: asymmetric drift.)
- Small galaxies could stream out of voids faster than large galaxies.
- Red giants in dwarf galaxies can have a screened core but unscreened envelope, i.e. effective G variation within the star, giving \( \Delta T \sim 100K \) at tip of RG branch.
- Lensing mass = dynamical mass for screened galaxy, but not for unscreened galaxy.

\[
S = S_{\text{GR}} + \int d^4x \left[ -\frac{1}{2} (\partial \varphi)^2 + \mathcal{L}_{\text{int}}(\varphi) \right] + \int d^4x \left[ h_{\mu\nu} T_{\mu\nu} + \varphi T_m \right]
\]
A generic test of scalar-tensor theories.

- Idea: assuming black holes have no scalar hair/charge, while normal stars do, let’s check for the difference in their rate of free fall (Nordvedt).
  (More generally, compact objects should have Q/M < 1, because M is dominated by gravitational binding energy rather than $T^\mu_\nu$.)

- This effect would be hopeless to detect, for classic Brans-Dicke theory, because solar system tests already tell us the Brans-Dicke scalar must be very weakly coupled i.e. the scalar force is much weaker than gravity.

- Recent versions of scalar-tensor theories offer a hope: they pass solar system tests, yet have interesting $O(1)$ effects elsewhere (e.g. in cosmology).

$$S = S_{GR} + \int d^4x \left[ -\frac{1}{2} (\partial \phi)^2 + L_{\text{int}}(\phi) \right] + \int d^4x \left[ h_{\mu\nu} T^\mu_\nu + \phi T_m \right]$$
Vainshtein screening - galileon theories, massive gravity.

Galileon: Nicolis, Rattazzi & Trincherini;
Massive gravity: de Rahm, Tolley & Gabadadze;
$L_{\text{int}}(\varphi)$ also gives self-acceleration.

- Large scale structure produces an unscreened scalar that can penetrate galaxies (LH, Nicolis).

(The galileon symmetry means: given any nonlinear solution, adding a linear gradient gives another solution.)
Vainshtein screening - galileon theories, massive gravity.

Galileon: Nicolis, Rattazzi & Trincherini;
Massive gravity: de Rahm, Tolley & Gabadadze;
$L_{\text{int}}(\phi)$ also gives self-acceleration.

- Large scale structure produces an unscreened scalar that can penetrate galaxies (LH, Nicolis).

(The galileon symmetry means: given any nonlinear solution, adding a linear gradient gives another solution.)
The idea is to look for the offset of massive black holes from the centers of galaxies.
The offset should be correlated with the direction of the streaming motion. The massive black holes can take the form of quasars or low luminosity galactic nuclei i.e. Seyferts.
The offset is estimated to be up to 0.1 kpc, for small galaxies.
Beware: other astrophysical effects (e.g. case of M87), compact star cluster.
Resonance detector for gravitational waves

Weber's Bar
Hulse-Taylor binary pulsar 1913+16

- Periastron separation: $1.1 \, R_\odot$
- Orbital eccentricity: 0.6
- Orbital period $P$: 0.3 day
- Pulsar spin period: 59 milliseconds
Figure 2. Orbital decay caused by the loss of energy by gravitational radiation. The parabola depicts the expected shift of periastron time relative to an unchanging orbit, according to general relativity. Data points represent our measurements, with error bars mostly too small to see.
Idea: focus instead on the scattering of a gravitational wave (GW) background by the binary. Order of magnitude estimates:

- Gravitational wave background causes the binary period to random walk. \( \Delta P/P \) per period \( \sim 10 h \)
- Strain \( h \) most effective at harmonics of the orbital period, \( f \sim 4 \times 10^{-5} \) Hz
- Over duration \( T_{\text{tot}} \), accumulated rms \( \Delta P/P \sim 10 h \sqrt{T_{\text{tot}}/P} \)
- Accumulated rms periastron time shift \( \Delta T \sim 10 h T_{\text{tot}} \sqrt{T_{\text{tot}}/P} \)

Useful numbers: \( T_{\text{tot}}/P \sim 4 \times 10^4 \), \( T_{\text{tot}} \sim 10^4 \) days

- Periastron time measurement accuracy \( \sim 10^{-7} \) day
- Constrain strain by \( \Delta T < 10^{-7} \) day \( \implies h < 5 \times 10^{-15} \)

Note: \( h \) here represents the square root of the tensor power spectrum per log freq i.e. \( h \) is the rms strain.

Early work: Mashhoon; Carr, Hu & Mashhoon
Some technical details:

- This is a stochastic process: compute two point correlation of orbital change, and relate it to the two point correlation of GW background.

\[ \langle h(t)h(t') \rangle = \int \frac{df}{f} h_{\text{rms}}^2 e^{2\pi if(t-t')} \]

- Optimal data weighting.

Remarks:

- How much better can we do?
- What other systems?
- From upper limits to detection: issues.
Summary:

1. Scalar-tensor theories, despite their universal microscopic coupling to matter, contain seeds of $O(1)$ equivalence principle violations, due to macroscopic renormalization of the scalar charge. Manifestations: stellar versus gaseous rotation curves in chameleon theories, off-centered black holes in Vainshtein/galileon theories, etc.

2. Binary random walk can be used to constrain/detect gravitational waves.