The Inverse Square Law of Light

Why the world gets dark so fast outside the circle of the campfire, and how astronomers measure distances to far away objects

Goals:

- To demonstrate that the brightness of a source of light is a function of the inverse square of its distance.
- To understand how the brightness of light could be used to measure distances, even to stars and far away galaxies.

Overview:

We all know that a light, such as a candle or a streetlight, looks dimmer the farther away from it we get. The question of how much dimmer it looks was answered a long time ago. This activity gives an easy way to repeat that discovery. Students use a simple light source and measure its brightness on a piece of graph paper at different distances. Then, they graph the data and discover the mathematical relationship between brightness and distance.

Once students discover the relationship, they can begin to understand how astronomers use this knowledge to determine the distances to stars and far away galaxies.
What You Need:

- **A Mini-Maglite™ flashlight.** No substitutes! A point source of light is required for this activity. (Or make your own economical light source with a square of heavy cardboard, a Mini-Maglite™ replacement bulb, two batteries - either AAA, AA, C, or D - and clip leads to connect them (see assembly for details).
- **Cardboard or foamcore**
- **Large file card, heavy construction paper, or poster board**
- **An exacto knife or scissors**
- **Transparent tape**
- **Ruler with centimeter markings**
- **Graph paper**
- **Calculator (or Brain)**
- **Pencil**

Getting Ready:

(15 minutes or less if you use MiniMaglite™, 30 minutes or less if you make the light source)

Create 4 sides of a shade box with cardboard or foamcore (see the picture on the first page). On the one vertical wall of the box, tape a piece of graph paper. Along the bottom of the box, use a ruler to draw lines of distance from the graph paper (in centimeters).

Now, cut a 1 x 1 cm square hole in the file card/poster board. Hold or mount the card 10 cm in front of the light source. The square of light made when the light shines through this hole will shine on the graph paper.

Unscrew the front reflector assembly of the Mini-Maglite™ to expose the bulb. The bulb on the MiniMaglite™ will come on and stay on when the reflector assembly is removed.

If you're making your own light source, you need a replacement bulb for the Mini-Maglite™, two batteries (either AAA, AA, C, or D), and clip leads to connect them. Using the clip leads, wire the bulb in series with the batteries (see the diagram). Cut a small hole in the cardboard. Push the bulb through the hole so that it fights tightly and gives you something to hold on to.
Procedure:

(15 minutes or more)

1) Keep the distance between the bulb and the card with the 1 cm square hole constant at 10 cm. Put the bulb at different distances from the graph paper, and count how many squares on the graph paper are lit at each distance. Be sure to measure the distance from the bulb, not the card. Record the number of squares illuminated in the data table.

2) Measure the size of the squares in the graph paper to determine the area of each square. If you use the graph paper provided with this activity they should be 1/2 cm on a side, and thus each has an area of 1/4 cm². Calculate the area illuminated at each distance measured, and record it in your data table.

3) What we are interested in knowing is how distance affects the amount of light that falls on each square. The amount of light received per area is called brightness. The amount of light given off by the bulb and passing through the hole in the card always remains constant. So, what we want to calculate is the brightness relative to some standard brightness (say the brightness of the bulb on the graph paper at 10 cm). Let’s look at the relationships mathematically. We call brightness B, Area A, and the amount of light (also called power or luminosity) L, and we can write the following:

\[ B = \frac{L}{A} \text{ for any distance and } B_0 = \frac{L}{A_0} \text{ for your standard distance (10 cm)} \]

So relative brightness is \( \frac{B}{B_0} = \frac{A_0}{A} \)
(L cancels out because it is the same for both)

But, at a distance of 10 cm the area illuminated was 1 cm²
So, \( A_0 = 1 \) and we have \( \frac{B}{B_0} = 1/A \)

Calculate the relative brightness for each distance, and record it in your data table.

4) Make a graph of the relative brightness as a function of distance. In examining your graph, can you determine how brightness depends on distance? Is it directly proportional, inversely proportional, proportional to the inverse square, etc.?
**Discussion Notes:**

The light from the Mini-Maglite™ spreads out equally in all directions. As the distance from the bulb to the graph paper increases, the same amount of light spreads over a larger and larger area, and the light reaching each square becomes correspondingly less bright. For example, adjust the distance from the bulb to the graph paper to 10 cm. At this distance, the graph paper touches the card. A 1 cm² area will be illuminated. When the graph paper is moved 20 cm from the card, 4 cm² will be illuminated on the graph paper. When the graph paper is moved 30 cm from the card, 9 cm² will be illuminated, and so on. The area illuminated will increase as the square of the distance.

The brightness of light is the power (energy per second) per area. Since the energy that comes through the hole you cut is constant but spreads out over a larger area, the brightness (or intensity) of light decreases. Since the area increases as the square of the distance, the brightness of the light must decrease as the inverse square of the distance. Thus, brightness follows the inverse-square law.

If you had two light bulbs and knew that they both give off the same amount of light (same luminosity/power), then you could calculate the relative distance between the two of them simply by measuring their relative brightness. If you also knew what the luminosity/power of the bulbs was, you would then be able to determine the distance to both bulbs. Or, if you knew the distance to one of the bulbs you could determine the distance to the other one.

This is how astronomers use the inverse square law of light to measure distances to stars or galaxies. They find stars that are the same kind (same size and temperature) and, therefore, have the same luminosity. They measure the brightness of the stars and can determined distances if they know either what the luminosity of the stars is or the actual distance to one of the stars by some other method.

**Going Further:**

The inverse-square law applies not only to the brightness of light but also to gravitational and electrical forces. The pull of Earth's gravity drops off at $1/d^2$, where $d$ is the distance from the center of Earth. The attraction or repulsion between two electric charges decreases with the distance at $1/d^2$, where $d$ is the distance between the two charges.

Note: This activity is based on a “Science Snack” activity from the Exploratorium (http://www.exploratorium.edu/snacks/inverse_square_law.html)
Data Table:

<table>
<thead>
<tr>
<th>Distance From Bulb (cm)</th>
<th># of Squares Illuminated</th>
<th>Area Illuminated (cm^2)</th>
<th>Relative Brightness (cm^-2)</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>
Example Data:

<table>
<thead>
<tr>
<th>Distance From Bulb (cm)</th>
<th># of Squares Illuminated</th>
<th>Area Illuminated (cm²)</th>
<th>Relative Brightness (cm⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>14</td>
<td>8.4</td>
<td>2.10</td>
<td>0.48</td>
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<tr>
<td>15</td>
<td>9.3</td>
<td>2.33</td>
<td>0.43</td>
</tr>
<tr>
<td>18</td>
<td>13.3</td>
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</tr>
<tr>
<td>20</td>
<td>16.4</td>
<td>4.10</td>
<td>0.24</td>
</tr>
<tr>
<td>24</td>
<td>23.5</td>
<td>5.88</td>
<td>0.17</td>
</tr>
<tr>
<td>25</td>
<td>26</td>
<td>6.50</td>
<td>0.15</td>
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<tr>
<td>28</td>
<td>34.8</td>
<td>8.70</td>
<td>0.11</td>
</tr>
<tr>
<td>30</td>
<td>36.6</td>
<td>9.15</td>
<td>0.11</td>
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</tbody>
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Note: The graph on the next page plots these data as points and also plots a line representing how the relative brightness should theoretically depend on distance: \( \frac{B}{B_0} = \frac{100}{d^2} \)

This is derived by assuming that the area illuminated is proportional to the square of the distance and solving for the constant of proportionality...

\[ A = kd^2 \]

For \( d = 10 \text{ cm} \), \( A = 1 \text{ cm}^2 \)

Thus, \( k = \frac{1}{100} \)