MEMORANDUM

To: Gang of Seven
From: Colin CRM
Subject: Design Study Memo #1 - Phase Calibration

Here is a memo discussing the difficulty of high frequency calibration. I intend that it should be part of a series of memos in which we document the progress of our design study.
Phase Calibrators for the SMA

At the high frequencies where the SMA will operate, calibrators for phase will be hard to find. In this note I consider what calibration sources are available, and whether any technical tricks could be useful for improving the calibration accuracy.

I. Array Sensitivity

The sensitivity of the SMA is obviously the most important factor in determining how many calibrators can be observed in a reasonable integration time—say 5 minutes. The instrumental stability is the important factor in determining how often calibration is required. Using the analysis on p. 85 of the Yellow Book, and assuming 6m antennas, 1500 K effective system temperature, 2 polarizations and a 5 minute integration, we have:

1 baseline r.m.s. = 0.26 Jy
Sum over 6 antennas = 0.15 Jy
Sum over 9 antennas = 0.09 Jy

For < 10% decorrelation, the r.m.s. phase error must be <0.3 radians, so the absolute minimum signal/noise ratio required is 3. A more reasonable value is 10, indicating that, for the above parameters, we need a minimum flux density of 1.5 Jy, for a 6 antenna array.

Under the best conditions, the system temperatures should be below 1000K at frequencies up to 345 GHz, but moderately bad weather and large zenith angles can easily put the system temperature up to 2000 K, even with a receiver whose
noise temperature is 400K (SSB) which is typical of the best current SIS designs at 345 GHz (e.g. Ed Sutton’s or Brian Ellison’s receivers both have best noise temperatures of ~300 K (SSB)). At higher frequencies, the receiver temperatures will be at least proportional to frequency, and the atmosphere will also be worse, so we must consider the possibility of system temperatures exceeding 10,000 K. High frequency receivers will also be harder to construct, so we may not have two polarizations available at all times. For 10,000K and 1 polarization, the required calibrator flux density is 10 Jy.

II. Source Counts

Since the writing of the Yellow Book, a survey of point sources has been carried out at 230 GHz and 90 GHz by Steppe et al (Astronomy and Astrophysics Supplement, 75, 317, 1988). Taking crude averages over the several measurements of each source, I find the following counts at 230 GHz:

\[ \begin{align*}
N(S>2 \text{ Jy}) &= 17 \\
N(2>S>1 \text{ Jy}) &= 26 \\
N(0.5>S> \text{ Jy}) &= 38
\end{align*} \]

giving a slope \( N(>S) \propto S^{-1.1} \)

The median spectral index is approximately - 0.3 between 90 and 230 GHz, so the source count as a function of \( S \) and \( \lambda \) is roughly

\[ N(S, \lambda) \approx 40 \ S_j \lambda^{-1.1} \lambda^{0.3}_{\text{mm}} \]
It is probably worthwhile applying for time to make a small survey of these sources at frequencies greater than 230 GHz, to test this extrapolation of the source counts.

The Steppe et al survey was not exhaustive, but it was quite comprehensive so it probably gives a fair estimate of the situation. Some sources have infrared excesses due to dust, but the brightness temperature is low, because the physical temperature is \( \sim 50 \text{K} \) and I show below that this is inadequate for calibration.

We can conclude that there are sufficient point source calibrators (\( \sim 25 \)) for frequencies up to 345 GHz. The typical separation of calibrators is about 0.6 radian so that only part of any geometric error will be removed by the calibration. By 492 GHz, with a 3500K system temperature the number of calibrators will be down to \( \sim 9 \) and at the highest frequencies, there may be almost no such calibrators of sufficient strength.

III. Alternative Calibrators - Thermal Sources

There is another class of sources with good potential as calibrators, i.e. those objects with dusty envelopes heated by a single central star. Such envelopes are associated both with young stars like L1551-IRS5 and old stars like IRC+10216.

Although they are not pointlike, such sources often have a strongly peaked distribution of emission and a flux density which increases with frequency. For example, IRC+10216 has a flux density of 4Jy at 1 mm wavelength and 73 Jy at
350μm. The envelope has a total extent of ≥ 1' and the temperature corresponding to the spectral maximum is ~ 500K.

The criterion for deciding whether such sources are viable as calibrators is the brightness temperature required for sufficient signal strength. This is a function of baseline length. The worst case is the longest baseline, which may be as much as 1km. If we assume that the largest allowable source size is half of the fringe spacing, then it is possible to calculate the flux density as a function of brightness temperature. At a baseline of 1km, the fringe spacing is 0''21λ mm, and the optimal source diameter for a calibrator is 0''15 λ mm, corresponding to ~ 2000 K/Jy. Even for a 1Jy calibrator, the necessary brightness temperature is very high. Stellar photospheres are the only thermal objects with sufficient brightness, and most of those are too small to provide a large enough flux density. In other words, it is impractical to use thermal sources to calibrate the longest baselines.

The brightness temperature criterion can be relaxed slightly if the short baselines in the array are used for the phase calibration, but this provides only a modest gain for small arrays. For circular crystalline arrays, the ratio of maximum/minimum baseline is ~ n/2, where n is the number of antennae. The largest spacing required to connect all the antennae is roughly twice as large as the shortest baseline. For a 6 element array, the longest baseline necessary for calibration is ~2/3 of the maximum baseline, and the brightness temperature requirement is reduced to ~ 1000 K/Jy. In a 9 element array, the gain is slightly larger, to ~ 500 K/Jy, but these temperatures are still very high. Similar
calculations will apply to Y arrays, with the longest necessary baseline being $\sim 1/2$ of the maximum for a 6 element array and $\sim 1/3$ for a 9 element array, reducing the requirements to $\sim 500 \text{ K/Jy}$ and $200\text{K/Jy}$ respectively. However, because of the longer integration time, self calibration will be possible if strong thermal sources are in the field of the program source.

At shorter baselines thermal sources would be much more practical. If the maximum baseline were 100 m, for example, the requirement would be only 80K/Jy, although there will still be few sources which could provide a 10 Jy signal.

IV. Other Options

From the preceding discussion it appears that it will be difficult to find calibrators for the highest frequency bands, and maybe impossible at the longest baselines, so we should consider other possibilities like local transmitters and simultaneous calibration at low frequencies.

a) Local transmitter.

In this scheme, a transmitter would be located somewhere near the center of the array and set to radiate a frequency within the observing band. To calibrate, all the antennas would point at the transmitter. This scheme has the advantage that the calibration signal would be very strong. A power of $1\mu\text{W}$ radiated isotropically over a distance of 1km would be equivalent to a source of $10^4$ Jy in a 1 GHz bandwidth. One disadvantage of the scheme would be the lack of correction for geometric errors, since the calibration transmitter would not be close to the program
source. Also, in general there would be systematic phase offsets due to effects such as the difference in wavefront curvature from the transmitter at each dish, but these could be measured and calibrated out. The greatest difficulty in principle is that this scheme provides no correction for atmospheric effects and might even introduce extra atmospheric errors due to the path between the transmitter and the antennas. In fairness, any astronomical calibrator may be so far from a typical program source that atmospheric effects would not be well calibrated in any case.

Because of the usefulness of a strong test signal for general system tests, it may be worthwhile providing such a transmitter in any case. For example, it would make holography much faster than if we were restricted to astronomical sources.

b) Low frequency calibration

Since calibrators are stronger at low frequencies, and receivers are better, we may profit by using low frequency measurements to calibrate the high frequency phase. The phase error required for the measurement is smaller, since any phase is multiplied in proportion to frequency and the required signal/noise ratio would be inversely proportional to frequency. If calibrators had flux density independent of frequency, and the antenna efficiency were also independent of frequency, then the choice of frequency would be arbitrary if system temperature were directly proportional to frequency. Because effective system temperature will probably increase faster than linearly with frequency, better calibration will usually be achieved by relying on low frequency measurements. Compared with the number
given earlier, the necessary flux density will be higher since a signal/noise ratio of 10 \( r \) is needed, where \( r \) is the ratio of frequencies.

There are two complications in principle. The first is that not all parts of the system are common to the two frequencies. It would therefore be necessary to make a measurement of some object at both frequencies to calibrate the phase offset. The l.o. system should of course be designed to make this difference as stable as possible. The second difficulty arises from dispersion in the atmosphere, which can change the refractive index by a few percent from one frequency to another. This is a serious impediment to calibration \textit{ab initio} from a lower frequency, but should not be too bad if the initial phase offset is determined by some direct calibration at the high frequency, and the lower frequency is used to calibrate changes in phase.

Two possibilities exist for making dual frequency measurements. In the simplest mode, the array would switch between frequencies as it moved between the calibrator and the program source. This requires no hardware beyond the minimum. In the second, the array would operate simultaneously at both frequencies. This would require an enlarged correlator, but would double the throughput of the array since objects could be observed simultaneously at two frequencies. An important advantage is that, if a continuum source is available for self-calibration at one frequency, then it should be possible to transfer that calibration to the other frequency. This may also be an argument for having an analog continuum correlator because it would be cheap to build more than one.

V. Summary
A sufficient number of point source calibrators exists for operation of the array up to a frequency of $\sim 400$ GHz. Beyond that point, the low flux densities of the calibrators and poor receiver sensitivities may preclude the use of such objects. For short baselines at high frequencies, $\leq 100m$, thermal emission from circumstellar dust sources may provide enough signal for calibration, but for longer baselines there may be no suitable calibration sources.

Two options for high frequency calibration are 1) the use of a local transmitter and 2) extrapolation from low frequency measurements of calibration sources. Both will have systematic errors which may be measurable. The test signal from (a) would be useful for checking many aspects of the system. If (b) is implemented by permitting simultaneous observations at two frequencies, then the correlator size would have to be increased. We should certainly consider this option in designing the array.