Antenna Surface Measurement for the SMA

Summary

I discuss the available techniques for measuring the surface of the SMA antennas (tape and theodolite, template, photogrammetry, low-frequency holography (10-40 GHz) with a satellite, phaseless holography, and holography with an astronomical source). For each one I give a crude analysis of the cost and accuracy. We will almost certainly use a combination of these techniques, with the final verification of the surfaces being done by astronomical holography, which is the only method which does not have systematic errors.

1 Introduction

Many different methods have been used or proposed for the measurement of radio antennas. Some of these are physical, in the sense that they measure the dimensions of the structure, while others are radiometric, using the properties of radio signals received by the antennas. Most of the methods require some auxiliary equipment and all take a finite time for measurement. The high accuracy (< 5 μm r.m.s.) required by the SMA antennas is difficult to achieve for all of the methods. An ideal method for the SMA would have high resolution (better than 30 cm, or 1 point per adjuster), high accuracy (1 μm), low cost, high speed (< 1 hr), and be usable at a range of elevations. No one method has all of these characteristics, but with a suitable combination, we should be able to make an adequate measurement.

In the discussion I will consider the following methods:

1) Surveying This is the traditional method for setting antennas, with tape and theodolites. It is slow and has low accuracy, but is reliable and a good way of getting an initial setting.
2) Template This is a method in which a parabolic template is inserted into the dish and the dish is measured with respect to the template. It can be tricky and the radial accuracy is limited by the quality of the template.
3) Astronomical holography This is the standard technique in which the completed interferometer is used to observe a bright object, usually a planet. One antenna is scanned to measure the primary beam, and the results are Fourier transformed to give the aperture pattern. This is the only final test of the radiomen-k quality of the dish, but accurate measurements are slow.
4) Low-frequency holography This uses signals from an artificial satellite in an interferometric setup to measure the amplitude and phase pattern of the primary beam. The accuracy is good, but
there are very large **diffraction** effects **from** the subreflector if the feed is not placed at prime focus.

5) **Phaseless holography** This uses the amplitude pattern of the primary beam, in conjunction with an algorithm which estimates the phase. The very high signal/noise required necessitates the use of a local transmitter.

The following methods will not be considered:

6) **Phorogrammetry** This involves taking very accurate photographs from a number of locations and solving for the 3D locations of markers on the antenna. The method is capable of high accuracy, although the processing of the photographs is slow.

7) **Shearing Interferometry** In this method optics are used to form an image of the aperture, the beam is split in two and the two parts are **refocussed** on the reflector, while one of them is scanned to make a complex measurement of the focal plane pattern. This has similar accuracy to (3), but can be done with individual antennas, at the expense of a fairly cumbersome piece of optical & mechanical equipment.

8) **Gated Aperture** In this method, an aperture with a rotating chopper is moved across the face of the antenna, while observing a beacon. Synchronous demodulation of the signal yields the amplitude and phase at the position of the aperture. The method requires an accurate framework for translating the aperture, and has few advantages over (5) if a suitable beacon is available.

9) **Laser Ranging** Laser methods have not been successful in the past for measuring dishes, although these have usually been based on angle measurements. Range measurements are likely to be better, but accuracy adequate for the SMA has not yet been demonstrated. The antenna under measurement would have to be outfitted with retro-reflectors.

### II Surveying

In the surveying method, a combination of theodolites and measuring tapes are used to measure to a set of points on the antenna. The measurement is slow and must be done at night to avoid temperature changes. At SEST the dish could be measured in one night and the surface accuracy achieved after a series of iterations was about 50 μm. The method does not use any auxiliary equipment, apart from the standard theodolites and tapes, and can be done anywhere, with an individual dish. For this reason, surveying could be used for initial setting, although it would have to be followed by a more accurate method. The accuracy is probably limited by temperature changes during the measurement period and by atmospheric refraction. The cost is probably low, since we will have theodolites and tapes for surveying the pads.

### III Templates

Several antennas have been measured with templates, including CSO and the 12 m. In this method, a parabolic template is set up along a radius of the antenna and the separation between the template and the dish is measured by some form of transducer. By rotating the template or the antenna, the whole antenna surface can be measured. The transducer errors can be very small (1 μm), but there are significant systematic errors, since the measurement result is only the difference between the template and the dish. Because the dish can be rotated accurately, the azimuthal structure can be measured very accurately, but the radial structure depends entirely on the
accuracy and stability of the template. The method requires construction of an accurate template and a system for measuring the separation between the template and the dish.

An accurate template, of the sort used for construction of the Leighton dishes, costs $\sim$250 k, and requires careful setting and use. It could, however, be accurate to $< 10 \mu m$, in a laboratory setting. A simpler template would be cheaper, say $50k$, but would be less accurate, say $-25 \mu m$. A template of either sort would be very good for looking at small-scale errors on the dish surface, and would be least accurate on large scales. Even a precision template would not remove the need for holography as an ultimate test, so there is little to gain by the extra effort and expense needed for its construction. The most appropriate use for a template would be for initial rough setting of the surface.

IV Astronomical Holography

This method uses two elements of the completed interferometer in conjunction with an astronomical source. While one antenna points directly at the source, the other is scanned across it, to measure the complex beam pattern, sampling slightly more often than 2 points per beamwidth. The beam pattern is then Fourier transformed to produce the aperture distribution. The surface error in the aperture map depends on the number of pixels, and the signal/noise ratio of the observed source, according to the formula (e.g. Morris 1985):

$$\delta S = (\lambda / 2 \pi) (N/R),$$

where $\lambda$ is the observing wavelength, N is the number of resolution elements across the dish diameter, and R is the signal/noise ratio when both antennas are pointing directly at the source. A factor of 2 has been allowed for the degradation in sensitivity at the edges of the antenna, where the illumination intensity is lower. With 4 rows of panels and 4 adjusters per panel, the minimum necessary value of N is about 20. At 230 GHz ($\lambda = 1.3$ mm), a conservative value for the system temperature is 500 K. With a bandwidth of 2 GHz, and a measurement time of 9 sec per point (total time $\sim 1$ hour), the noise level is about 0.36 Jy. The strongest extragalactic source is -10 Jy at 225 GHz, not nearly enough for an accurate measurement at full resolution. For a fixed total measurement time, R is inversely proportional to N, so the surface error, $\delta e$, is proportional to $N^2$. The result is

$$\delta e = 270 (N/20)^2 (T_s / 500 K) (10 \text{ Jy} / \text{Flux}) (T / 1 \text{ hr})^{-0.5} \mu m.$$

With a 10 Jy source, only a very low resolution map ($N = 4$) could be made with fair accuracy ($11 \mu m$).

In practice, therefore, planets are used for holography. If the planet is not small compared with the beam of the antenna, then the sensitivity at the edge of the dish is reduced, but the surface measurement is otherwise unaffected. It is important that planets be measured at the smallest practicable spacing, since the resolved flux falls rapidly with baseline. The total flux density, $S$, of the planet is given by the formula $S = 2 k T \Omega / \lambda^2$, where k is Boltzmann’s constant, T is the temperature of the planet, and $\Omega$ is the solid angle subtended by the planet. As a rough example, a
planet might have a diameter of 15” and a temperature of 100K, giving a flux density of 680 Jy at 230 GHz. The first zero of the planet’s visibility would fall at a spacing of 22 m. With a spacing of 10 m, we should get a resolved flux density of about 400 Jy, allowing a much more detailed map (N-20) at an accuracy of 7 μm. Smaller planets avoid the resolution problem, but the flux density drops as (diameter)², so they are not much better than extragalactic point sources. Larger planets have more flux density, but suffer heavily from resolution. The best compromise is a planet which is slightly resolved (-0.5 x peak flux) at the smallest baseline. Of course, there are very few planets in the sky and they are not all of the correct size and in the right place when we will want them. The consequence is that observatories plan their holography campaigns around the seasonal availability of the appropriate planet(s). Despite these difficulties, the planets are essential for the final mm-wave holography of the antennas and under the right conditions they are just about strong enough to do the job.

Masers could possibly be used as point sources for holography since they are very bright. However, submm masers are not yet well understood and it is likely that the brightness will be offset by their narrow linewidth. At -3 mm, for example, the strongest SiO masers give comparable signal/noise to the strongest extragalactic point sources, and planets are the sources of choice for holography.

There are probably limitations on accuracy imposed by the atmosphere. A rough estimate is that errors of a given percentage in amplitude or a given fraction of a radian on each measured point will translate into errors of a similar magnitude on the surface. The surface errors should be slightly lower in magnitude than the measurement errors, since the surface determination depends primarily on the far-out parts of the beam, where the sidelobe level is low, compared with the peak of the beam. From the atmospheric measurements, I estimate that the typical r.m.s. path at 10 m baseline is -35 μm, so the limit on any holographic map should be a few times lower, say -5 μm. Pointing probably has a similar effect, even after the mean pointing error has been removed from the aperture map. These errors are no more than comparable with the noise level, and they can be reduced by averaging several maps. Further work is needed to quantify the errors more accurately. The best strategy, therefore, is to make several maps as quickly as possible and average them to build up the total integration time.

There is no extra cost for astronomical holography, since it uses parts of the interferometer itself. By the same token, however, it cannot be employed until the interferometer, or at least a baseline, is complete. It will be the final test of surface accuracy, since it is the only system which does not introduce its own systematic errors.

V Low frequency holography

In view of the difficulties in doing holography with weak cosmic radio sources, many observatories have used signals from artificial satellites, principally communications satellites at 12 GHz or the Lincoln Labs satellite at 34 GHz. There will soon be a NASA satellite with a beacon at 27 GHz, but I do not know if this will be visible from Hawaii. The obvious way for us to make use of satellites is to reuse the system built for site testing, which looks at a satellite at 11.7 12 GHz. The sensitivity can be calculated fairly easily by extrapolating from the experience of the site-testing
system. With 2 x 1.8 m dishes pointing at the satellite in the site-testing system, the r.m.s. path length is about 5 \( \mu \text{m} \) in 1 second of integration. If one antenna is 6 m diameter then the central r.m.s. is 1.5 \( \mu \text{m} \) in path, or 0.75 \( \mu \text{m} \) in surface error. For an N x N map with a total measurement time of 1 hour, the achievable accuracy is

\[
\delta e = 5 \left( \frac{N}{20} \right)^2 \left( \frac{T}{1 \text{ hr}} \right)^{-0.5} \mu \text{m}.
\]

This is adequate for a rapid measurement system, which can operate in a stand-alone mode, using one of the 1.8 m dishes as a reference.

The largest problem is that the dish cannot be fed in Cassegrain mode because of diffraction at the secondary. At the operating wavelengths (< 1 mm), the diffraction angle from the secondary is less than 1/300 radians and the first ~ 15 diffraction rings are lost in the central blockage of the antenna, for a secondary to primary distance of 3 m. At 11.712 GHz, the diffraction spot of the secondary is as large as the central blockage and the perturbations are significant. There is a second set of diffraction rings at the edge of the antenna, whose spatial scale is comparable with a panel size. To avoid these effects, the dish must be fed from the prime focus, necessitating the removal of the secondary during the measurement.

The Lincoln labs satellite would have similar diffraction problems, albeit at a lower level due to its higher frequency, but it has its own difficulties, since it moves in the sky by a significant amount, both during a day, and from time to time as the orbit is changed. Another possibility is a soon-to-be-launched NASA satellite, ACTS, but I do not yet know much about it. In all cases, however, diffraction at the secondary would be a problem and the best solution would be removal of the secondary, to put the feed at the prime focus.

Atmospheric and pointing effects will be similar to those in astronomical holography, perhaps 5 \( \mu \text{m} \), but these should be removed again by averaging maps. There may be systematic errors due to the characteristics of the low-frequency feed, but it is hard to know how large these would be.

A second possibility is to use satellite holography simply as a rough setting tool. In this case, we could tolerate some errors from the diffraction effects, provided that calculation would allow us to offset them to some extent. This would allow us to simplify the feed arrangements and use the subreflector, or a separate plane mirror to couple the feed horn to the primary. According to the calculation which Rachael Padman (1990) made, the peak phase error is about 20\(^\circ\), or a surface error of 710 \( \mu \text{m} \), but the r.m.s. across most of the dish is closer to 250 \( \mu \text{m} \). With a calculated correction good to 20\%, we should be able to set the surface to 50 \( \mu \text{m} \).

### 6 Local transmitter (Phaseless Holography)

Phaseless holography has been employed by the JCMT with great success. The signal/noise requirement for phaseless holography is greater than for regular holography and it is only practical with a local transmitter. When such a transmitter is available, however, it has the advantage of using the standard 230 GHz receivers and the standard optical arrangement. If we consider a simple transmitter radiating 0.1 mW isotropically at a distance of 1 km, 6 m dishes with 50 \( \% \)
aperture efficiency, and the 500 K receivers, with a 1 kHz bandwidth, the central signal/noise is $S/N_0 = 2 \times 10^7$, which is more than adequate.

There are many practical complications. Because the dish cannot be focused accurately on the nearby transmitter, there is a minimum number of points which must be measured across the beam, of order $D^2 / (\lambda U)$, where $D$ is the dish diameter, $\lambda$ is the wavelength, and $U$ is the distance to the source. JCMT uses 100 x 100, but we might get away with 40, even using a shorter wavelength of 1.3 mm. A good detector must be used in order to avoid compromising the high signal/noise by effects such as non-linearity.

Two maps are required so the time per point is 1800 N-2 seconds. According to the calculation by Dave Morris (1985), with these parameters we should get

$$\delta \epsilon = 0.7 \frac{(T / 1 \ hr)}{(N / 40)^{1.5}}.$$

Richard Hills (1991) says that the s/n requirement is less than that calculated by Dave Morris, so this formula may be pessimistic. However, the $< 1 \mu m$ accuracy is not achieved in practice, because the measurement is limited by atmospheric effects. The JCMT experience is that the repeatability of the measurements is 10 $\mu m$. With our smaller antennas and an operating frequency of 230 GHz, we might be able to achieve 5 $\mu m$. Under these conditions, the best strategy again is to make many maps as quickly as possible and average them. At JCMT, they take 5 points per second and integrate only 10 milliseconds per point. If we can achieve the same rate, we could make a pair of 40 x 40 maps in 15 minutes. Assuming that each one is limited to 10 $\mu m$ by atmospheric effects, we might achieve 5 $\mu m$ in 1 hour, so my estimate is

$$\delta \epsilon = 5 \frac{(T / 1 \ hr)}{1 hr}^{0.5} \mu m.$$

If we can scan faster then we can achieve even better accuracy. This is obviously a very promising technique once the receivers are up and running. The transmitter is easy to arrange. If it is desired to set the antennas before the whole receiver system is set up, it may be desirable to buy a separate room temperature receiver system, which would have slightly lower sensitivity, but not enough to compromise the measurement. To use this system, we need a good linear narrowband detector, such as the vector voltmeter used at JCMT or our own correlator, and a clear sightline to a transmitter (may be difficult at Haystack).

A big advantage is that the optical train is used in the same way as during normal operation. Any doubts about near-field corrections, etc., can be dealt with by a final round of regular holography. Since the frequency of operation is high, the surface r.m.s. should be less than 1 radian (200 $\mu m$) before starting, to avoid ambiguities, although the JCMT experience is that even large errors ($h/2$) can be corrected. There are requirements for focus travel to permit the edge rays to focus on the transmitter and many aspects of the data taking and reduction must be planned carefully.

An extension of the technique is possible when the interferometer is running properly, since we can use one of the other antennas as a phase reference. This should improve the quality of the data, although there is a possibility that the extra atmospheric path to the second antenna will introduce
more errors. However, this should enable us to avoid some of the possible errors in the phase-recovery algorithms, but we would still be relying on calculations to deal with the near-field effects.

7 Discussion

The characteristics of the various methods are summarized in the table below. The cost numbers are little better than guesses and the accuracy of some methods, such as the template, depends heavily on the quality of their execution.

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There are a number of other characteristics of the methods which have not yet been discussed. All of the radio methods, with the exception of the satellite at prime focus, cannot distinguish between the subreflector surface and the primary. None of the others accounts for the subreflector. It may be useful to be able to rotate the subreflector easily to separate subreflector errors from primary errors.
In general the radio methods will require a fair amount of computational development, but the advantage is that repeated measurements will go quickly and smoothly once the system is developed. A template requires significant development, while surveying takes almost none. The mechanical methods, however, always require a lot of time and care to make the measurements.

The only method which we can use for the final determination is astronomical holography, since it has no systematic errors and it covers a range of elevation angles. However it is very slow and limited to certain times of year, so it is not a suitable tool for setting the surfaces. The best bet for initial accurate setting is to use a local transmitter at 230 GHz, either with or without phase. The measurement should be fast enough to be convenient, but considerable work is probably required on the phase-recovery algorithms, and we will have to be able to scan and acquire data quickly. A suitable sightline may be a problem at Haystack. The question of how much initial accuracy is required is still open, but life will presumably get harder if the surface is worse than \( \lambda / 2 \), or 0.6 mm. Thus a rough setting step is required to go from the starting surface of perhaps 6 mm down to 0.6 mm. The quickest method is to use a satellite, which should get to about 0.1 mm in less than 1 hour, even when using the secondary. Some work will be required, however, to build the data-taking system. Surveying requires less development and might be less labor for one dish, but probably not for 6, set up at 2 locations, when several iterations are need for each dish.

References

