Professional Development Workshops Guide

Mathematics: What's the Big Idea?

The Annenberg/CPB Math and Science Collection

Professional Development Workshops Guide

Mathematics: What's the Big Idea?

An Eight-Part Workshop Series for K-8 Teachers of Mathematics

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Series Overview

Most math teachers have read, talked about, or heard about the NCTM Standards and, in general, what they recommend. Many of you are changing the way you teach mathematics or considering how you might. Change is always intimidating, especially if you have worked hard on the way you currently teach. This workshop series will support your efforts to change in three ways:

(1) by helping you learn some mathematics yourself in new and exciting ways;
(2) by suggesting how you might *teach* mathematics in new and exciting ways; and
(3) by showing you some classrooms that are changing, and letting you watch and learn from the teachers who work in these classrooms.

Some of you probably didn't enjoy your mathematics education. For many people, math class involved memorization, timed tests, and quiet, solitary work. The picture of successful math classrooms today is different. Students may work in groups, find several different ways to solve a problem, create their own representations of mathematical situations, do math in conjunction with language arts or science, and share their strategies with the rest of the class. They work with geometric figures, collect and analyze data, make graphs, and write their own stories to give meaning to numerical relationships.

Using these workshop materials, we hope that you *will* enjoy doing mathematics, that you will learn about yourself as a learner and as a teacher, and that you will work with colleagues at your school discovering the "Big Ideas in Mathematics."

This workshop series is based on an approach to teaching and learning that emphasizes students doing mathematics themselves and constructing their own mathematical strategies, rather than following rote recipes that someone else has designed. Much research has shown that students who approach math in this way develop a deeper and more flexible understanding that supports their ongoing mathematical learning. Students learn to describe their strategies to their peers and to prove that their approach is sound.

This approach to learning math does not mean that students neglect basic mathematical relationships such as addition and multiplication. Rather, students begin by establishing a basic understanding of an operation, often through the use of manipulatives. They then expand their knowledge by looking at more and more complex number relationships, explore the connections among these relationships, and develop a fluency that enables them to work with more complex mathematics. If students develop this kind of conceptual understanding, they will learn and retain the basic facts, as well as develop the foundation for more complex mathematics.

What kinds of problems fit with this approach? A good problem leads students to consider important concepts in mathematics. It should have more than one approach, so that students can share a variety of strategies and so that students with different mathematical strengths are able to approach the problem. While it is important for a problem to be somewhat open-ended, it is also important that there be enough

structure to give students direction. A good problem is an opportunity for students to explore and use a variety of forms of representation (for a single mathematical model, equation, relationship, etc.) including verbal, pictorial, numerical, tabular, graphic, and symbolic. During these workshops, we will include as many such problems as possible to give you a chance to see how this works.* For example, as we hear from sites around the country, we will see diverse approaches to both mathematical and pedagogical problems.

We also explore assessment. By assessment, we mean discovering and evaluating what students know at a given time and using that information to plan future lessons. (The problem of actually assigning *grades* is separate from assessment in this discussion.) With problems that invite individual approaches and discussion, teachers can learn much more about their students than when strictly numerical tests provided most of the information. With such rich information, ranking students is not the point; mathematical understanding is much more complex than a single grade can convey. Assessment should take place within the context of actual lessons, as well as during explicit assessment problems. In this workshop series, we will consider opportunities for assessment within specific content areas.

Technology—calculators and computers—has an important place in the math classroom. The key to solving a numerical problem with a calculator is knowing what numbers to enter, what operation to perform, and how to interpret the answer. Students will work with calculators all their lives, no matter what career they choose—it's important that they don't just take the answer that appears on the calculator at face value without considering whether or not it is reasonable.

Our hope is that by participating in this series of math workshops you will gain a better appreciation for the meaning, value, and importance of mathematics in school and in society, and you will have a better idea of where mathematics came from and, perhaps more importantly for your students, where it is headed.

– The Content Guides

* To solve a good problem, you need the right tools. Workshop participants should be prepared to do activities that require some supplies. **For every workshop you should have available pencils, paper, scissors, rulers, calculators, tape, and a variety of colored markers.** Specific workshops may require a few additional supplies.

Workshop Synopses

Workshop 1 — Patterns and Functions: What Comes Next?

Guide: Andee Rubin

Mathematics is about patterns waiting to be found. Patterns surround us from birth — in sight, in sound, in words, in music. This program sets the stage for the whole series, demonstrating how students' explorations of patterns grow richer and more complex as they move through school.

Workshop 2 — Data: Posing Answers and Finding Questions

Guide: Timothy Erickson

From the earliest grades, students learn to connect situations, data, and graphs. We'll see data displays as they develop through the grades — and we may even solve a mystery or two.

Workshop 3 — Geometry: Castles and Shadows

Guide: Andee Rubin

Every day, when we look at shadows around us, we interpret flat representations of three-dimensional objects. When we wrap packages, we create three-dimensional objects from two-dimensional representations. We will see that the relationship betwen two and three-dimensional representations is at the very core of geometry.

Workshop 4 — More Geometry: Quilts and Palaces

Guide: Beryl W. Jackson

Geometry appears in beautiful works of art, architectural wonders, and physical structures. We will explore geometrical figures, transformations, and connections to art and science.

Workshop 5 — Whole Numbers: Memory and Discovery

Guide: Timothy Erickson

What does it take to develop fluency with whole number calculation? We will compare algorithms and explore mental math strategies.

Workshop 6 — Ratio and Proportion: When Is a Third More Than a Half?

Guide: Beryl W. Jackson

Our students often develop misconceptions about fractions which affect their mathematical development. We will work with rational numbers and activities dealing with ratio, proportions, and that big stumbling block, equivalent fractions.

Workshop 7 — Algebra: It Begins in Kindergarten.

Guide: Monica Neagoy

The teaching of algebra is moving away from the static manipulation of skills to the dynamic exploration of relations. We will trace the fundamental concepts that students can develop the rough the grades to help them enjoy and be successful with new approaches to algebra.

Workshop 8 — The Future of Mathematics: Ferns and Galaxies

Guide: Monica Neagoy

The advent of new technologies in the classroom creates new visions of school mathematics. We will discover amazing mathematics that could not exist today without the power of computers, and we will discuss possible directions for mathematics in the 21st century.

About the Content Guides

Timothy Erickson

Timothy Erickson received his Ph.D. in Science and Mathematics Education at the University of California, Berkeley in 1987. He has worked as a curriculum developer, writer, and in-service teacher in the EQUALS program in Mathematics and Computer Education at the Lawrence Hall of Science. Dr. Erickson currently serves as a consultant on many mathematics projects including most recently the Edmonds (Washington) Mathematics Project (secondary school mathematics), WGBH Boston (videos and print material for staff development), Theatrix Interactive (multimedia math software design), Key Curriculum Press (high-school statistics curriculum materials), and New Standards (assessment item design).

Beryl W. Jackson

Beryl Jackson is a veteran mathematics educator in the District of Columbia Public Schools (DCPS), where she taught math to secondary students for 19 years, and then served as the Assistant Director of Mathematics and as an Instructional Supervisor of Mathematics. She worked for several years with a mathematics program at Public Broadcasting Service (PBS) called MATHLINE and is currently managing a new Star Schools distance learning program. She has been the recipient of the Presidential Awards for Excellence in Mathematics and Science Teaching (PAEMST) and the GTE Gift Teachers Fellowship. She has served as a member of the board of the National Council of Supervisors of Mathematics, consultant for the National Science Foundation, and past member of the editorial panel of the "Teaching Mathematics in the Middle School," a journal of the National Council of Teachers in Mathematics.

Monica Neagoy

Monica Neagoy has been teaching mathematics and innovative teaching strategies to pre-K–12 teachers at Georgetown University since 1985. She designs, directs, and teaches workshops on a host of topics and approaches. In addition, she provides services to public and private schools in the Washington, D.C. area, including workshops for teachers, interactive presentations for students, and district-wide teacher education programs on math reform. She has a Ph.D. in Mathematics Education from the University of Maryland. Her exposure to many cultures, her mastery of several languages, and her professional involvement in the arts and sciences provide her with a unique perspective on the teaching and learning of mathematics.

Andee Rubin

Andee Rubin is a senior scientist at Technical Education Research Centers (TERC)in Cambridge, MA, where she has played a leading role in several major curriculum development and research projects. These include VIEW: Video for Exploring the World; Investigations in Number, Data and Space, a K-5 math curriculum project; and Through the Glass Wall, an investigation of math-based computer games that appeal to girls. She has worked extensively in teacher training, and has consulted for the National Council of Teachers of Mathematics, the WGBH Teaching Math Video Libraries Project, the Educational Development Center, Cornell University, and the University of California at Irvine along with many other organizations.

Workshop Components

Guest Teachers

Several teachers from schools in the Boston area appear throughout the workshop series. These teachers engage in group discussions throughout the workshop just as you will discuss ideas with participants at your own school.

Activities and Discussion

Throughout each workshop, the content guide will present activities and questions for you to do and discuss with the other participants. The questions will be displayed on the screen for approximately 15 seconds. We suggest that you pause the tape when these questions come up, and take about 5 or 10 minutes to discuss them with your colleagues.

Pre-Workshop Assignments

At the end of each workshop, the content guide will present an assignment for you to do in preparation for the following workshop. Instructions for these assignments are listed in the print material for each workshop. (Please note that the Pre-Workshop Assignment for Workshop 1 appears before the Workshop 1 materials.)

Puzzler

At the end of each workshop, the host will present a math puzzler just for fun! These puzzlers are not assignments, but rather fun challenges. We enjoyed solving them and wanted to share them with you. Solutions to the puzzlers will be discussed in the following workshop.

Post-Workshop Questions

In addition to the questions that will be posed during each workshop, we will provide you with several questions to think about after each workshop. You might discuss these questions with other participants at your school, ponder them on your own, or answer them in your journal.

Suggested Classroom Activities

The print material for each workshop includes a list of related activities for you to do with your students. We have tried to provide a variety of activities — some may be more age-appropriate for your students than others.

Journal

We recommend keeping a journal throughout the workshop series so that you can record and comment upon your own learning over the eight sessions of the series. As the series progresses, pay particular attention to changes in your thinking and the implications of the changes.

If you're stuck for what to write about, you may want to answer the Post-Workshop Questions listed in the print material for each workshop. Here are some suggestions to get you started on your first journal entry:

Think about your experiences learning mathematics. Did you enjoy learning math in school? Why or why not? Do you feel confident about your mathematical ability? Did you enjoy algebra? Geometry? What was your favorite thing about math class? Least favorite? Think about one math learning experience that you remember particularly well. (You might want to share your answers to these questions with the other participants when you gather for the first workshop.)

Pre-Workshop Assignment for Workshop 1

Spend some time during the day or two before the first workshop noticing patterns in your life — visual, pictorial, auditory. Write down three or four patterns that you find particularly intriguing.

- What makes these patterns?
- How are they the same?
- How are they different?

Workshop 1

Patterns and Functions: What Comes Next?

Content Guide: Andee Rubin

Supplies Needed for Workshop 1

Cuisenaire Rods®,

pencils, paper, scissors, rulers, calculators, tape, a variety of colored markers

About the Workshop

What is the theme of the workshop?

In a sense, doing mathematics requires looking for patterns everywhere and expecting to find them — believing that the world is often predictable. This workshop explores how the idea of predictability forms the basis of mathematics, and introduces some mathematical activities in which patterns are central.

Whom do we see? What happens in the videoclips?

We'll see students from pre-kindgergarten through middle school working with patterns, and we'll consider what kinds of pattern-related activities support their development — activities ranging from simple-patterned songs and stories to generalized functions represented by mathematical expressions and graphs.

What issues does this workshop address?

One issue we'll consider is how to make activities accessible to students at different points in their mathematical development. We'll do several activities together that we hope you will find challenging, and we'll think about how to modify these activities for younger and older students.

What teaching strategy does this workshop offer?

We'll discuss the role of group work in math class — many of the students we'll watch will be working in groups, and the teacher will be playing the role of active observer and questioner.

To which NCTM Standards does this workshop relate?

In grades K-4, this workshop is closely related to Standard 13: Patterns and Relationships. The standard emphasizes a variety of ways students can interact with patterns: "recognize, describe, extend, and create a wide variety of patterns." In grades 5-8, the corresponding standard is Standard 8: Patterns and Functions, which recommends that students, in addition, be able to use tables, graphs, and rules to represent functional relationships. In this workshop, we see examples of all of the above ways of working with patterns, and we see how students progress from pictorial representations to more algebraic forms of functions.

In addition, this workshop is related to the general standards: Mathematics as Problem Solving and Mathematics as Communication. The students are solving problems and communicating about them verbally, in pictures, in graphs, and even in song.

Suggested Classroom Activities

Guess My Rule

You may structure this activity as presented in the Workshop: have one student (or a group of students) make up a rule and give a three-number set that follows the rule, and have other students in class try to guess the rule by suggesting other three-number sets that they think follow the rule. Each guess must be accompanied by a reason. Other ways to play the game include:

- Pair students (or groups) up and have them alternate making up and guessing a rule.
- For younger students, the rule can be simpler — just taking one number and giving back a different one. For example, what is the rule if I give you two and you give me five; then, I give you three and you give me seven, etc. You can act this out, with a student being the function machine that changes any number to another based on a rule.
- For older students, rules can be quite complex, involving more than one operation and including operations such as squaring, dividing, etc.

This activity provides a strong basis for the concept of functions, as it captures one of the most important characteristics of a function: that it is defined by a rule that will always give one — but only one — answer for every number you put into it.

Patterns in Poems and Songs

Challenge students to come up with songs that have the same kind of pattern as a selected song. Examples from the program include: rounds (e.g., "Row, Row, Row Your Boat"), repeat with variation (e.g., "Hokey Pokey"), and growing (e.g, "There Was an Old Lady Who Swallowed a Fly"). You might also try structures such as verse and chorus (e.g., "Puff the Magic Dragon"), or shrinking (like "B-I-N-G-O"). Look for similar patterns in children's stories.

Patterns in poems usually have to do with the rhyme scheme. Choose a poem and have students find others with the same rhyme scheme. Some verses of songs count as poems, too. See if all of the verses of a song follow the same rhyming pattern.

These activities are accessible to younger students and introduce them to the idea of predictability in an entertaining way.

Secret Codes

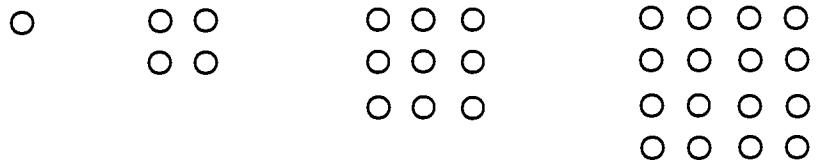
Students make up a code that matches numbers to letters or letters to other letters. They write messages in code to other students, who must decode them in order to read them. The decoder might be told what the code is, or for a more advanced activity, might have to figure it out. For example, a code might be: a=c, b=d, c=e, . . . ,y=a, z=b. Some students may be able to figure out the code based on common letter patterns; others might need some clues.

These codes are actually functions — the rule specifies how a letter is transformed to another. The rule works in the same way for all letters, just as functions work in the same way for all numbers.

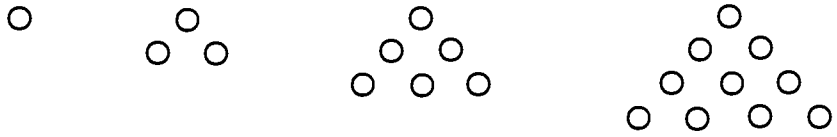
Figurate Numbers

Figurate numbers are those that can be described geometrically. Square numbers are 1, 4, 9, 16, etc. Triangular numbers are 1, 3, 6, 10, etc. "L" numbers are 1, 3, 5, 7, etc. Finding a pattern for square numbers, for example, could be noticing that the differences between them are all the odd numbers (3, 5, 7, 9, etc.)

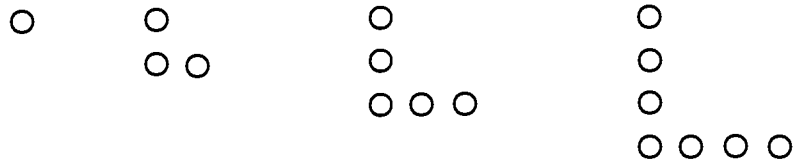
These kinds of patterns are especially interesting because they can be looked at both numerically and geometrically; different students are likely to approach the problem in different ways. There are many other problems of this general type, such as the Tiles problem from the Workshop.



Square numbers *1* *4* *9* *16*



Triangular numbers *1* *3* *6* *10*



L numbers *1* *3* *5*

Suggested Strategy

The Guess My Rule activity puts students in both the "expert" role and the "guesser" role. This is an important shift away from students always trying to answer the teacher's questions. Try this strategy for some math activity — either the Guess My Rule activity or some other — and watch what students can learn in each role.

Post-Workshop Questions

1. Bonnie Edwards connects patterns with addition and multiplication when she asks a student how many balloons it would take if the pattern repeated twice. How might you use patterns to work on other topics such as multiples, factors, even and odd numbers, measurement, fractions, etc.?

2. Many of the students in Lilia Olivas's class had trouble with the valentine exchange problem. How might you have made the problem simpler and/or given students some help with it? What have you done when a problem that you have posed seems too difficult for most of your students?

3. Both Bonnie and Lilia have their students work in groups. Is this an appropriate arrangement for these particular problems? In general, what kinds of problems work well in groups? How do you bring closure to a problem that groups have been working on separately?

Pre-Workshop Assignment for Workshop 2

Administer the Survey

Please administer the "really quick" survey (see page 16) to your class. The sheet has four copies of the form on it. Copy enough for your class (e.g. seven if you have a class of 28) and cut them into individual forms.

Pass out the forms, asking students to fill them in. Collect the forms. This should take less than five minutes.

Important: If students should ask whether they can mark more than one box per question ("What if we like both dogs and cats?") or none ("I only like soccer. What if I hate both basketball and football?"), DO NOT answer the question directly. Instead, PLEASE say simply, "Follow the instructions on the survey as best as you can." If necessary, explain that your directions indicated that you can only tell them to follow the instructions on the form. In a pinch, show them these instructions!

Afterwards, you may, if you wish, share the data with the class and have them try to figure out a good way to display it.

If you have students who are non-readers or ESL students, you may adapt this questionnaire any way you think will give us the same data. The easiest way is to read it aloud or to make a new questionnaire with pictures.

If you are an administrator or do not have direct access to a classroom, we encourage you to poll your family, friends, or colleagues.

Submitting Your Data

Enter your poll results in the following tally chart:

	Male	Female
Cats / Basketball		
Cats / Football		
Dogs / Basketball		
Dogs / Football		

If any forms were not completed correctly (i.e. had more or less than three answers per form), enter that number here: ____.

Are you surprised by the results? In Workshop 2, you will be able to compare your results to those of teachers across the country, and examine various ways of graphing these results.

A really quick survey

Please mark one box in each line.
Thanks!

I am a	❒ male	❒ female
If I have to choose between dogs and cats, I prefer	❒ cats	❒ dogs
If I have to choose between football and basketball, I prefer	❒ basketball	❒ football

A really quick survey

Please mark one box in each line.
Thanks!

I am a	❒ male	❒ female
If I have to choose between dogs and cats, I prefer	❒ cats	❒ dogs
If I have to choose between football and basketball, I prefer	❒ basketball	❒ football

A really quick survey

Please mark one box in each line.
Thanks!

I am a	❒ male	❒ female
If I have to choose between dogs and cats, I prefer	❒ cats	❒ dogs
If I have to choose between football and basketball, I prefer	❒ basketball	❒ football

A really quick survey

Please mark one box in each line.
Thanks!

I am a	❒ male	❒ female
If I have to choose between dogs and cats, I prefer	❒ cats	❒ dogs
If I have to choose between football and basketball, I prefer	❒ basketball	❒ football

Workshop 2

Data: Posing Answers and Finding Questions

Content Guide: Timothy Erickson

Supplies Needed for Workshop 2

pencils, paper, scissors, rulers, calculators, tape, a variety of colored markers

About the Workshop

What is the theme of the workshop?

Data-rich activities enliven the mathematics classroom. In this workshop, we'll explore the relationship among situations, data, and graphs, and discuss why we should care about statistics and data in the math curriculum.

Whom do we see? What happens in the videoclips?

We'll see students from kindergarten through middle school doing age-appropriate activities with data, ranging from the very beginning—learning to represent quantities with symbols—to the beginnings of statistical inference. We'll see kids use data from the real world and estimation as well as "pure mathematical" data.

What issues does this program address?

This workshop addresses a range of issues. For example, how does understanding develop over the years? What role does data play in helping more students succeed in mathematics? How do you decide whether to use "real" or "made-up" data?

What teaching strategy does this program offer?

In this workshop, we see teachers using good general principles for using data in the math classroom: using data children collect and generate themselves; using data related to students' lives, either through experience or interest; asking children both to create and to interpret representations of data; and using data to support other areas of the curriculum (e.g., by plotting estimates on a number line, which supports developing number sense and estimation.)

To which NCTM Standards does this workshop relate?

This workshop focuses primarily on Standard 11: Statistics and Probability (elementary) or Standard 10: Statistics (middle grades). Naturally, other content strands appear, notably Estimation (Elementary 5, Middle 7). All four "process" standards are well-represented, but you will especially see Standards 2 and 4: Communication and Connections.

Suggested Classroom Activities

Collecting and Displaying Data: Surveys

In order to be intelligent–and skeptical–consumers of data, students need experiences collecting and displaying data themselves. Surveys are natural venues for this. Students design their own surveys and (with teacher approval) administer them. Then they make a display of their data and report on what they found. This can be more or less sophisticated depending on the experience of the students: kindergartners can make picture plots, surveying their own class, and dictate "one thing you can say from the graph." Eighth-graders can make two-way tables with row and column percentages, list questions you can answer based on the data, list possible causes and alternative hypotheses, and comment on possible bias in the sample.

Collecting and Displaying Data: Measurements

Continuous data has a different character from typical, categorical, survey data (in which the subject answers questions with a finite number of choices, e.g., yes/no or vanilla/chocolate/strawberry). Displaying and analyzing these data involve point plots, scatter plots (to see trends and relationships), box plots, and using summary statistics such as mean, median, and interquartile range. Again, students can make a display and a write-up. Topics might include: comparing before and after estimates; comparing heights; how far balls bounce or paper airplanes fly; how long you can hold your breath or stand on one foot blindfolded; how many minivans are in the parking lot at the mall; or repeated trials of doing ten free-throws in a row.

Analyzing Published Data

Get graphs or tables from anywhere (USA Today is one good choice) and let students analyze them. At first, the point is simply to tell in your own words what the graph or table is saying. Later, students can figure out the context and what point the author is trying to make. Then they can come up with alternative explanations, and alternative representations with the same data.

Peer Assessent

Any of these small projects are perfect fodder for peer assessment. If students make posters that they put up around the room, they might also be asked to look at three other posters and comment on them. Consider applying "writers' workshop" commenting guidelines, e.g., students write answers to "What did you like about this poster/presentation?" and "What additional questions do you have?" to help make the situation safe for criticism. In this way, each student gets feedback and has the experience of giving it; all of this supports the goal of fostering communication skills in mathematics.

Suggested Strategies

We'll see teachers using a variety of instructional techniques. During the workshop, pay special attention to how the teachers used cooperative groups and student presentations. How would these work in your own classroom?

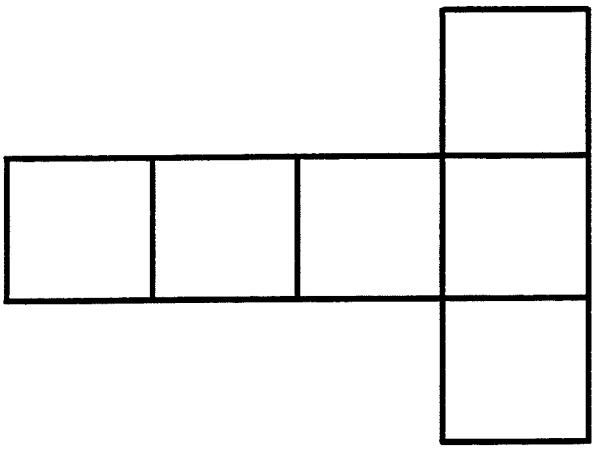
Post-Workshop Questions

1. Students will naturally become more sophisticated thinkers as they grow older. But when, exactly, do some data-oriented ideas become accessible? In particular, when can students begin to make and interpret various kinds of graphs (e.g., point plots, circle graphs, scatter plots, box-and-whisker plots)? When can students begin to reason about differences among groups in their data? When should we expect students to begin using statistical measures such as mean, median, etc., spontaneously? And how does their understanding develop over time?
2. How do you decide if it's worth the class's time—and your effort—to do an activity that may span more than one period? What do you do afterwards to evaluate for yourself whether it was worth it?
3. When students give a wrong answer or use faulty reasoning in a presentation, how do you deal with that?
4. How do you feel about your own skills and experience with data and statistics?

Pre Workshop Assignment for Workshop 3

A "net" is a set of attached squares. How many different nets can you find that can be folded along the lines to make a cube?

Each net should have six squares so there's no overlap. Here is one net that works:



In the workshop, you will be asked to describe your nets using numbers from the grid below. The solution we described could be called, "4-7-8-9-10-16."

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18

Note that a net can have more than one numerical description. For example, the net described above could also be described as "1-7-8-9-10-13."

We are attaching a page of grids for you to work with. Copy it if you need more.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18

Workshop 3

Geometry: Castles and Shadows

Content Guide: Andee Rubin

Supplies Needed for Workshop 3
snap cubes,
pencils, paper, scissors, rulers, calculators, tape, a variety of colored markers

About the Workshop

What is the theme of the workshop?
For some people, geometry was the most frightening part of mathematics; for others, it was the only part that made sense. We will try to shed some new light on the study of geometry by inviting you to use your hands and eyes to explore both familiar and unusual geometric objects.

Whom do we see? What happens in the videoclips?
We'll see students at a range of grade levels investigating relationships between two-and three-dimensional objects. In all of these classrooms, we'll see students working with their hands: building three-dimensional models, cutting and gluing paper shapes, making perspective drawings, and folding flat paper into solid objects.

What issues does this workshop address?
One issue we will explore is how teachers help students who are having trouble with a geometric task. What is the balance between helping students develop independence and guiding them — sometimes subtly, sometimes more explicitly — toward the mathematical goals of the lesson?

What teaching strategy does this workshop offer?
In two of the videos, the teachers have asked students to work in groups and then to report their work to the whole class. We will consider how teachers might comment on student work in these situations to make the experience meaningful both for the presenters and for the rest of the class.

To which NCTM Standards does this workshop relate?
In grades 1-4, this workshop is related to Standard 9: Geometry and Spatial Sense, and in grades 5 -8 to Standard 12: Geometry. Both standards stress description, modeling, and comparison of geometric shapes; exploring the results of transformations on these shapes; and, in general, the development of spatial sense. You will also see Standard 1: Mathematics as Problem Solving and Standard 3: Mathematics as Reasoning in Action.

Suggested Classroom Activities

Design Your Own Instructions

Each student or group of students builds an object (e.g., a building) out of interlocking cubes. They then write and/or draw instructions for making the object and pass them on to another group. The second group follows the instructions and compares the results with the original object. This activity focuses on mathematics as communication and encourages students to think about the meaning of mathematical vocabulary.

More on Silhouettes

Each group of students builds a building with interlocking cubes, then draws the front, right, and top silhouettes. They trade their silhouettes with another group of students, who tries to reconstruct the original building. Interesting questions to discuss: Is there more than one building that fits a given set of silhouettes? What determines whether a set of silhouettes will fit only one building? How many buildings can you make that will have all three silhouettes look the same?

Folding Nets

Do the pre-workshop activity (See pp. 19-21) with your class. Extend the activity by exploring all the nets for 2 x 1 boxes — there are a lot more. Can you (or your students) come up with a labeling system that makes it easy to determine whether any two nets are the same?

Drawing from Memory

Make transparencies of simple designs (some examples are given below). Use an overhead projector to show students one of the designs for a few seconds, then take it away and ask them to draw it. Put the design up again for a few seconds, then remove it and let students revise or complete their drawing. Finally, show the design once more — leaving it up this time — so that everyone can check their work. Ask students to describe how they remembered the picture: What shapes did they see? How did each drawing relate to the previous one? This activity can be done as early as kindergarten with very simple two-dimensional figures; it can be challenging for middle school students if the drawings are of three-dimensional objects.

Suggested Strategies

All of the lessons we will see in the videos require additional materials other than the standard pencils, paper, rulers, etc. Consider how you use different kinds of materials in your classroom. How do they affect students' learning? What, if any, classroom management issues can arise when you use special materials, and how might you deal with them?

Post-Workshop Questions

1. Marco Ramirez spends a long time with one student clarifying the meaning of the word "side." How might you have handled the same situation?

2. Language plays a major role in Marco Ramirez's bilingual classroom. In addition to the discussion referred to in Question 1, there are several situations in which Marco Ramirez encourages students to connect language and mathematics. For example, as students present shapes, he labels the shapes with their formal names. When students create an unusual shape, he allows them to name it (e.g., a Z with 2 heads). He encourages students to write the names of shapes in the best way they can, even if they don't know the exact spelling. How do you react to these techniques? Would you use them in your classroom? More generally, what do you think about the issue of mathematical vocabulary?

3. There is great variety in the mathematical sophistication with which students in Nan Sepada's classroom classify the hexominoes. While we do not see her offer comments to any of the groups, how do you think she might have responded? What would you have done? In particular, how would you have responded to the group that used letters of the alphabet to classify the shapes?

Pre-Workshop Assignment for Workshop 4

In preparation for Workshop 4, please conduct the following survey:

There are two groups of rectangles: Group 1 and Group 2. Show both groups to 20 people, and ask them to select one rectangle from each group which is the best looking or most pleasing. Record their responses on the tally chart (p. 25). You may also make a note of the age range of your respondents. Bring your results with you to Workshop 4.

There may be some surprises in your data. During the workshop, we'll give you an opportunity to call in and share the results of your survey. In this way, you can see how your results compare to results from teachers in other parts of the country.

Directions: Show subjects the two groups of rectangles (*Group 1* and *Group 2*). Ask them which rectangle in each group is the best looking or most pleasing. *Mark one rectangle per group for each person you survey*. Tally the results and enter them under *Totals*.

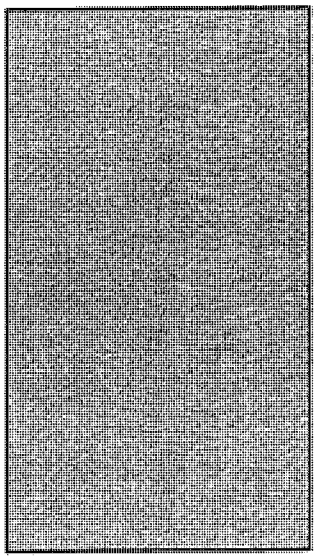
Rectangles

		Group 1			Group 2				
	Age	1 - a	1 - b	1 - c	2 - a	2 - b	2 - c	2 - d	2 - e
Person 1									
Person 2									
Person 3									
Person 4									
Person 5									
Person 6									
Person 7									
Person 8									
Person 9									
Person 10									
Person 11									
Person 12									
Person 13									
Person 14									
Person 15									
Person 16									
Person 17									
Person 18									
Person 19									
Person 20									
Totals:									

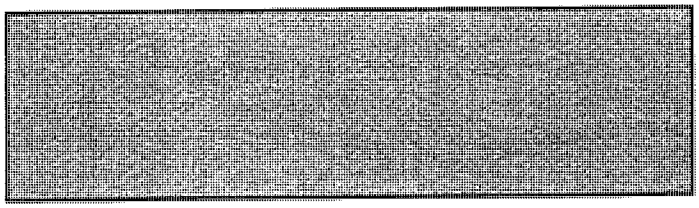
Persons

Group 1

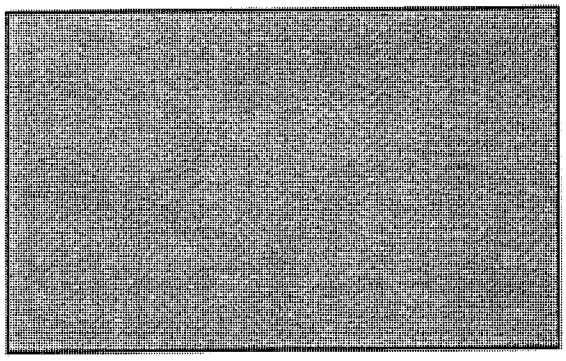
1-a.



1-b.

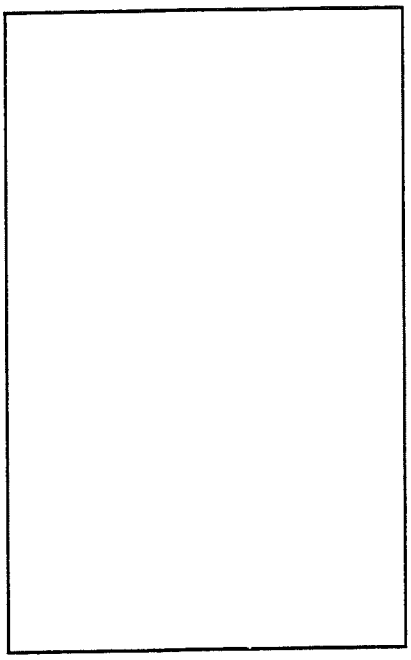


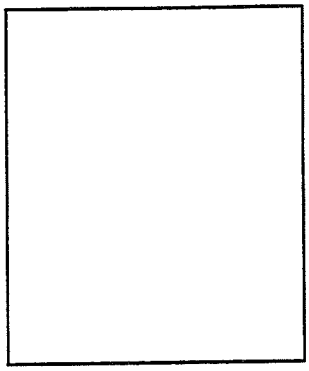
1-c.

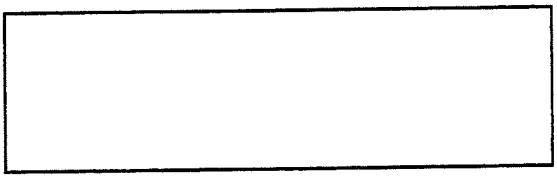


Group 2

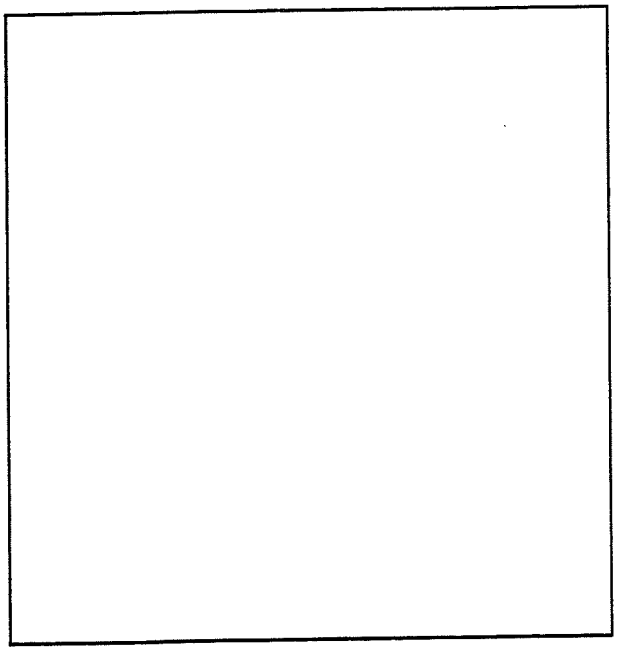
2-a.

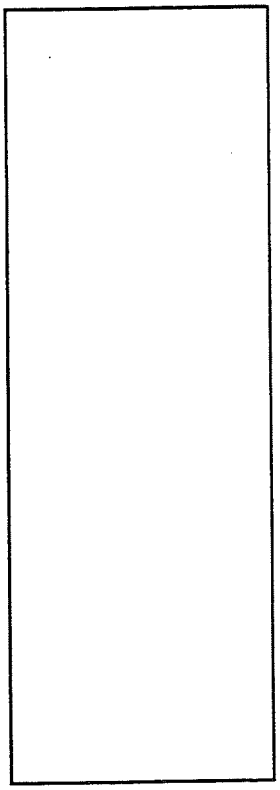






2-d.





Workshop 4

More Geometry: Quilts and Palaces

Content Guide: Beryl Jackson

Supplies Needed for Workshop 4

pattern blocks, graph paper, geoboards, metric tape measures,
masking tape, small rectangular mirrors (2 mirrors for every pair of participants)
pencils, paper, scissors, rulers, calculators, tape, a variety of colored markers

About the Workshop

What is the theme of the workshop?
No other topic in mathematics demonstrates its relationship to the real world as concretely as geometry. All around us we see geometric figures in all shapes and sizes. We walk on planes with straight or curved sidewalks, study in rectangular classrooms, shield ourselves from the rain with dome shaped objects, and model both simple and complex mathematical and scientific phenomena with geometric interpretations. We find beauty in the quilt and are in awe of the majestic structure of the palace. This workshop will focus on the mathematics in art and nature as they relate to two-dimensional geometry.

Whom do we see? What happens in the videoclips?
We will see students, at both the elementary and middle school levels, engaged in tasks which illustrate the development of geometric ideas. These ideas range from the recognition of whole shapes, to analyzing the relative properties of shapes, to investigating relationships between shapes. Ultimately, students are making conjectures about these relationships.

What issues does this workshop address?
This workshop will address how children form basic geometric concepts in mathematics. Tasks are designed to help teachers help students understand some of the underlying concepts in geometry, their connection to other strands of mathematics, and the role geometry plays in our physical world.

What teaching strategy does this workshop offer?
The teacher's and students' roles in discourse and the learning environment will all be addressed during this workshop. Each of these roles is discussed extensively in the NCTM Professional Teaching Standards.

To which NCTM Standards does this workshop relate?
Most specifically, this workshop addresses Standard 9: Geometry and Spatial Sense of the K-4 standards and Standard 12: Geometry for teachers of grades 5-8. There is also an inherent focus on Standards 2, 3, and 4: Mathematics as Communications, Mathematics as Reasoning and Mathematical Connections as we encourage students to use the language of geometry, understand and apply geometric properties and relationships, and link these relationships to both the real world and the world of mathematics.

Suggested Classroom Activities

The Platonic Solids

A tessellation is a design that covers a flat surface without leaving gaps and without overlapping. A regular tessellation is a design whereby only one shape is used to create the tiled pattern. As a task related to the activity presented in the workshop, have your students construct models of the Platonic Solids — so named for the Greek philosopher, Plato. Plato presented a complex ideology relating these solids to the world.

There are five Platonic solids. This means that there are only five solids in which all of the faces are congruent regular polygons. The Platonic Solids are the tetrahedron (4 equilateral triangles as faces), the hexahedron (6 squares as faces), the octahedron (8 equilateral triangles as faces), the dodecahedron (12 pentagons as faces), and icosahedron (20 equilateral triangles as faces).

The solids can be constructed from paper cut-outs of squares, equilateral triangles, and regular pentagons. These cut-outs should be tabbed so that the pieces can be glued together to form the polyhedra. Another way to have students construct these solids is to use toothpicks and gumdrops (or tiny marshmallows) — the toothpicks are the edges and the gumdrops are the vertices. Still another way of building these solids is to cut straws so that they are about 2 inches in length, and then thread these straws using string to build the faces. Tie the faces (i.e. 4 equilateral triangles) together to create the solid.

After the solids have been constructed, instruct students to investigate the differences and similarities between the regular tessellations and the Platonic solids. Also, what do the tetrahedron, octahedron, and icosahedron have in common? Can students find a pattern that exists when they go from one to the other? Why are there only five of these types of solids? Middle school students may want to research Plato's ideology about these solids.

Symmetry

Since symmetry appears so abundantly in nature — the flower, the human figure, and crystals — it is not surprising that the use of symmetry in art, architecture, and even in of the design of national flags is not by accident. Symmetry provides a naturally pleasing view of both the man-made world and the world of nature.

Collect and display copies of some familiar logos. These can be product logos and/or service logos. Be certain that you select logos which have some form of symmetry. However, also include a few logos that do not have any forms of symmetry. In small groups, allow students to examine several of these logos to determine if they are symmetric in any way. That is, do they have reflection symmetry (vertical, horizontal, and/or rotational)? Reconvene students in a large group and facilitate a discussion about their findings.

Next, working in pairs, instruct students to create their own logos for products of their choice. Stipulate that their logos must have symmetry and should, in some way, relate to the product selected.

Understanding how objects are symmetrical involves spatial sense. Investigations in symmetry help to develop this sense in children and provide them with opportunities to look at the world differently.

Where's the Gold?

If the ratio of the measure of the longer side of a rectangle to the measure of the shorter side of a rectangle approximates 1.6:1 (or, more accurately, 1.6180:1), then that rectangle is said to be "golden." The rectangles formed with this special relationship were considered by the ancient Greeks to be the most aesthetically pleasing of all of the rectangular shapes.

Familiarize your students with examples of these rectangles. Have them look for objects in their world which they think are "golden." Instruct them to verify their visual instincts by actually measuring/researching to find out the actual measurements of these objects. Finally, instruct them to create the ratios of the measurements of the sides to determine if the figures possess the golden proportions. This would be a good time to encourage students to use the calculator. An example of an item possessing this relation is a playing card.

If a camera is available, allow students to photograph some of the larger items they have found. Using the pictures and the actual smaller items, create a visual display. Students will be able to see (and hopefully appreciate) the attention given to the manufacturing and production of everyday items. This activity will encourage students to begin to notice their physical world and to question why things are put together as they are. Does the shape of an item entice one to purchase that item?

There are other special geometric shapes which are also designed to have the golden proportion. Have students investigate this.

Suggested Strategies

Whenever possible, bring out the relationships between geometry and measurement, number, and patterns. As illustrated with Rose Christiansen's class, it is not necessary to wait until a special time to begin building concepts with fractions. Design tasks so that there is a natural connection and flow from one strand of mathematics to another. Geometry easily fosters these relationships and connections.

Post-Workshop Questions

1. Despite some reform in the teaching and learning of mathematics, there is still too often major emphasis placed on arithmetic skills at the elementary level, while those skills related to geometry are viewed as less important. Discuss your view on the role of geometry in the K-8 mathematics curriculum.

2. Often, tasks are selected based on the amount of time it should take to complete them rather than on the quality of the activity itself. True geometric investigations consume larger chunks of time because they require the use of physical materials and require a higher level of thinking on the part of the students. Children need time to put the pieces together and to communicate their findings. Share your thoughts on time verses task.

3. Investigations in geometry help students develop inductive reasoning skills. When students begin a formal study of geometry, typically they experience difficulty when asked to make generalizations from their observations (inductive reasoning) or to reach logical conclusions (deductive reasoning). Identify some examples of activities related to geometry that have proven to be successful in helping students think and reason inductively and/or deductively.

Pre-Workshop Assignment for Workshop 5

In preparation for Workshop 5, please answer these questions.

Some people say that students are not learning as much math as they need to.

- What's an example of something students coming into your class don't know as well as they need to?

- How have you seen students changing over the years with regard to their mathematics learning?

Workshop 5

Whole Numbers: Memory and Discovery

Content Guide: Timothy Erickson

Supplies Needed for Workshop 5

- six-sided dice (5 dice for every 2 participants)
- copies of Workshop 5 Worksheet (See p.35) for every participant

pencils, paper, scissors, rulers, calculators, tape, a variety of colored markers

About the Workshop

What is the theme of the workshop?
Nothing says "math" to the public louder than computation with whole numbers. But what role should whole numbers play in today's mathematics classrooms? What should students memorize? When should they use calculators? There are no easy answers to these questions; we'll work together to address them as best we can.

Whom do we see? What happens in the videoclips?
We'll see elementary school students doing a variety of activities either focusing on or requiring basic computation. These range from doing number puzzles with multiple solutions to elaborate estimation activities.

What issues does this workshop address?
This program addresses a range of issues. For example, what are "the basics"? How might we distinguish basic number skills such as the times tables from basic paper-and-pencil algorithms such as borrowing and carrying? What should the role of mental math and estimation be for today's students?

What teaching strategy does this workshop offer?
We'll see a number of effective ways to introduce and reinforce number skills in the classroom. We'll see ways to practice basic number relationships (e.g., 3x3=9) in rich, engaging contexts, and also "pure math" lessons that have no immediate real-world counterparts. We'll discuss games as a medium to reinforce facts. And we'll see how some teachers have used artifacts from real life and activities from other curriculum areas to give their students opportunities to do a lot of estimation and mental math.

To which NCTM Standards does this workshop relate?

This workshop focuses on Standards 5–8: Estimation; Number Sense and Numeration; Concepts of Whole Number Operations; and Whole Number Computation (middle grades Standards 5 and 7). All four "process" standards are represented, but we will especially see Standard 3: Mathematics as Reasoning.

Suggested Strategies and Classroom Activities

Seven-Minute Boogie

Playing basic-fact games can be both fun and effective, as long as they are either non-competitive or "self-competitive." In Seven-Minute Boogie, students roll two dice, make a multiplication problem out of the numbers, write it down (with the answer), and repeat the process as many times as they can in seven minutes. Their score for the day is the number of multiplication sentences they got right in the seven minutes. The next day, they try to improve their scores. With regular dice, they practice low numbers, which are the most useful facts anyway. But you can alter dice to cover more facts: try "2-3-4-6-7-8" and "3-5-6-7-8-9," which miss only a few of the non-trivial facts.

Comparing Procedures

Have students explain to the group how they do various computational procedures. Have students discuss what they like or would change about each procedure. This helps students see, first, that which procedure is best may depend on context, and second, that there is room for disagreement and flexibility in which procedure they use. Some examples: in single-digit addition (for example, as part of a larger algorithm), do you find "friendly and complementary numbers" (e.g., 6 and 4), or just add one after another or count on with your pencil? When you multiply 25 x 36, do you first convert it to 25 x 4 x 9 (even in your head), or do you start with the 5 x 6 is 30, put down zero, and carry three? Be sure to ask what the dangers are of using a "trick."

Peer Assessment

Any of these small projects are perfect fodder for peer assessment. If students make posters they put up around the room, they can also be asked to look at two or three other posters and comment on them. Consider applying "writers' workshop" commenting guidelines. For example, students write answers to "What did you like about this poster/presentation?" and "What additional questions do you have?" to help make the situation safe for criticism. This way, each student gets feedback and has the experience of giving it, which supports the goal of fostering communication skills in mathematics.

Finding Estimation Opportunities

Chances to do the kind of estimation that requires mental math and calculation — especially "back-of-the-envelope" problems — arise frequently and from all sides. How many cars pass the school in a day? How many people do you think are in a newspaper picture of St. Peter's Square in Rome? How long will it be until the school runs out of copier paper? How many little dots are there on the ceiling tiles of this room? Remember: the point is not to guess, but to calculate!

Post-Workshop Questions

1. Some teachers let students use calculators at any time — but only after they've gotten their "calculator license." A student gets this license by proving that he/she can calculate well enough without the calculator. Do you like this idea? What do you think would be a reasonable requirement for getting and maintaining one's calculator license?

2. Think about your own computational strategies in your daily life. For example, when do you use long division? When do you use a calculator? Do you estimate by saying, for example, "29 is almost 30, so..."? Do you look for friendly or complementary numbers (e.g., 7 and 3) when adding? When you're faced with subtraction, do you line up the columns and borrow? You might ask your colleagues the same questions: Is there much variation in the strategies you use?

3. Ignoring the requirements of standardized tests for a moment, at what grade do you think students really need to know the times tables up to 10 times 10? When do these multiplication facts become genuinely useful?

4. In this workshop, we saw teachers using a variety of contexts, ranging from rich contexts to no context at all. Think about your mix of contexts and where you want it to be. Do you usually struggle to fit everything into a context? Or do you always reach for "naked numbers?" And what do you think is best?

5. Debate this argument pro and con: "There's not a lot you can do in mathematics (outside of geometry) without number skills. Number permeates and supports mathematics. So it doesn't make sense to have students do open-ended problems in rich contexts until they master the basics. It's a waste of their time and only leads to frustration and misconceptions."

6. Being a math teacher, you probably learned the traditional algorithms and memorized your times tables pretty early. You also probably have pretty good number sense. Do you think that you developed number sense from your computational ability, or did you learn to compute well because you had number sense? Or were you just the kid who "got it" no matter what? What helped you learn math well, and how does that apply to your instruction and your students?

Pre-Workshop Assignment for Workshop 6

Look at the large triangle on Worksheet 2 for Workshop 6 (p. 42).

If the large triangle is equal to one whole unit, what is the value of:

- a small triangle
- a parallelogram
- a trapezoid
- a hexagon (made up of 2 striped trapezoids)

Workshop 5 Worksheet

You'll need one of these pages for each group of four in your workshop.

▲ Kids and Gerbils

If Carlos and Denise put their Gerbils in the same cage, there would be ten gerbils altogether.

How many gerbils does each kid have?

▼ Kids and Gerbils

Densie brought her gerbils over to Ferhiz's house to visit Ferhiz's gerbils. When they were all together, there were eleven gerbils.

How many gerbils does each kid have?

◄ Kids and Gerbils

Denise, Ferhiz, Carlos, and Emmett are the four kids who started The Gerbil Club. Carlos and Emmet have seven gerbils between them.

How many gerbils does each kids have?

► Kids and Gerbils

Emmett and Ferhiz have gerbils. If you put their gerbils together, you would have eight gerbils.

How many gerbils does each kid have?

Carlos | **Denise** | **Emmett** | **Ferhiz**

The upper part of page consists of four clues. During the workshops, we'll cut out the clues (and, if you like, the name labels), pass them out, and solve the problem as a group.

Please don't solve it ahead of time!

Source

Problems like these are designed especially for group work. This one is original for this program, and you may reproduce it for use in your classes. Additional problems in the same format are widely available, for example in:

Get It Together (EQUALS, 1989) (grades 4+)
Group Solutions (GEMS, 1992) (K–4)
United We Solve (eeps media, 1996) (grades5+)

Format adapted with permission from United We Solve (eeps media, 1996)

Workshop 6

Ratio and Proportion: When Is a Third More Than a Half?

Content Guide: Beryl Jackson

Supplies Needed for Workshop 6

worksheets 1–4 (pp. 42-45),
pattern blocks, tangram (or tangram cut-outs), centimeter grid paper (several sheets per participant),
tape measure, square tiles, 1 bag of M&M's® per teacher (20.9 or 47.9 gram bag),
pencils, paper, scissors, rulers, calculators, tape, a variety of colored markers

About the Workshop

What is the theme of the workshop?

Encounters with mathematics in the real world are not always limited to the neatness of the whole number system. We realize that we need a way to express quantities related to a part of a dollar, divisions of land, the likeliness of rain, or the percentage of decrease in crimes against society. We further realize that music, maps, recipes, and architectural and engineering designs provide settings that emphasize the necessity for numbers other than whole numbers. This workshop will address some issues and provide some ideas related to the teaching and learning of rational numbers and proportional relationships.

Whom do we see? What happens in the videoclips?

We will see teachers and students at the elementary and middle school levels engaged in activities that range from students representing one-half of the same whole in multiple ways to students representing "messy fractions" on the number line. All of the activities incorporate mathematical tasks designed to reinforce concepts of rational numbers and ratios which build basic concepts while making learning fun.

What issues does this workshop address?

Historically, the study of rational numbers has been a real turnoff for many learners. Many teachers have dreaded pulling out the old unit on teaching fractions. The good news is that there are many strategies and activities that can be used which help students visualize and conceptually understand what fractions are and how they relate to each other. Furthermore, combining the study of rational numbers with that of proportional reasoning illustrates the connectivity of mathematics.

What teaching strategy does this workshop offer?

We will see that communication is key to assessing student understanding. In each of the video clips, the teachers encourage open communication among the students. Students are encouraged to state their answers, provide justification for their answers, and even suggest alternative solutions, where possible. One teacher says, "One of the benefits to having students share their strategies is that. . . they question and challenge each other in a different way so that they can get to a truer meaning of the math."

To which NCTM Standards does this workshop relate?

This workshop relates to Standard 12: Fractions and Decimals, Standard 8: Computation, and Standard 5: Estimation of the K-4 Standards, and extends to include Standard 5: Number and Number Relationships and Standard 6: Number Systems and Number Theory of the 5-8 Standards. Additionally, Mathematics as Communication is essential to building concepts with rational numbers and proportional reasoning. Also interwoven in this workshop is Standard 1: Mathematics as Problem Solving, Standard 3: Reasoning, and Standard 4: Connections.

Suggested Classroom Activities

Are you a Square, a Long Rectangle, or a Wide Rectangle?

Have each child, with the help of a partner, determine his/her height (H). Next, have each student determine his/her arm span (A). Students then form a ratio, comparing their height to their arm span (H:A).

If H:A is greater than (>) 1, the student is a LONG RECTANGLE.

If H:A is less than (<) 1, the student is a WIDE RECTANGLE

If H:A is equal to (=) 1, then the student is a SQUARE.

Discuss with the students the conditions that will make each of these relationships true. For example, if H:A > 1, then the height is greater than the arm span. If H:A < 1, then the height is less than the arm span. If H:A = 1, then the height and arm span are the same.

You may have students use non-standard or standard units for measuring. As a follow-up, prepare a class graph classifying the students in the three categories.

The Lilliputians' Measurements

When the Lilliputians measured Gulliver for a suit of clothes, they had their own special way of computing measurements. To determine if their way will work for us, the following relationships should be true:

- Twice around the base of the thumb = once around the wrist
- Twice around the wrist = once around the neck
- Twice around the neck = once around the waist

To do this activity, provide each student with a piece of string (long enough to go around the waist) and centimeter rulers or tape measures. Have students construct ratios which represent the relationships described above. Students can collect class data, construct scatter plots, and view for correlation. This is a good graphing calculator activity for middle school students.

A Fraction Equivalent to a Third

Challenge students to make a fraction (five digit number over five digit number) equivalent to 1/3 by using the numbers 0 through 9 exactly once:

$$\frac{\text{XXXXX}}{\text{XXXXX}} = \frac{1}{3}$$

Visual Representations of Fractions

The tangram, pattern blocks, square tiles, geoboards and Cuisenaire Rods® are all excellent tools to use to develop fractional concepts. The workshop provided you with a few ways to incorporate their use in the instructional program. Here are a few additional ideas:

Geoboards: Use the geoboards to show different ways of representing fourths and eighths.

Tangram: Assign a particular value to a piece of the tangram. Based on that value, find the value of all of the other six pieces. The values assigned can be values less than one or greater than one. Since there is a proportional relationship among the pieces, it does not matter.

Pattern Blocks: Like the tangram, a proportional relationship exists among the trapezoid, triangle, hexagon and blue parallelogram. Use these relationships to develop fractional situations.

Cuisenaire Rods®: Establish one rod as the whole. Have students establish the relationship of the other rods to this whole.

Suggested Strategies

It is more difficult for students to acquire conceptual understanding once they have learned rote procedures. Thus, it is essential to focus initial instruction related to fractions, decimals, ratios, and other multiplicative-based relationships on building conceptual understanding. These play a major role in the development of proportional reasoning which is said to be the cornerstone for much of the mathematics in the secondary years, and merit whatever time and effort it takes to ensure careful development. Therefore, whenever possible, use concrete materials, games, pictorial representations, and real situations to assist in the conceptual development processes.

Post-Workshop Questions

1. How has the way that you have engaged your students in the study of rational number concepts changed over the course of your teaching career? With what results?

2. The major manufacturers of calculators have all introduced calculators which are able to perform operations with fractions. This even includes calculators with capabilities for providing different representations for a given fraction. What is your opinion on using calculators to perform computations and other operations with fractions? Should they be used in elementary and middle school classrooms?

3. During the workshop, four ways of interpreting rational numbers were highlighted: part-whole meaning, quotient meaning, ratio meaning, and operator meaning. Where developmentally appropriate, describe activities that you have used in your instructional program which address these rational number levels of meaning.

4. The Standards suggest that increased attention be placed on the meaning of fractions and decimals, and decreased attention be placed on fraction computation using paper and pencil. Do you agree or disagree with this suggestion? Why or why not?

5. In the video segments highlighted during the workshop, were there any particular segments which you felt provided a model for instruction that would be most appropriate for your current teaching situation? Describe. If not, what modifications would you make to one of the lessons to make it more conducive to addressing the needs of your students?

6. Many teachers begin the year with a study of ratio and proportion because there are so many opportunities for capturing the interest of students early in the year, and because there are many rich explorations related to this topic. If this is a topic that you introduce early in the year, how has it helped or hindered the study of other strands taught later in the year?

Pre-Workshop Assignment for Workshop 7

Your Definition of Algebra

How do you define algebra? If you had to write a definition of algebra for an upcoming mathematics dictionary, what would you write?

Algebra Magic: "Think of a Number"

Think of a number, then follow these instructions:

1. Add 4
2. Multiply by 3
3. Subtract 9
4. Multiply by 2
5. Divide by 6
6. Subtract the original number

(Note: Each instruction is to be applied to the previous answer.)

Now think of a different number, and follow the instructions again. Try it several times with several different numbers. What do you find? Is it surprising? Why?

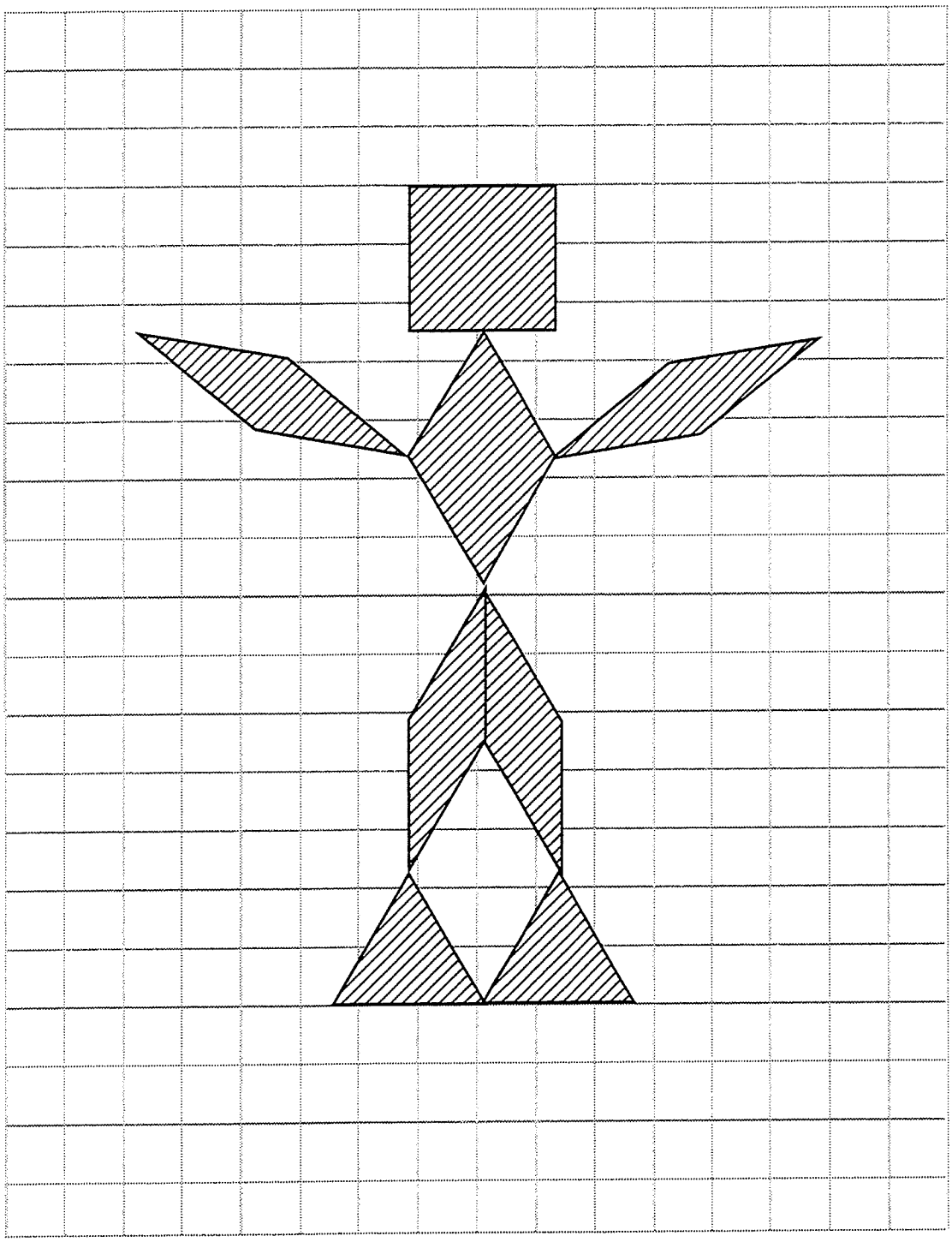
Challenge #1

Try to devise a similar "Think of a Number" magic trick in which the outcome is the same regardless the starting value.

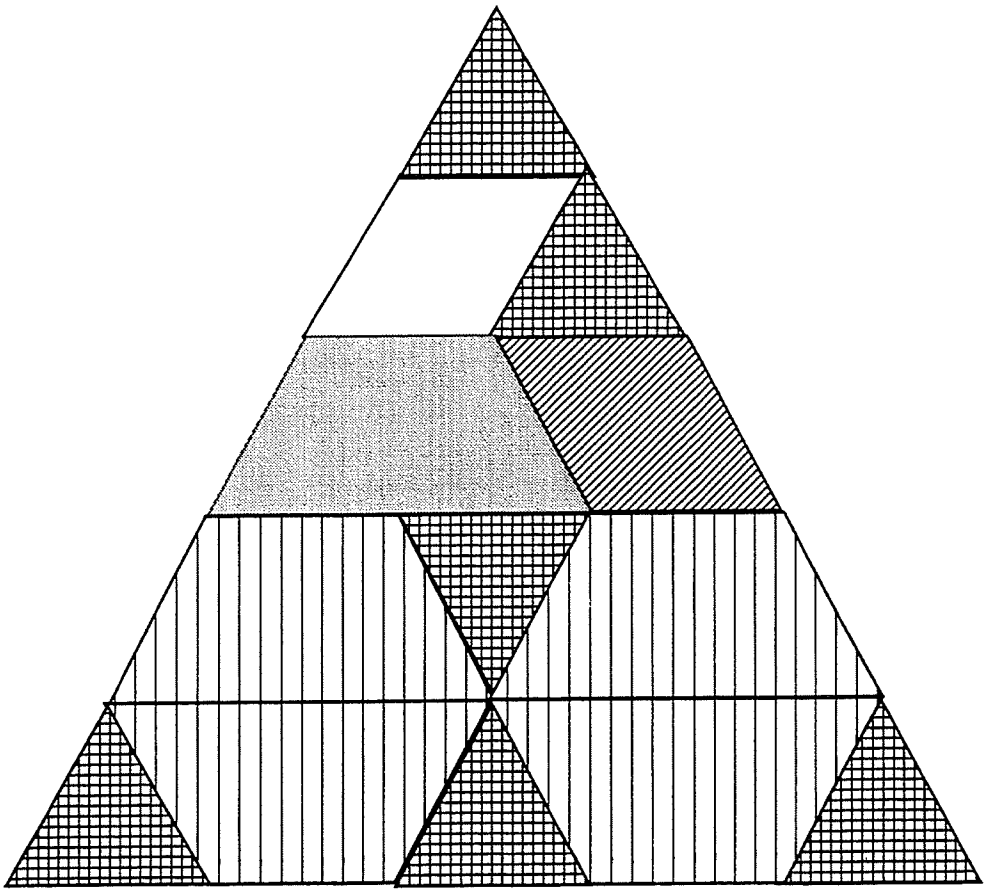
Challenge #2

Try to devise an algebra magic trick in which different starting values yield different outcomes, but you can guess the outcome if you know the starting value. You are a magician!

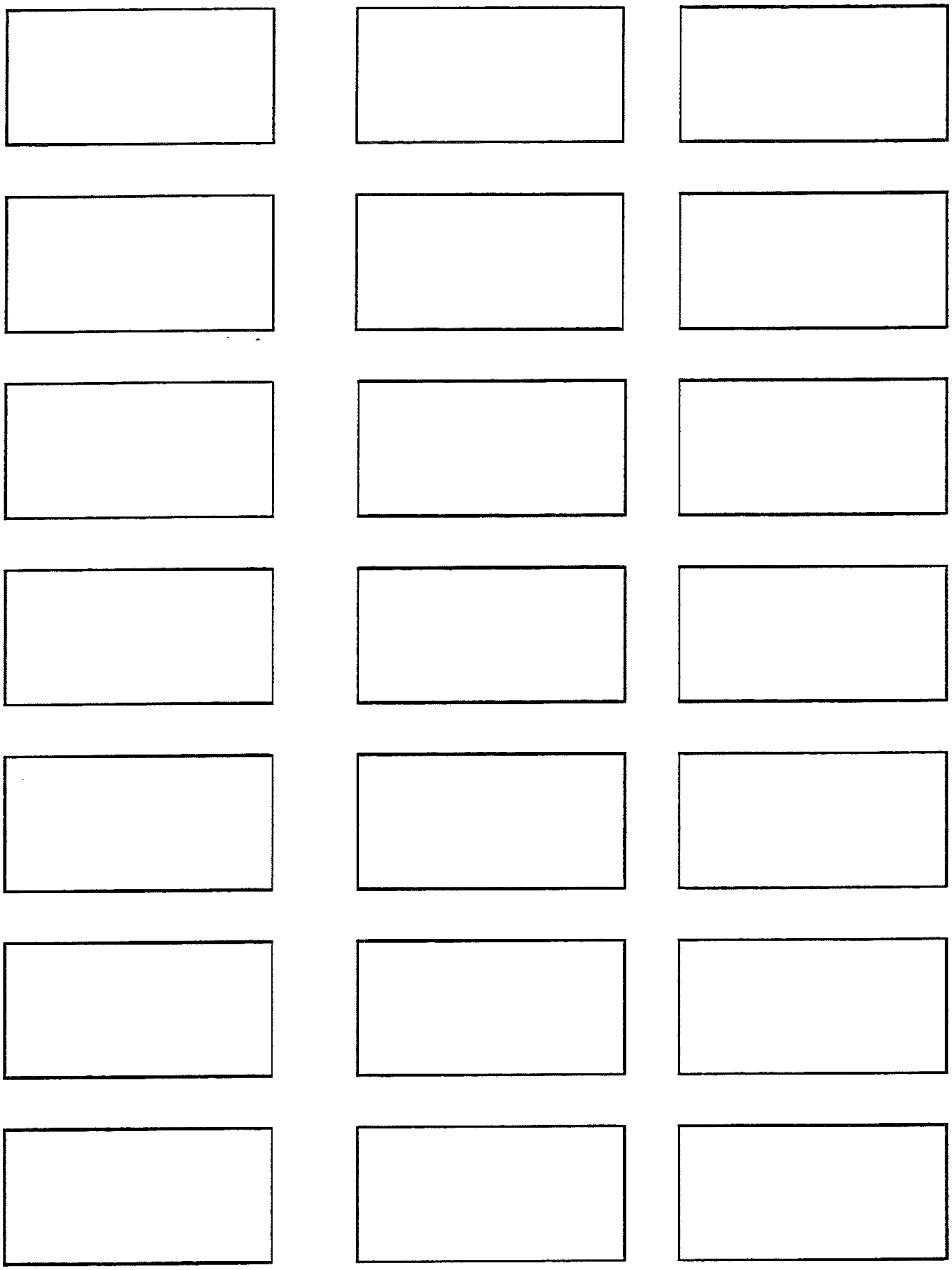
Worksheet 1



Worksheet 2



Worksheet 3



Worksheet 4

A unit and equivalent fractional parts of a unit

Directions: Please color each strip a different color. Then cut strips and fractional units apart prior to Workshop 6.

1 Unit											
1/2					1/2						
1/3				1/3				1/3			
1/4			1/4			1/4			1/4		
1/5		1/5			1/5			1/5			
1/6		1/6		1/6		1/6		1/6		1/6	
1/7		1/7		1/7		1/7		1/7		1/7	
1/8		1/8		1/8		1/8		1/8		1/8	
1/9		1/9		1/9		1/9		1/9		1/9	
1/10		1/10		1/10		1/10		1/10		1/10	
1/12		1/12		1/12		1/12		1/12		1/12	

Workshop 7

Algebra: It Begins in Kindergarten.

Content Guide: Monica Neagoy

Supplies Needed for Workshop 7

graphing calculators (TI-81, 82, 83, or something similar),
flat toothpicks, colored tiles, worksheet 1 (p.53),
pencils, paper, scissors, rulers, calculators, tape, a variety of colored markers

About the Workshop

What is the theme of the workshop?
President Clinton's education agenda calls for all American students to be competent in algebra by the end of the eighth grade. This can only be accomplished if algebraic concepts are introduced and developed throughout the grades, beginning in kindergarten.

Whom do we see? What happens in the videoclips?
We will see students from kindergarten through eigth grade using variables, studying relationships, exploring multiple representations, and making generalizations, all of which are at the heart of algebra.

What issues does this workshop address?
Algebra is not the meaningless abstraction nor the symbolic manipulation that math leaders are pushing to incorporate into the early curricula. Nor is it what we studied it in high school. What, then, constitues algebra in the primary and middle grades? The answer to this question is the main issue of this workshop.

What teaching strategy does this workshop offer?
A variety of teaching strategies will be modeled and discussed. Of note is the integration of graphing calculators in the teaching and learning of algebra. We will also see students working alone, in pairs, and in small groups and using a variety of manipulatives.

To which NCTM Standards does this workshop relate?
This workshop will stress number sense and spatial sense (Standards 6 and 9 in the K-4 content standards). The activities we will explore involve finding patterns and exploring relationships (Standard 13 in the K-4 standards, Standard 8 in the 5-8 standards), and measurement (Standard 10 in the K-4 standards, Standard 13 in the 5-8 standards). The four process standards (Mathematics as Problem Solving, Communication, Reasoning, and Making Connections) will also figure into this workshop.

Suggested Classroom Activities and Strategies

Developing Algebraic Reasoning Through Literature (K-2)

As Jerry Kincaid does in Teaching Math K-4, you can read Pat Hutchins' *The Doorbell Rang* (or a similar story) to your students, and then help them discover the relationship between the number of children and number of cookies each child gets. Encourage your students to use any of a variety of forms of representation to express this relationship (e.g., concrete, verbal, numerical, tabular, pictorial, graphical, or symbolic).

Suggested Strategies

This activity can be done with the entire class. Invite a volunteer to come up in front of the class and model the solution. This first solution may be in the form of a sentence or picture. Thereafter, invite others to give alternative representations for the same relationship (assuming it is correct). Explore multiple representations with the class (concrete, numerical, tabular, graphical, symbolic), pointing out the similarities and differences among them.

What Comes Next? (Grades 2-3)

Give students a variety of sequences (e.g., numerical, color, shape, texture). Provide at least the first five terms of each sequence, and have students figure out the following three terms. Then concentrate on numerical sequences. Ask students the following:

(a) Do they see the "change" from one term to the next? Can they articulate it?

(b) Can they find the 10th, 20th, 50th, 100th term?

(c) Can they generalize for the nth term in their own way (using either words, manipulatives, or symbols)?

Suggested Strategies

This activity could begin as a whole-class activity for one or two sequences. After stressing what is important (i.e., looking at the "change" from term to term, devising generalization techniques, etc.), have students make up their own sequences and quiz their peers. It is important, every now and then, for students to be problem writers, not simply problem solvers. They invest more energy if the problem is their own creation. Also, creating a good problem can be more involved and challenging than solving one.

Discovering the Meaning of Pi (grades 4-8)

Arrange your class in small groups. Give a circular object to each group (e.g., a bottle cap, a soda can, a trash can, a flower pot). Have them measure the diameter (D) and the circumference (C). Have them also compute the ratio C/D for their object. When all the groups have completed the task, have them complete the following chart together:

	D	C	C/D
Group 1			
Group 2			
Group 3			
Group 4			
and so on...			

Ask students to verbalize their findings. Encourage them to express the relationship between C and D symbolically.

The Cube Counting Problem

Have students build 2x2x2, 3x3x3, 4x4x4, and 5x5x5 cubes with multilink or snap cubes. Then have students mark the outside of the cubes thus formed (they could paint or color the outside of the cubes, or could simply use a washable marker to dot each and every visible face). When the faces have been marked, have the students take the cubes apart.

1. In each case, how many cubes have three faces painted? Two faces? One? Zero?

2. Can students discover the patterns in each of the four cases? Can they express the patterns algebraically?

3. Can students make sense of these algebraic expressions by relating them to the geometric structure of the cubes?

Suggested Strategies

This activity (as well as the previous one) could be assigned to small groups of students. Divide the tasks beforehand. When the groups are done, have group reporters take turns sharing their work with the rest of the class. Encourage the "listening" groups to be attentive to the presentations by requiring an "intelligent question" of them. Finally, conclude by articulating the "big ideas."

Post-Workshop Questions

1. Many people, young and old, think of algebra as that "obscure, abstract, meaningless subject one studies in high school." How did you feel about algebra before this workshop? Do you feel differently about algebra now? How will that difference affect your math lessons?

2. What do you think of the use of technology in the teaching and learning of mathematics in general? Of algebra in particular? Have your thoughts about the issue changed since participating in this workshop series?

Suggested Resources

Edwards, Ronald. *Algecadabra! Algebra Magic Tricks*. Pacific Grove, CA: Critical Thinking Press and Software, 1992.

Lappan, Glenda, et. al. *Variables and Patterns.* Palo Alto, CA: Dale Seymour Publications, 1997.

Laycock, Mary. *Algebra in the Concrete*. Hayward, CA: Activities Resources Co., Inc., 1997.

National Council of Teachers of Mathematics. *Mathematics Teaching in the Middle School.* NCTM. February, 1997.

National Council of Teachers of Mathematics. *Algebra for Everyone*. Reston, VA: NCTM, 1990. (Book and Videotape).

National Council of Teachers of Mathematics. *The Ideas of Algebra, K–12*. Reston, VA: NCTM, 1988. (Yearbook).

National Council of Teachers of Mathematics. *Patterns: K-4 Addenda Series*. Reston, VA: NCTM, 1993.

National Council of Teachers of Mathematics. *Patterns and Relationships: 5-8 Addenda Series*. Reston, VA: NCTM, 1991.

Teaching Children Mathematics. NCTM. February, 1997.

Pre-Workshop Assignment for Workshop 8

The main purpose of this assignment is to become familiar with Pascal's Triangle (p.52). If you are already familiar with it, see if you can discover patterns within the triangle which you have never found before.

Let n denote the row number, beginning with:
n = 0 for row "1", and then
n = 1 for row "1 1",
n = 2 for row "1 2 1",
n = 3 for row "1 3 3 1", and so on.

Questions:

1. Add up the numbers in each row. Do these sums form a pattern? Can you express the nth sum in terms of n, the row number?

2. If you are given a certain row of Pascal's Triangle, how can you derive the row beneath it?

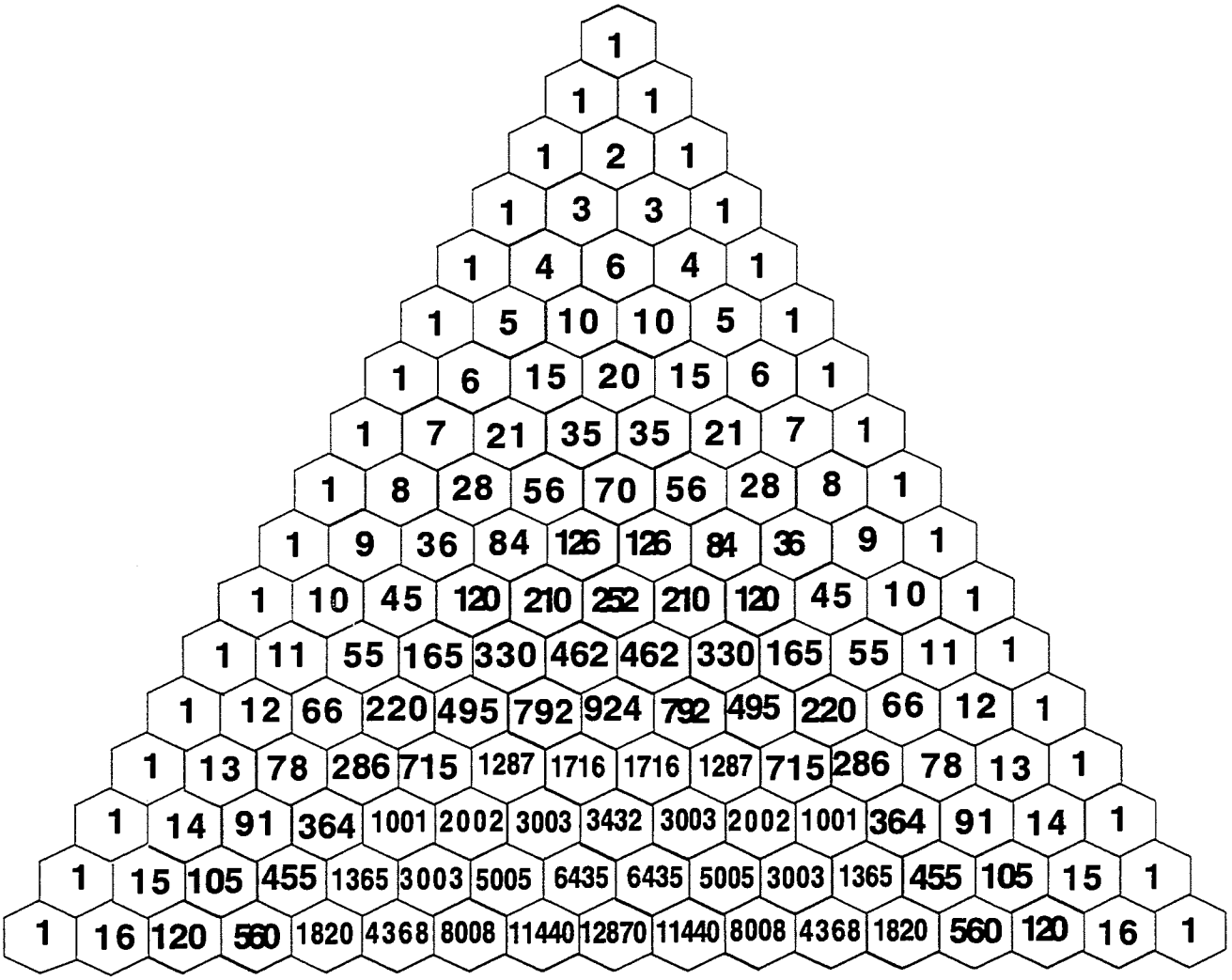
3. Can you find the famous Fibonacci sequence lurking within Pascal's Triangle?

4. Can you find the consecutive powers of 11, starting with 11^0, inside Pascal's Triangle? What happens after 11^4? Can you explain?

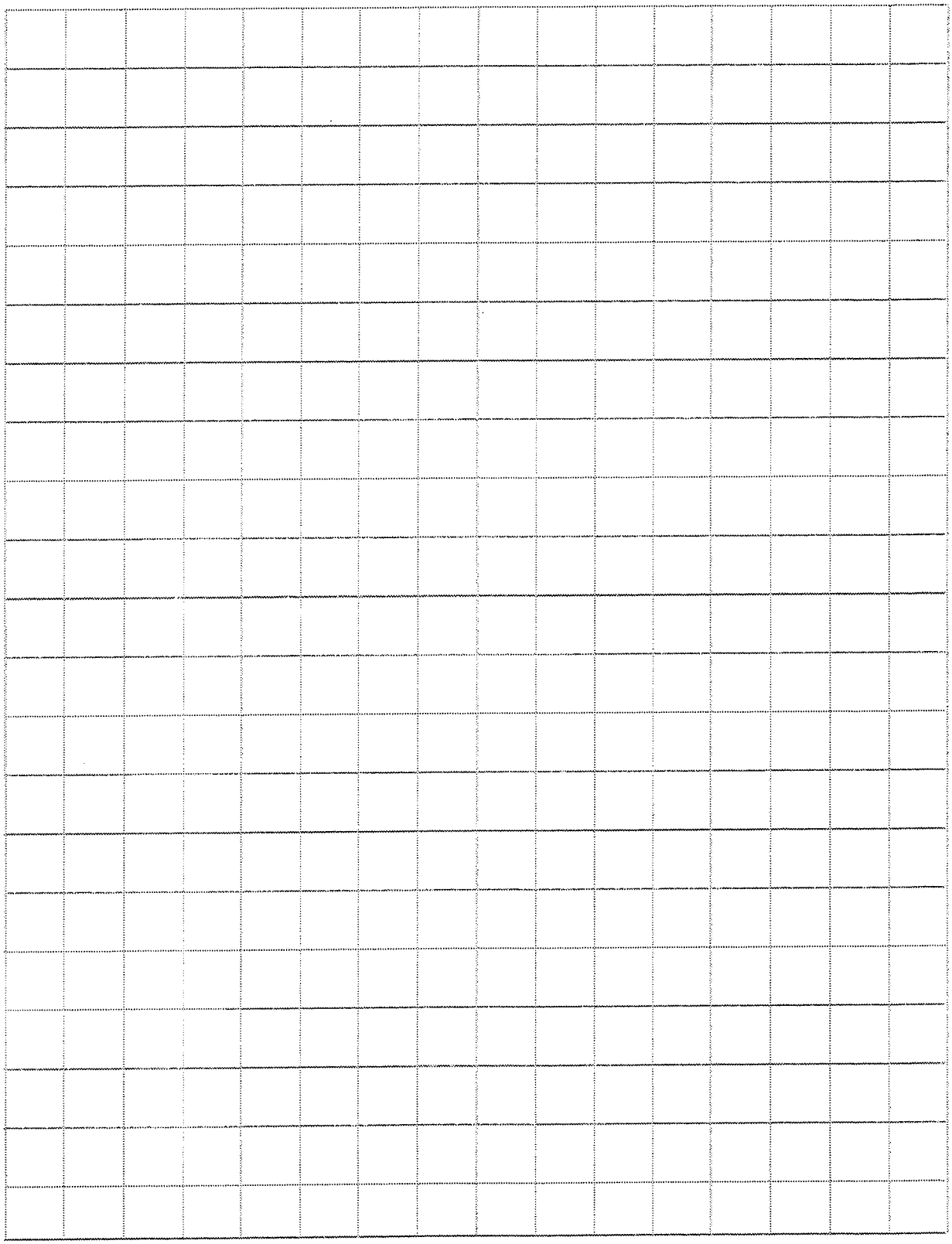
5. Notice the positions of even numbers within the triangle. Is there a pattern?

6. What other numerical patterns can you find?

Pascal's Triangle



Worksheet 1



Workshop 8

The Future of Mathematics: Ferns and Galaxies

Content Guide: Monica Neagoy

Supplies Needed for Workshop 8

1 die and 1 ruler per pair of participants, worksheets 1–4 (pp. 61-64), pencils, paper, scissors, rulers, calculators, tape, a variety of colored markers

About the Workshop

What is the theme of the workshop?

The advent of new technologies in the mathematics classroom is rapidly changing how, what, and when we teach. Not only do higher-level mathematics become accessible to students at a younger age, but new and fascinating worlds of mathematics can open up to them — in their own classrooms — thanks to the power of computers. We will examine two new mathematical tools, recursion and iteration, and introduce a new geometry, Fractal Geometry, which complements the much older and more familiar Euclidean Geometry.

Whom do we see? What happens in the videoclips?

We will see many amazing and beautiful things in this workshop. First we will see three different approaches to the creation of a famous fractal, the Sierpinski Triangle. Second, we will see multiple representations of fractals: fractals on paper, on a computer monitor, in nature, and built with manipulatives. Finally, we will touch on the connection between fractals and chaos, and see how beautifully patterned shapes can result from "chaotic" systems.

What issues does this workshop address?

This workshop will stress the importance of familiarizing both teachers and students with new, burgeoning concepts in mathematics if we are to produce young citizens who will be competent players on the cutting edge of mathematics and science in the 21st C entury. Mathematics scholar and internationally renowned author, Ian Stewart, warns that the scientist/mathematician of the future "will need to combine, in a single integrated world view, aspects of traditional mathematics, modern mathematics, experimentation, and computation (i.e., computer "know-how").

What teaching strategy does this workshop offer?
A variety of teaching and learning strategies will be modeled and discussed. Of note is the use of a variety of media — paper and pencil activities, concrete models, transparencies, graphing calculators, video clips, and computer software — to introduce a completely new topic.

To which NCTM Standards does this workshop relate?
This workshop will stress number sense and spatial sense (Standards 6 and 9 in the K-4 content standards). The activities we will explore involve finding patterns and exploring relationships (Standard 13 in the K–4 Standards, Standard 8 in the 5-8 Standards), and measurement (Standard 10 in the K–4 Standards, Standard 13 in the 5-8 Standards). The four Process Standards (Mathematics as Problem Solving, Communication, Reasoning, and Making Connections) will also figure into this workshop.

Suggested Classroom Activities and Strategies

Exploring Recursion and Iteration in the Primary Grades

A. Think of counting as an iterative process. Ask students to articulate what rule is repeated over and over again as we count from one whole number to the next.

B. Have students find nursery rhymes and folk songs that are recursive in nature, stories that contain the telling of stories within them, movies that show movies inside movies, or plays that illustrate a play within a play.

C. Have students find recursive pictures or labels, such as the well-known Morton Salt box.

D. Have students bring in recursively-built toys or puzzles, such as Russian dolls or the Tower of Hanoi.

E. For those students who use word processing applications at home or in school, explain to them that a file inside a subdirectory, inside a directory, inside a drive, inside the hardware is yet another example of netting and embedding, similar to a small Russian doll inside a medium Russian doll, inside a large Russian doll, inside an extra large Russian doll, etc.

Suggested Strategies
In each example, make sure to point out the element of "permanence" and the element of "change." In other words, pose the questions "What changes?" and "What stays the same?" In example (A), the method of "adding 1" is permanent, and the outcome (i.e., the new and bigger number) after each iteration changes. In example (D), the size of the dolls changes (i.e., becomes smaller as we open them up one by one), but the shape remains the same. After explaining a few examples, have students write a sentence for what changes and what remains the same in each new example. For homework, have them find additional illustrations of recursion in everyday life.

Other Visual Patterns in Pascal's Triangle for Middle School (p. 59)

In this workshop we created an amazing visual pattern when coloring in the even numbers in Pascal's Triangle. Another phrase for even numbers is "multiples of two." Divide your class into seven groups to further explore the visual patterns created by coloring in multiples of 3, 4, 5, . . ., 9. Assign a whole number n, from 3 to 9, to each group. Have each group color in the multiples of n only. When the groups are done, they can share their visual patterns with the whole class. Have them verbalize and then record at least three observations. Challenging: Are any of these new patterns fractals? How can you justify your answer?

Suggested Strategies

This activity can be assigned to pairs of students within each group. One student uses a calculator and divides each number by the assigned whole number n to see if the remainder is 0. If it is, his or her partner proceeds to color in the hexagon that contains that number. The students take turns using the calculator and coloring the hexagons. This method helps students see the connection between "multiple" and "divisor," a connection which is not always clear to students. Obviously, when the number is small, no calculator is necessary.

Exploring Other Geometric Fractals in Grades 4-8 (Activity Sheet 2)

1. Show the Box Fractal to your students. Ask them to verbalize the iterative process from one stage to the next. (You will find that they will provide a variety of explanations.)

2. Ask students to suppose that the side length of the initial square is 1 linear unit. Have them write a sequence for the common side length of the squares at stages 0, 1, 2, 3, . . . , n.

3. Then ask students to suppose that the area of the initial square is 1 square unit. Have them write a sequence for the total area of the Box Fractal at stages 0, 1, 2, 3, . . . , n.

4. Have students try to find other fractals at various Web sites on the Internet. In particular, have students find the (Von) Koch Snowflake or the (Von) Koch Curve. Have them formulate investigative questions.

Suggested Strategies

This activity could be done in a variety of ways – individually, in pairs, in small groups, or with the class as a whole. After students find the two sequences in parts (2) and (3), have them articulate additional questions on this — the box fractal — or other fractals. For instance, "Write a sequence for the total perimeter of the Box Fractal at stages 0, 1, 2, 3, 4,..."

Finding Fractals in Nature

Have students collect natural fractal objects or bring in pictures of fractal objects in nature. Stress the similarities and differences between geometric and natural fractals. Ask students to create their own fractals, either with paper and pencil or with the aid of a computer.

Post-Workshop Questions

1. Did you have any idea that there were other geometries besides Euclidean geometry, the type we learn in grades K-12? If not, how do you think you might incorporate aspects of this new geometry (fractal geometry) into your math lesson plans?

2. Graphing calculators are slowly but surely trickling down from high school to middle school, and from middle school to primary school. In general, today's children are raised in a much more technological world than many of us were, and therefore are much less technophobic than we are. How do you feel about allowing students to use state-of-the-art technologies in your math class? What are your thoughts about how these technologies may help or hinder their conceptual development?

3. Rare is the student who is not captivated by the beauty and surprise of fractal geometry. Students will want to take the subject further, and will probably ask questions that neither they nor you can answer. What will you do if such questions arise?

Suggested Resources

Briggs, John. *Fractals: The Patterns of Chaos*. New York: Simon and Schuster, 1992

Briggs, J. and F. D. Peat. *Turbulent Mirror*. New York: Harper Row, 1989.

Gardner, Martin. *Fractal Music, Hypercards and More....* New York: W.H. Freeman and Co., 1992.

Gleick, James. Chaos: *Making a New Science*. New York: Penguine Books, 1987.

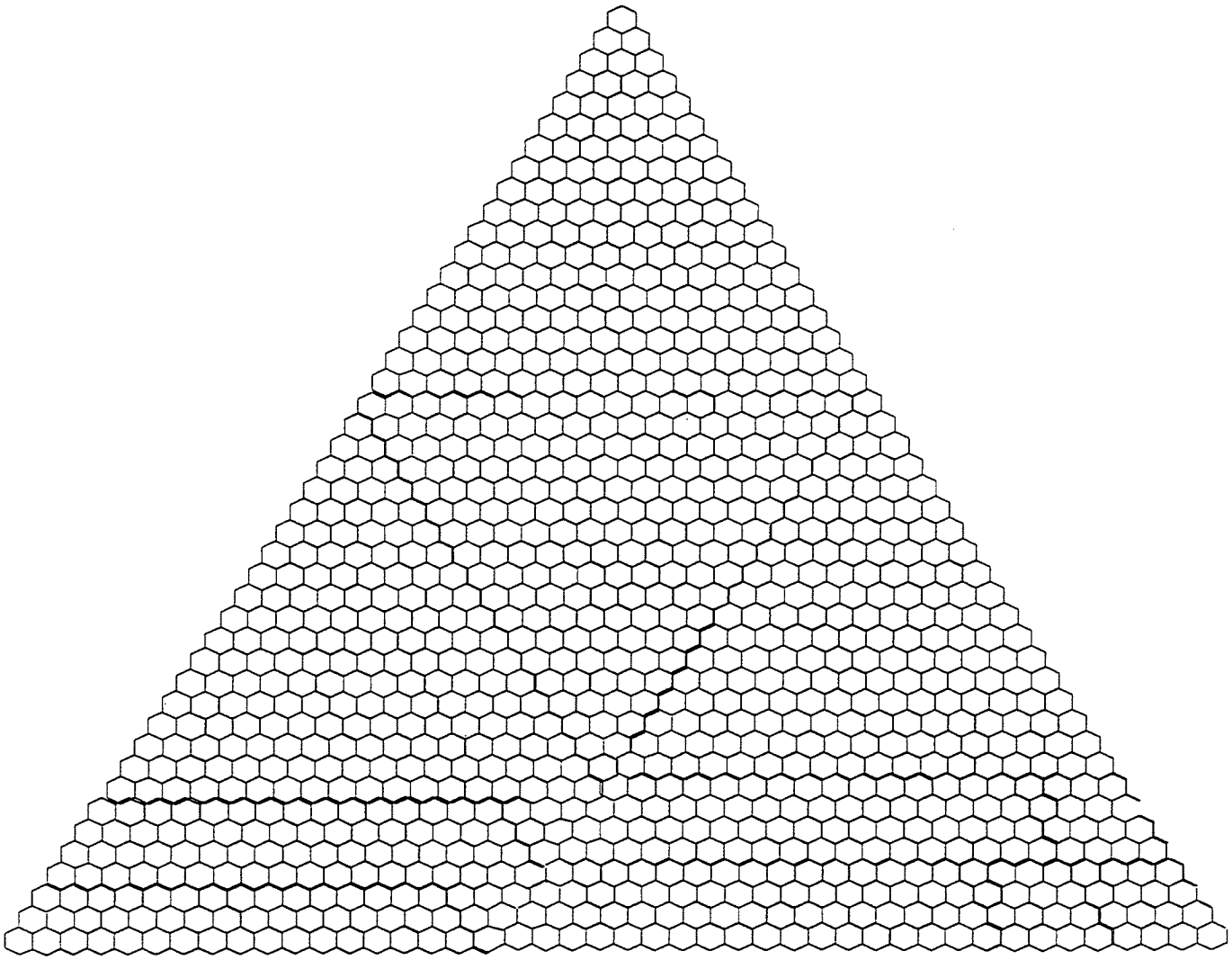
McGuire, Michael. *An Eye for Fractals*. Reading, MA: Addison Wesley, 1991.

Peitgen, H.O. and P.H. Richeter. *The Beauty of Fractals*. New York: Springer-Verlag, 1986.

Wahl, Bernt. *Exploring Fractals on the Macintosh*. Reading, MA: Addison Wesley, 1995. (Book and Software),

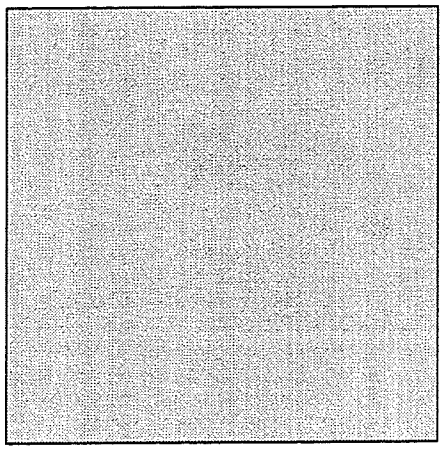
Activity Sheet 1

Multiples of n

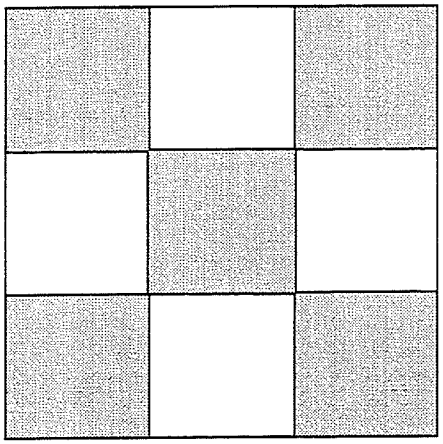


Activity Sheet 2

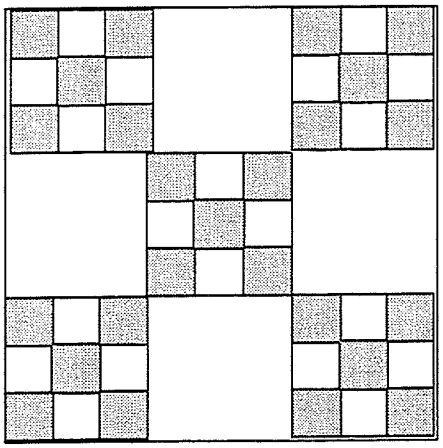
Box Fractals



n=0

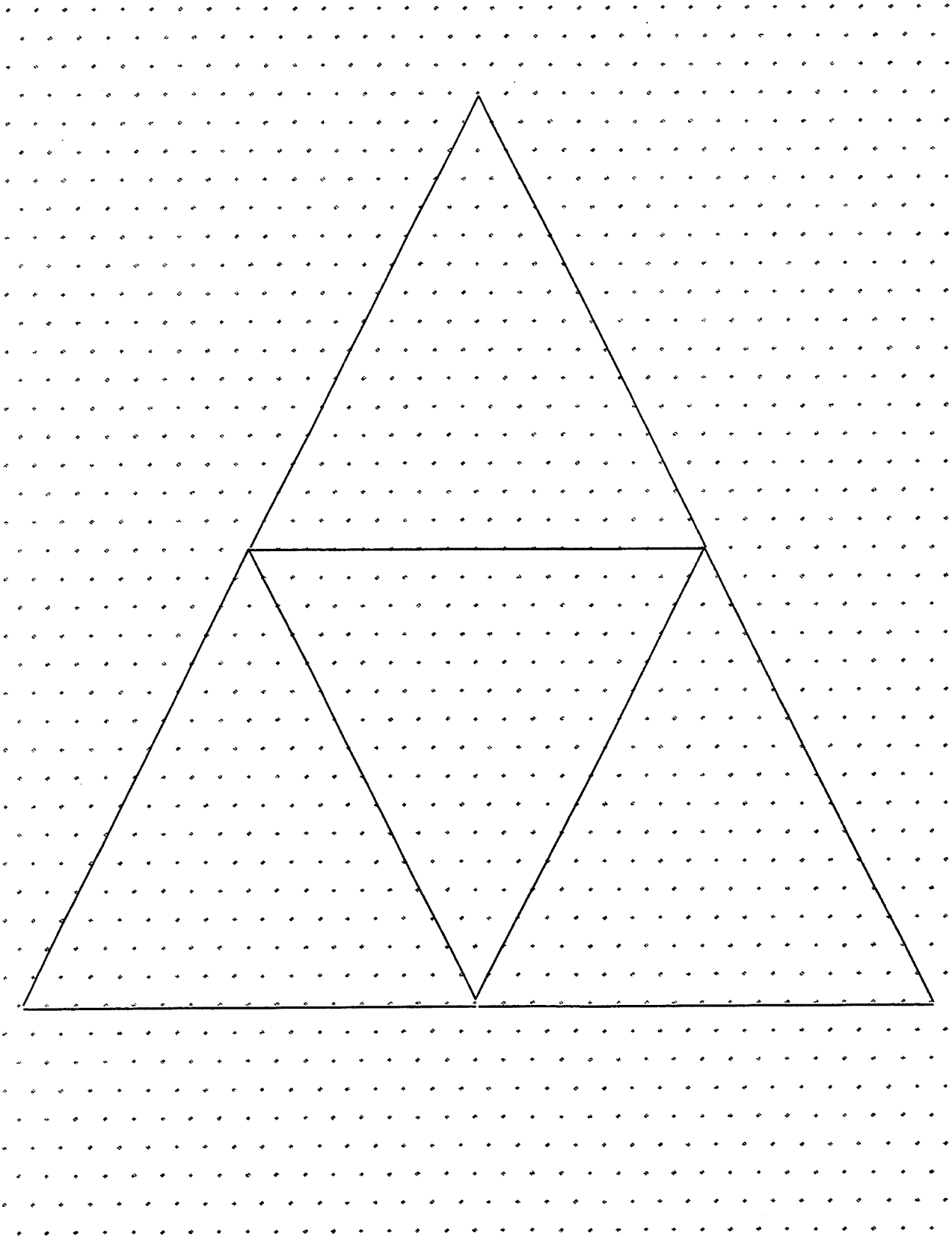


n=1



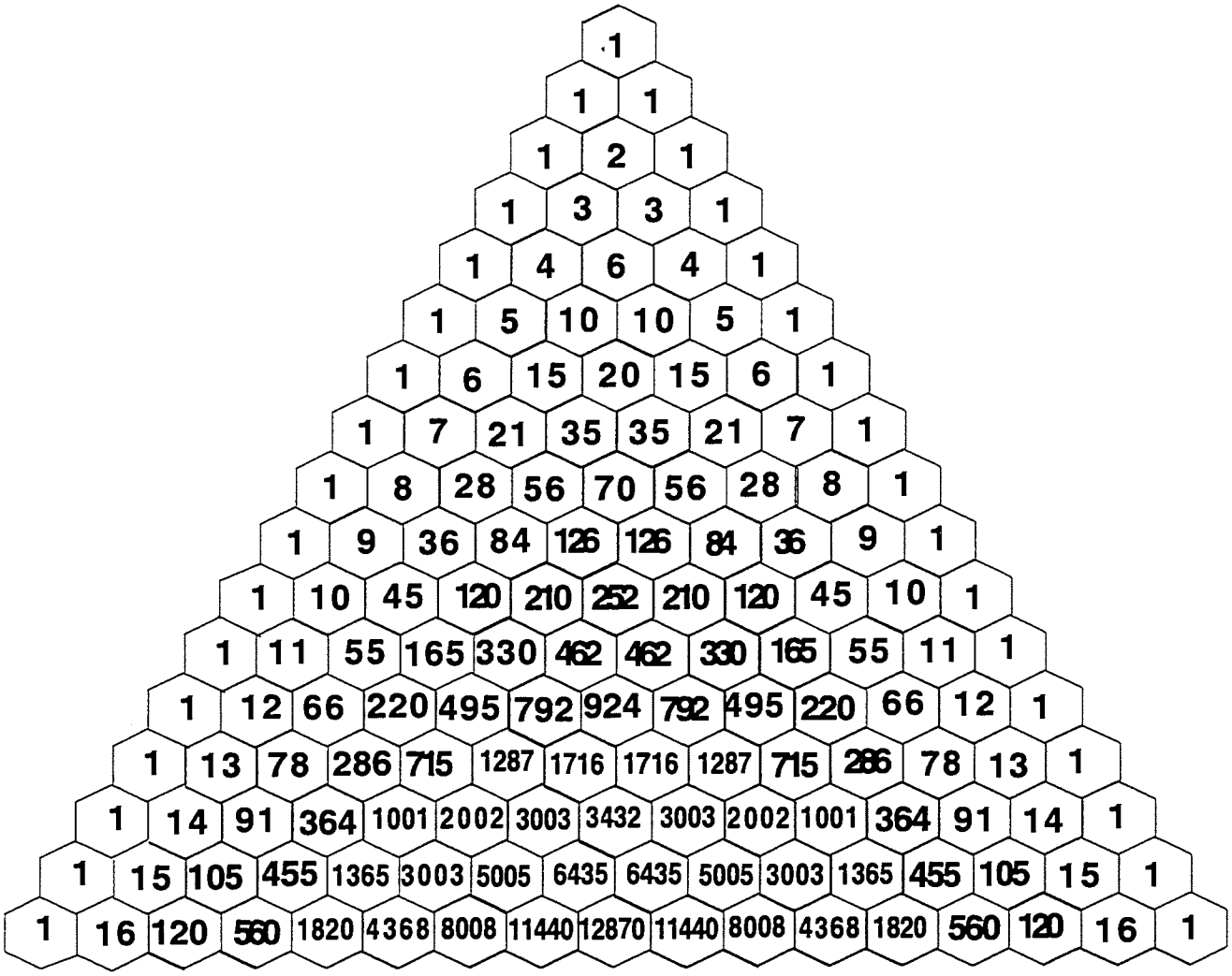
n=2

Worksheet 1

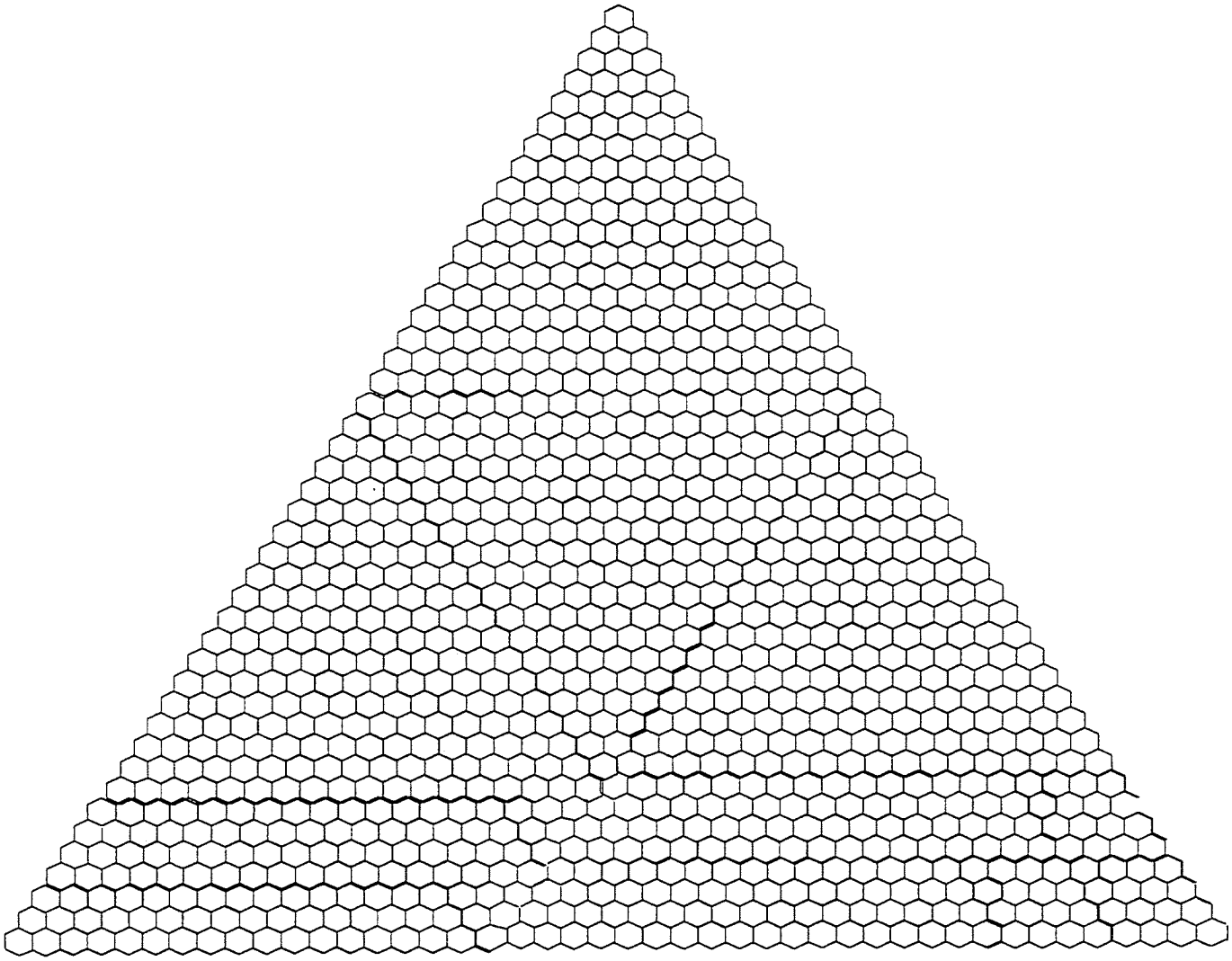


Worksheet 2

Pascal's Triangle



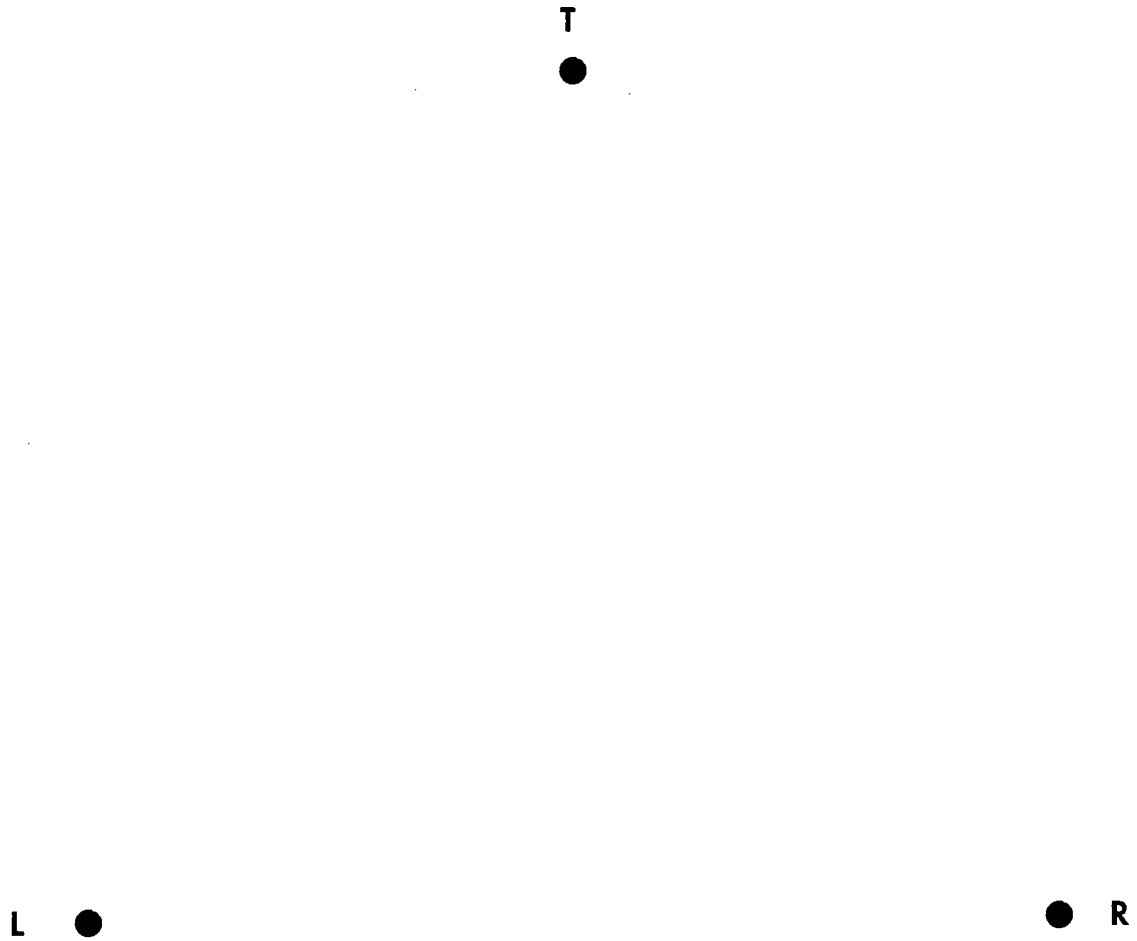
Worksheet 3



Worksheet 4

The Chaos Game

- **If you roll a 1 or 2, move halfway to L.**
- **If you roll a 3 or 4, move halfway to T.**
- **If you roll a 5 or 6, move halfway to R.**



Suggested Resource List

Burns, Marilyn. *About Teaching Mathematics: A Kindergarten Through Eighth Resource.* Portsmouth, NH: Heinemann, 1993.

Burns, Marilyn. *What Are You Teaching My Child?* New York: Scholastic, 1994 (Videotape)

Corwin, Rebecca, et al. *Talking Mathematics: Supporting Children's Voices.* Portsmouth, NH: Heinemann, 1995.

Countryman, Joan. *Writing to Learn Mathematics: Strategies That Work, K-12.* Portsmouth, NH: Heinemann, 1992.

Hart, Diane. *Authentic Assessment: A Handbook for Educators.* Menlo Park, CA: Addison-Wesley, 1993.

Kamii, Constance, Barbara A. Lewis, and Sally Jones Livingston. "Primary Arithmetic: Children Inventing Their Own Procedures." *Arithmetic Teacher,* December, 1993.

Mokros, Janice R., TERC and Jan Mokros. *Beyond Facts and Flashcards: Exploring Math With Your Kids.* Portsmouth, NH: Heinemann, 1996.

Mokros, Jan, Susan Jo Rusell, and Karen Economopolous. *Beyond Arithmetic: Changing Mathematics in the Elementary Classroom.* Palo Alto, CA:Dale Seymour Publications, 1995.

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: NCTM, 1989.

National Council of Teachers of Mathematics. *Standards Professional Standards for Teaching Mathematics.* Reston,VA: NCTM, 1991.

National Council of Teachers of Mathematics, Assessment Standards Working Group. *Assessment Standards for School Mathematics.* Reston, VA: NCTM, 1995.

National Research Council. *Everybody Counts: A Report to the Nation on the Future of Mathematics Education.* Washington, DC: National Academy Press, 1989.

Parker, Ruth. *Mathematical Power: Lessons from a Classroom.* Portsmouth, NH: Heinemann, 1993.

Schifter, Deborah, and Catherine T. Fosnot. *Restructuring Mathematics Education: Stories of Teachers Meeting the Challenge of Reform.* New York, NY: Teachers College Press, 1993.

Stenmark, Jean, Virginia Thompson and Ruth Cossey. *Family Math.* Berkeley, CA: Lawrence Hall of Science, University of California, 1986.

Web Resources

The Math Forum
http://forum.swarthmore.edu

Math
An extensive list of sites with lesson plans, activities, puzzles, etc.
http://www.csun.edu/~vceed009/math.html

Science and Math Initiatives (SAMI)
Science and math database and teacher help service
http://www.learner.org/sami

Internet Resources for Elementary Mathematics Educators
Elementary math lesson plans, teacher resources, software, etc.
http://www.math.ttu.edu/~dmettler/title.html

Math Central
A K-12 math education Website
http://MathCentral.uregina.ca

Centre for Innovation in Mathematics Teaching
Website for the Center that focuses on research and curriculum development in Mathematics teaching and learning
http://www.ex.ac.uk/cimt

Teachers Helping Teachers
Resource for teachers featuring live teacher chats and math related information
http://www.pacificnet.net/~mandel

A Homepage for New Math Teachers
Resources, suggestions, and lesson plans for new math teachers
http://www.clarityconnect.com/webpages/terri/terri.html

Teacher's Lounge
Live chats with teachers of varying grade levels and subjects
http://wbs.net/webchat3.so?cmd=cmd_doorway:Teachers_Lounge

INSTRUCT
Implementing the NCTM School Teaching Recommendations Using Collborative Telecommunications
http://instruct.cms.uncwil.edu