Pointing Errors due to the Effect of Telescope Tilt as Measured by a Pair of Arbitrarily Oriented Tiltmeters

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Xiaolei Zhang

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On the AST/RO telescope, the pair of x-y tiltmeters are mounted with an orientation angle θ, which is not an integer multiple of 90 degrees, with respect to the encoder north. The standard pointing equations (see, e.g., Condon's memo for the GBT telescope) give only the pointing corrections due to the tilts as measured by tiltmeters that are co-aligned with the north-south and east-west directions (assuming that the encoder zero is at nominal north). In order to apply time-dependent tilt correction to the AST/RO star-pointing data, we need to derive the expressions of pointing corrections due to the telescope tilts as measured by a pair of arbitrarily-oriented tiltmeters, with the only constraint that the axes of the x and y tiltmeters are perpendicular to each other.

In the following derivation, we adopt the same convention for the coordinate systems as that used in Condon. In the so-called "horizon system" H centered on the observer (Figure 1), the x_h axis points to the south, the y_h axis points to the east, and the z_h axis points to zenith. For a source with azimuth A and elevation a, the unit vector $\vec{H}$, which points at the source, has its direction cosines in the horizon system as

$$\vec{H} = \begin{pmatrix} x_h \\ y_h \\ z_h \end{pmatrix} = \begin{pmatrix} -\cos a \cos A \\ \cos a \sin A \\ \sin a \end{pmatrix}. \quad (1)$$

An az-el mounted telescope, originally pointing towards zenith and with its azimuth encoder reading zero (north), points to a source by first rotating clockwise around z_h axis by an angle A, with the effect that the original unit vector $\vec{H}$ now appear in the rotated coordinate system R as vector $\vec{R}$, where

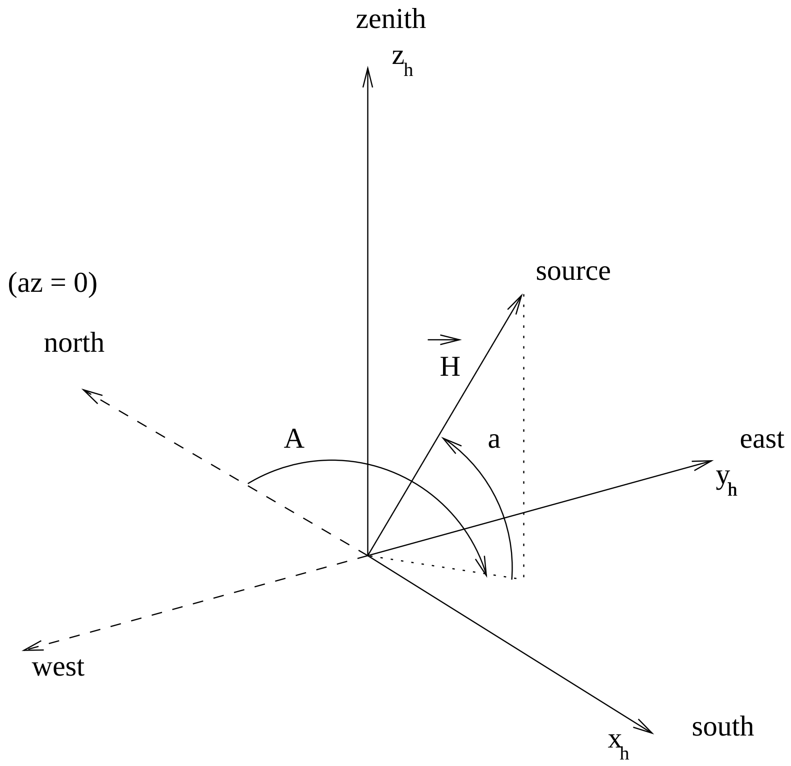

Figure 1: Horizon coordinate system as used by Condon

$$\vec{R} = \begin{pmatrix} x_r \\ y_r \\ z_r \end{pmatrix} = \begin{pmatrix} \cos(-A) & \sin(-A) & 0 \\ -\sin(-A) & \cos(-A) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_h \\ y_h \\ z_h \end{pmatrix} = \begin{pmatrix} -\cos a \\ 0 \\ \sin a \end{pmatrix}. \quad (2)$$

To point the telescope exactly at the source, the previous azimuth rotation is followed by a rotation of the coordinate axes (fixed on the telescope) by an angle $(\pi/2 - a)$, clockwise about the y_r axis. The unit vector pointing to the source in this new coordinate system T becomes the unit vector $\vec{T}$, with

$$\vec{T} = \begin{pmatrix} x_t \\ y_t \\ z_t \end{pmatrix} = \begin{pmatrix} \cos(a-\pi/2) & 0 & -\sin(a-\pi/2) \\ 0 & 1 & 0 \\ \sin(a-\pi/2) & 0 & \cos(a-\pi/2) \end{pmatrix} \begin{pmatrix} x_r \\ y_r \\ z_r \end{pmatrix}$$

$$= \begin{pmatrix} \sin a & 0 & \cos a \\ 0 & 1 & 0 \\ -\cos a & 0 & \sin a \end{pmatrix} \begin{pmatrix} -\cos a \\ 0 \\ \sin a \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \quad (3)$$

Now if the azimuth track does not lie on the horizontal plane, but rather is tilted, the unit vector of the source in the horizon system will no longer be $\vec{H}$, but will become $\vec{H} + \Delta\vec{H}$. Assume that the amount of the azimuth track tilt is measured by a pair of tiltmeters which are orthogonal to each other, and which are oriented with respect to the encoder north as shown in Figure 1, i.e. the tiltmeter axes x' and y' are rotated counterclockwise from the south and east direction by an arbitrary angle θ. If the tilt measured by the x' tiltmeter is Δx, which represents the amount of counterclockwise rotation around x' axis, and that measured by y' tiltmeter is Δy, which represents the amount of counterclockwise rotation around the y' axis, and if both Δx and Δy are small rotation angles, the unit source vector in the tilted horizon system $H + \Delta H$ now becomes

$$\vec{H} + \Delta\vec{H} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & -\cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \Delta x \\ 0 & -\Delta x & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -\Delta y \\ 0 & 1 & 0 \\ \Delta y & 0 & 1 \end{pmatrix} \cdot$$
$$\cdot \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_h \\ y_h \\ z_h \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 & -\Delta y\cos\theta - \Delta x\sin\theta \\ 0 & 1 & -\Delta y\sin\theta + \Delta x\cos\theta \\ \Delta y\cos\theta + \Delta x\sin\theta & \Delta y\sin\theta - \Delta x\cos\theta & 1 \end{pmatrix} \begin{pmatrix} x_h \\ y_h \\ z_h \end{pmatrix}. \tag{4}$$

Therefore, after the azimuth rotation of the telescope by A clockwise, the unit vector pointing to the source in the $R + \Delta R$ system becomes

$$\vec{R} + \Delta\vec{R} = \begin{pmatrix} \cos A & -\sin A & 0 \\ \sin A & \cos A & 0 \\ 0 & 0 & 1 \end{pmatrix} (\vec{H} + \Delta\vec{H}) =$$
$$\begin{pmatrix} -\cos a \\ 0 \\ \sin a \end{pmatrix} + \begin{pmatrix} \cos A & -\sin A & 0 \\ \sin A & \cos A & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -\Delta y\cos\theta\sin a - \Delta x\sin\theta\sin a \\ -\Delta y\sin\theta\sin a + \Delta x\cos\theta\sin a \\ \{(\Delta y\cos\theta + \Delta x\sin\theta)(-\cos a\cos A) \\ + \\ (\Delta y\sin\theta - \Delta x\cos\theta)(\cos a\sin A)\} \end{pmatrix}. \tag{5}$$

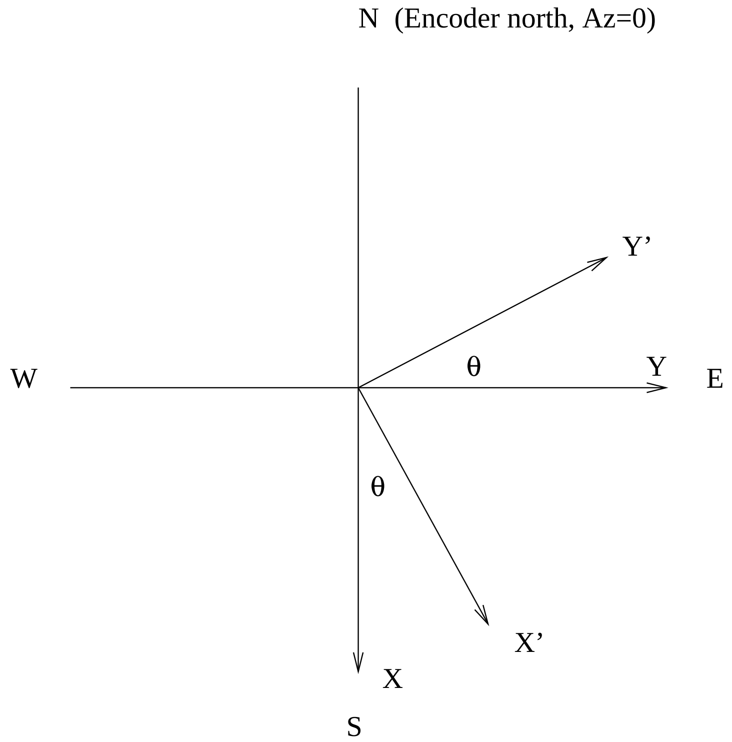

Figure 2: Tiltmeter Orientation With Respect to the Encoder North

After the rotation in the elevation direction downward (clockwise around the new y_r axis) by $\pi/2 - a$, the unit source vector in the tilted telescope system $T + \Delta T$ becomes

$$\vec{T} + \vec{\Delta T} = \begin{pmatrix} \sin a & 0 & \cos a \\ 0 & 1 & 0 \\ -\cos a & 0 & \sin a \end{pmatrix} (\vec{R} + \vec{\Delta R}) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{pmatrix}, \quad (6)$$

where the vertical pointing error ΔT_1 is given by

$$\begin{aligned} \Delta T_1 = & -\Delta y(\cos A \cos\theta - \sin A \sin\theta) \\ & +\Delta x(-\cos A \sin\theta - \sin A \cos\theta), \end{aligned} \quad (7)$$

and the horizontal pointing error ΔT_2 is given by

$$\Delta T_2 = -\Delta y(\sin A \sin a \cos\theta + \cos A \sin a \sin\theta)$$

$$+\Delta x(\cos A \sin a \cos\theta - \sin A \sin a \sin\theta), \tag{8}$$

and finally,

$$\Delta T_3 = 0. \tag{9}$$

In the limiting case of $\theta = 0$, the above pointing error expressions become the standard pointing terms in Condon (compare with his equation 5, with the substitution of $\Delta x = \Delta W$, and $\Delta y = -\Delta N$). Note that especially at the South Pole, the nominal north direction used in the above derivation is identified with the encoder zero direction.

On the actual AST/RO telescope, the positive y' axis of the tiltmeter is pointed rather in the opposite direction from that shown in Figure 2, i.e. the two tiltmeter axes form a left-handed system. In such a case, the Δy factor in the above pointing correction terms need to have a sign change (there might have been orientation changes in the most recent years, so this information needs to be updated).

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Reference

Condon, J. J. "GBT Pointing Equations," GBT technical memo, No. 75