Spectral Energy Distributions & Masses

Flux at λ for $\mathcal{N}$ particles of size a:

$$F_\lambda = \mathcal{N}\left\{\frac{\pi a^2}{D^2}\right\} Q_\lambda B_\lambda(T) \qquad B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{\exp(hc/\lambda kT)-1} \qquad Q_{\mathrm{FIR}} \propto \lambda^{-\beta} \begin{cases} \beta=0 & \text{blackbodies} \\ \beta=1 & \text{amorphous, lattice-layer materials} \\ \beta=2 & \text{metals \& crystalline dielectrics} \end{cases}$$

Flux at λ for a distribution with $\mathcal{N}_a$ particles of each size a:

$$F_\lambda(obs) = \sum_{\text{grain distribution}} \mathcal{N}_a \left\{\frac{\pi a^2}{D^2}\right\} Q_{\lambda,a} B_\lambda(T)$$

(Note: same flux can be achieved using different combinations of size distribution and emissivity law!)

Mass determination:

$$M_{dust} = \frac{4sF_\lambda D^2}{3B_\lambda(T_{dust})}\left\{\frac{a}{Q_\lambda}\right\}$$

Using "appropriate average" of a/Q_λ.

$$M_{dust}(true) = \sum_{\substack{\text{grain \& } \\ \text{temperature distribution}}} \frac{4sF_\lambda D^2}{3B_\lambda(T_{dust})}\left\{\frac{a}{Q_{\lambda,a}}\right\}$$

"Wien's Law":

$$\lambda_{\mathrm{peak}} \approx 3000\left[\frac{5}{\beta+5}\right]T^{-1}$$

Peak flux moves to longer λ for smaller β.

$\mathcal{N}$ = number of grains
a = "typical" grain size
D = distance from observer
T_{dust} = dust temperature
Q = emissivity
s = density of grain material