Spectral Energy Distributions & Masses

Flux at λ for N particles of size a:

\[ F_\lambda = N \left( \frac{\pi a^2}{D^2} \right) Q_\lambda B_\lambda (T) \]

\[ B_\lambda (T) = \frac{2hc^2}{\lambda^5} \exp\left( \frac{hc}{\lambda kT} \right) - 1 \]

\[ Q_{\text{FIR}} \propto \lambda^{-\beta} \]

\( \beta = 0 \) blackbodies
\( \beta = 1 \) amorphous, lattice-layer materials
\( \beta = 2 \) metals & crystalline dielectrics

Flux at λ for a distribution with \( N_a \) particles of each size a:

\[ F_\lambda (\text{obs}) = \sum_{\text{grain distribution}} N_a \left( \frac{\pi a^2}{D^2} \right) Q_{\lambda,a} B_\lambda (T) \]

(Note: same flux can be achieved using different combinations of size distribution and emissivity law!)

Mass determination:

\[ M_{\text{dust}} = \frac{4sF_\lambda D^2}{3B_\lambda(T_{\text{dust}})} \left\{ \frac{a}{Q_\lambda} \right\} \]

Using "appropriate average" of \( a/Q_\lambda \).

\[ M_{\text{dust (true)}} = \sum_{\text{grain & temperature distribution}} \frac{4sF_\lambda D^2}{3B_\lambda(T_{\text{dust}})} \left\{ \frac{a}{Q_{\lambda,a}} \right\} \]

"Wien's Law":

\[ \lambda_{\text{peak}} \approx 3000 \left[ \frac{5}{\beta + 5} \right] T^{-1} \]

Peak flux moves to longer λ for smaller \( \beta \).

\( N = \) number of grains
\( a = \) "typical" grain size
\( D = \) distance from observer
\( T_{\text{dust}} = \) dust temperature
\( Q = \) emissivity
\( s = \) density of grain material