1. Non-LTE Effects on Line Emission

In calculating the emission and absorption coefficients for a two-level electronic transition, we expressed the relation between the populations of the two levels in terms of an excitation temperature, $T_{ex}$. In calculating the relative populations of the two levels involved in a Hydrogen recombination line transition, this excitation temperature can be equated with the electron temperature $T_e$, assuming the H II region is in LTE. However, if LTE does not exist, we must somehow parameterize the deviation from equilibrium if we wish to continue describing the line emission in terms of $T_e$. One way is to introduce corrective factors into the Boltzmann equation:

$$\frac{n_u}{n_l} = \frac{b_u g_u}{b_l g_l} e^{-\hbar \nu / k T_e}$$

where the departure coefficient $b_n$ is defined as the ratio of the actual number density of atoms with electrons in level $n$ to the number that would be expected if the gas were in LTE with temperature $T_e$. (We defined this near-LTE situation as ETE, or "equivalent thermodynamic equilibrium," in class.)

a.) Show that the absorption coefficient for line absorption, $\alpha_L$, can be approximated as:

$$\alpha_L \cong \frac{h^2 \nu^2}{4 \pi k T_e} \phi(\nu) n_u^* B_{lu} \left[ 1 - \frac{d \ln b_u}{dn} \frac{\Delta n}{\hbar \nu} k T_e \right]$$

if $\hbar \nu \ll k T_e$. Here, $n_l$ is the number density of atoms in the lower energy state that would exist in the case of LTE, $\phi(\nu)$ is the line profile function, and $\Delta n$ is the number of energy levels separating $u$ and $l$; assume for a radio recombination line transition that $\Delta n \ll u$

b.) Define the LTE absorption coefficient as

$$\alpha_L^* \equiv \frac{h^2 \nu^2}{4 \pi k T_e} \phi(\nu) n_u^* B_{lu}$$

and the coefficient $\beta$ as:

$$\beta \equiv 1 - \frac{d \ln b_u}{dn} \frac{\Delta n}{\hbar \nu} k T_e$$

Show that the ratio of the peak line intensity to the continuum intensity is given by:

$$\frac{I_L}{I_C} = \left( \frac{\tau_c + b_u \tau_L^*}{\tau_c + b_u \beta \tau_L^*} \right) \left[ 1 - \exp\left(-\tau_c + b_u \beta \tau_L^*\right) \right] \left[ 1 - \exp\left(-\tau_c\right) \right]^{-1}$$

where $\tau_c$ is the continuum optical depth in the absence of line absorption (i.e. the integral of $\alpha_{gf}$ along the line of sight) and where $\tau_L$ is the LTE line optical depth (i.e. the integral of $\alpha_L$ along the line of sight). This equation corresponds to equation (2) in Garay et al. 1994.

c.) Explain (in words) how Garay et al. (1994) estimated the actual (non LTE) electron temperature in the objects they observed. What specific measurements did they have to make? What assumptions? Explain the reasoning behind their statement: “The LTE approximation is not a good assumption for the sources investigated here, particularly for those with the highest emission measures.”
2. H II Region Sizes

In class, using the “Strömgren Sphere” analysis, we calculated the radius of an H II region for a single star embedded in a uniform-density medium. In this problem, you will consider the effects of three stars on three different arrangements of ambient gas. The three stars are of type 05, 08 and B0. The stars can be considered close enough to each other so that the distance between them is small compared to the size of the ionized region they create. Cross sections through the mid-plane of three different (very) hypothetical H II regions are shown below.

For all parts of this problem, \( n_{\text{low}} = 1 \, \text{cm}^{-3} \), and \( n=50 \, \text{cm}^{-3} \), and the composition of the gas can be considered to be pure Hydrogen.
a.) What is the radius of the Strömgren sphere, $R_S$, for case A?

b.) For case B, assume that the filling factor shown, $f$, is achieved by having blobs with radius 0.1 pc, each with density, $n$, embedded in a medium of uniform density, $n_{\text{low}}$. This situation can be considered to extend to infinite radius. Assuming an equilibrium size is quickly achieved after an H II region is formed by the three embedded stars, estimate the size of this H II region. In order to do this problem, you will want to go back to the Strömgren sphere analysis, and see where it needs to be modified in order to accommodate non-uniform ambient material. In your answer, show how the calculations are modified. To do the problem right, a computer simulation might help. Please clearly state any simplifying assumptions you make along the way.

c.) The geometry shown in case C may be most realistic. As shown in the illustration, the high-density material is actually made of what was the “average” material (with filling factor $f$) from case B. And this material only exists over a limited region relatively close to the star. You can approximate its extent as $2R_S$, and its filling factor as 50%. The rest of the region (up to infinite radius) can be considered to have uniform density, $n_{\text{low}}$. Discuss, the following with regard to this scenario:

• How will the geometry of the region change?
• How will the overall volume of ionized gas compare with your answers in a.) and b.)?
• Will the recombination line emission from the region in C be stronger of weaker than in B? Why?

Be sure to state any additional assumptions you make in answering these questions.

For extra credit, give a quantitative extension of your analysis for case B to accommodate the geometry in C.

d.) Comment on the sizes you have estimated in comparison to the sizes of some real, comparable, H II regions. (Some literature/WWW research may be necessary for this part.)

References