

## AY145 Problem Set 4

Due Wednesday, March 9, 2005

### Problem 1. Hydrostatic Equilibrium

- a Show that the equation of hydrostatic equilibrium,

$$\frac{dP}{dr} = -G \frac{M_r \rho}{r^2}, \quad (1)$$

can also be written in terms of the optical depth  $\tau_\nu$ , as

$$\frac{dP}{d\tau_\nu} = \frac{g}{\kappa_\nu}. \quad (2)$$

- b Consider the function

$$f(r) = P(r) + \frac{GM_r^2}{8\pi r^4}. \quad (3)$$

By differentiating  $f(r)$  with respect to  $r$  and using the equation of hydrostatic equilibrium, show that  $f(r)$  decreases outward with increasing  $r$ .

- c Now consider a spherically symmetric star of mass  $M$  and radius  $R$ . Assuming zero pressure at  $R$ , use the result from part (b) to demonstrate that

$$P_c > \frac{GM^2}{8\pi R^4}, \quad (4)$$

where  $P_c$  is the central pressure. Note that you must show that  $M_r^2/r^4$  goes to zero as  $r \rightarrow 0$ . This sets a lower limit on the central pressure of a star.

- d Compute this lower central pressure limit for the Sun and compare it to the standard atmospheric pressure on Earth.

### Problem 2. Gravity and Mass

Attached is a data table for the 20 brightest galaxies in the Local Group. The distances and velocities are the distance from the Milky Way, which for this exercise we will assume is at the center of the group, and the radial velocity difference between the galaxy and the MW.

- a Using the Virial Theorem, estimate the mass of the Local Group (in Solar Units).
- b Compare that to the estimate of the LG mass from the projected mass method (assuming isotropic orbits).
- c Using the absolute magnitudes given, what then is the mass-to-light ratio of the Local Group in Solar units? Remember the absolute V magnitude of the Sun is +4.76, and that luminosities add but magnitudes do not!

**Table 1. Local Group Galaxies**

Name		Coordinates B1950		Type	D(kpc)	$M_V$	$V_o$ (km/s)
M31	NGC 224	00 40.0	+40 59	Sb	725	-21.1	-299
Galaxy		17 42.4	-28 55	Sbc	0	-20.6	0
M33	NGC 598	01 31.1	+30 24	Sc	795	-18.9	-180
LMC		05 24.0	-69 48	Irr	49	-18.1	270
IC 10		00 17.7	+59 01	Irr	820	-17.6	-343
NGC6822	DDO 209	19 42.1	-14 56	Irr	540	-16.4	-49
M32	NGC 221	00 40.0	+40 36	E2	725	-16.4	-205
NGC205		00 37.6	+41 25	E5	725	-16.3	-239
SMC		00 51.0	-73 06	Irr	58	-16.2	163
NGC3109	DDO 236	10 00.8	-25 55	Irr	1260	-15.8	403
NGC185		00 36.2	+48 04	E3	620	-15.3	-208
IC1613	DDO 8	01 02.2	+01 51	Irr	765	-14.9	-236
NGC147	DDO 3	00 30.5	+48 14	E4	589	-14.8	-193
Sextans A	DDO 75	10 08.6	-04 28	Irr	1450	-14.4	325
Sextans B	DDO 70	09 57.4	+05 34	Irr	1300	-14.3	301
WLM	DDO 221	23 59.4	-15 45	Irr	940	-14.0	-116
Sagittarius		18 51.9	-30 30	dE7	24	-14.0	140
Fornax		02 37.8	-34 44	dE3	131	-13.0	53
Pegasus	DDO 216	23 26.1	+14 28	Irr	759	-12.7	-181
And VII	Cas Dw	23 24.1	+50 25	dE3	760	-12.0	-307
Leo I	DDO 74	10 05.8	+12 33	dE3	270	-12.0	285