AY 145: HW 5 Draft

Due Wednesday, March 16th

1. **Stellar Atmospheres** *Carroll & Ostlie #10.11*

   If the temperature of a star's atmosphere is *increasing* outward, what type of spectral lines would you expect to find in the star's spectrum at those wavelengths where the opacity is greatest?

2. **Stellar Structure**

   Consider a star with the following equation of state

$$P = K\rho^2 \tag{1}$$

   where K is a constant.

   - Write down the structure equations for this star and show that the density profile

$$\rho(r) = \rho_c \frac{\sin(\pi r/R)}{(\pi r/R)} \tag{2}$$

     satisfy the equations, where ρ_c is the central density and R is the outer radius of the star.

   - Solve for R in terms of the constants of the problem, and show that your answer is dimensionally correct

   - Show that the total mass of the star satisfies a relation of the form

$$M_{tot} = C_1 \rho_c R^3 \tag{3}$$

     where C_1 is a constant. How does your solution for C_1 differ from that of a constant density star? How does R depend on M_{tot}? Is there anything odd about this result?

3. - **Energy generation** *Carroll & Ostlie #10.11*

     The Q value of a reaction is the amount of energy released (or absorbed) during the reaction. Calculate the Q value for each step of the PP I reaction chain (Eq. 10.46). Express your answers in MeV. The masses of ${}^2_1$H and ${}^3_2$He are 2.0141 u and 3.0160 u, respectively.

   - Estimate the number of PP I reactions that are needed to boil a cup of water (call it 0.25 liters) that is initially at room temperature (say 25 °C). Assume that the energy released in a PP I reaction is 20 MeV. Also, the heat capacity of liquid water is 1 kcal kg^{-1} $°C^{-1}$ = 4,184 J kg^{-1} $°C^{-1}$ and 1 J = 6.2 x 10^{12} MeV.

   - Also, use an order of magnitude calculation to convince yourself of John's statement that a Pedro Martinez fastball has 10^{21} ev of energy.

4. **Energy Generation II** *Carroll & Ostlie #10.10*

Beginning with Eq 10.56, and writing the energy generation rate in the form

$$\epsilon(T) = \epsilon'' T_8^{\alpha} \tag{4}$$

show that the temperature dependence for the triple alpha process, given by Eq. 10.57, is correct. ϵ'' is a function that is independent of temperature.

Hint: First take the natural logarithm of both sides of Eq. 10.56 and then differentiate with respect to $\ln T_8$. Follow the same procedure with your power law form of the equation and compare the results. You may want to make use of the relation

$$\frac{d \ln \epsilon}{d \ln T_8} = \frac{d \ln \epsilon}{\frac{1}{T_8} dT_8} = T_8 \frac{d \ln \epsilon}{dT_8} \tag{5}$$