AY 145: HW 6

Due Wednesday, March 23rd

1. Stars are optically thick objects. We want to find the specific intensity, I_ν, emerging from the star along a path measured by coordinate s. We can measure the distance into the star in terms of the optical depth, τ_ν, where $\tau_\nu = 0$ at the star's surface and increases toward the center of the star. Starting with the equation of radiative transport, you will show that the observed brightness temperature of a star is equal to the temperature at a depth in the star where $\tau_\nu = 1$. Because τ_ν varies with frequency and the temperature profile of most stars changes with radius, the measured blackbody temperature of a star will depend on the frequency of observation.

   - The equation of radiative transport is

     $$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu \tag{1}$$

     wher α_ν is the absorption coefficient and j_ν is the emission coefficient at frequency ν. Show that the equation of radiative transport can be rewritten as

     $$\frac{d}{d\tau_\nu}\left(I_\nu e^{-\tau_\nu}\right) = -S_\nu e^{-\tau_\nu} \tag{2}$$

     where τ_ν is the optical depth, and $S_\nu \equiv j_\nu/\alpha_\nu$ is called the source function. Note, you must take care with the sign of τ_ν because in the case of stellar atmospheres, the opacity is measured *opposite* to the direction of radiation propagation.

   - $S_\nu = a_\nu + b_\nu \tau_\nu$ (the first two terms of the power series) is a good approximation near the surface of the star. Using this approximation and integrating Equation 2, show that

     $$I_\nu = S_\nu\left(\tau_\nu = 1\right) \tag{3}$$

   - Use Equation 3 and the definition of brightness temperature, explain why, for thermal radiation, the brightness tempeature of an observation at frequency ν will be equal to the temperature of the star at the depth where $\tau_\nu = 1$.

   - Does this result have any implications for the apparant angular size of a star as a function of frequency? Include a diagram with your explanation.

2. *Collapse time for gas clouds*

   In class, we derived the time that it takes for a gas cloud to collapse into hydrostatic equilibrium. We called this the free-fall time, τ_{ff}. I skipped many steps in class. In this problem, you will do a complete derivation. Start with the equation for collapse under gravity (assuming that the gas pressure is negligible):

$$\frac{d^2r}{dt^2} = -\frac{GM(r)}{r^2} \tag{4}$$

   and integrate twice to arrive at the gas cloud radius as a function of time. Then solve for the time it takes for the gas cloud to completely collapse. The integrals can and should be done on paper (i.e. don't use Mathematica). The trickiest parts for the integration were to determine the variable substitution, which were included in the lecture.

   Once you've derived the free fall time, compute how long it would take the Orion Molecular Cloud to collapse. Use a density of 10^{-15} kg/m^3 for the OMC density.

3. *Fragmentation of Collapsing Gas Clouds*

   We also discussed that collapsing gas clouds can fragment into several collapsing sub-clouds. An argument of what eventually halts this fragmentation was briefly mentioned in class.

   By following the description of adiabatic collapse on pages 453-455 in Carroll & Ostlie, derive Equation 12.18

$$M_{J_{min}} = 0.03 \left(\frac{T^{1/4}}{e^{1/2}\mu^{9/4}} \right) M_{\odot} \tag{5}$$

   Show all work, explicitly define all variables used, and describe the assumptions that you make. Compute the Jeans' mass when the adiabatic assumption takes effect. Is this consistent with the minimum mass of stars that are observed? Finally, explain in words why a Jeans' mass that increases with density can stop fragmentation.

4. *Degeneracy Pressure and Neutron Drip*

   - At a density just below neutron drip, assume that all of the neutrons are in heavy neutron-rich nuclei such as $^{118}_{36}$Kr. Estimate the pressure due to relativistic degenerate electrons.
   - At a density just above neutron drip, assume (*wrongly!*) that all of the neutrons are free (and not in nuclei). Estimate the speed of the degenerate neutrons and the pressure they would produce.

5. *Neutron Star Rotation*

   Determine the minimum rotation period for a $1.4M_{\odot}$ neutron star (the fastest it can spin without flying apart). For convenience, assume that the star remains spherical with a radius of 10 kilometers.