Working with Magnitudes and Color Indices

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Fluxes add, Magnitudes do not
We've heard this statement before, but what are the implications when computing the color index of a galaxy? I'll review some concepts about magnitudes and introduce what a color index is and why it's a useful quantity. Several worked examples are included for your benefit.

1 A quick review of notation

- m_i is the apparent magnitude of an object measured in band i.
  It is a logarithmic measure of the detected flux from that object, in band i, as seen from the earth.[1]
- M_i is the absolute magnitude of the object, measured in band i.
  It is a logarithmic measure of what the detected flux from that object *would be* if the object were 10 pc away from earth.
- The magnitude scale increases with decreasing brightness. Negative values are brighter than positive values. For example, a 4th magnitude star is brigher than a 6th magnitude star, and a -1 magnitude star is brigher than a 0th magnitude star.

OK, now we can begin.

2 The magnitude of two identical stars

Let's start with a straightforward example that relates the flux and magnitude of a star. Consider a star named Star A. It has an apparent magnitude of +5 ($m_A = +5$). For now we do not specify the band in which the measurement was taken. You can assume that this is the total, or bolometric, magnitude if you like. Now let's take two copies of Star A and make their angular separation on the sky so small that we cannot resolve their separation with our telescope. This means that we will be tricked into thinking that there is only one star

[1] See the appendix for more information about the origins of the magnitude system.

there, let's call it "Star" B, but we will think that the flux of "Star" B is twice that of Star A.

Calculate the measured magnitude, m_B, of "Star" B.

Note, that it is absolutely *incorrect* to say that $m_B = 2 * m_A = +10$. This would mean that "Star" B appeared less bright (by a LOT) than Star A. This is the statement that "Magnitudes do not add." To compute m_B we must turn to the definition of the magnitude scale.[2] It turns out that, *by a definition established in 1856*, a step of 5 in magnitude corresponds to a factor of 100 in flux, F (if magnitude grows, flux shrinks and vice versa). A step of 1 in magnitude corresponds to a factor of $100^{1/5} = 2.512$ in flux. Thus we have

$$\frac{F_A}{F_B} = 100^{(m_B - m_A)/5}. \tag{1}$$

If you internalize this relationship, you can compute any flux or magnitude you'd like. If you add to that the fact that the flux decreases like the square of the distance to the source, you can then relate the flux of an object to its distance from us. We will not cover that here.

So we can use this to compute the magnitude of "Star" B. Let's first rearrange the equation a bit by taking the logarithm of both sides

$$m_B = m_A + 2.5 \log \frac{F_A}{F_B}. \tag{2}$$

Thus, in our example $F_B = 2 * F_A$ and so

$$\begin{aligned} m_B &= m_A + 2.5 \log \frac{F_A}{2F_A} && (3) \\ &= 5.0 + 2.5 \log 0.5 && (4) \\ &= 4.25 && (5) \end{aligned}$$

As expected, the change in magnitude is slighly less than one because the ratio of the fluxes is slightly less than 2.512 (see Appendix A).

3 This also works for absolute magnitudes

The relationship between magnitude and flux applies for absolute and apparent magnitudes. For example, we can solve the following problem:
What would is the V band magnitude of a galaxy that is composed entirely of sun-like stars?

The mass of a galaxy is $\sim 10^{12}$ solar masses.[3] If we assume that the entire galaxy is composed of stars like the sun (let the mass of each star be 1.1 solar masses), then we can say that there would be 10^{12} of these stars. I chose 1.1 solar masses because our "Basic Stellar Data" table lists the absolute V band magnitude of a 1.1 solar mass star. If we call this typical star, "Star A", then we know that the V band absolute magnitude of star A is: $\mathrm{M}_{A_V} = 4.4$. So we can use the relationship between flux and magnitudes to determine the V band absolute magnitude of the entire galaxy.

$$M_{gal_V} = M_{A_V} + 2.5 \log \frac{F_{A_V}}{F_{gal_V}} \tag{6}$$

[2]See Appendix A for the details of the origin of the magnitude scale.

[3]I'm avoiding the use of M_{sun} here because I don't want to confuse the symbol for mass with the symbol for magnitude. Consequently, I will always spell out the word "mass" and reserve the symbol, M, for magnitudes, until I reach Section 8.

$$
\begin{aligned}
&= M_{A_V} + 2.5 \log \frac{F_{A_V}}{10^{12} F_{A_V}} && (7)\\
&= 4.4 - 30 && (8)\\
&= -25.6 && (9)
\end{aligned}
$$

4 What if the galaxy has 2 types of stars?

Let's invent a new, 10^{12} solar mass galaxy. This one has two types of stars: 1.1 solar mass stars and 60 solar mass stars. There are 10^{11} of the lighter stars (call these Star A) and the rest of the galaxy mass is in the heavier stars (call these Star B). We can then ask:

What is the absolute V band magnitude of the galaxy?

To solve this we must know the absolute magnitude of a 1.1 solar mass star and that of a 60 solar mass star (look this up in our "Basic Stellar Data" sheet) and we must also determine how many 60 solar mass stars there are in the galaxy.

The data sheet tells us that $M_{A_V} = 4.4$ and $M_{B_V} = -5.7$. This means that the ratio of the V band flux from Star B to the V band flux from Star A is:

$$
\begin{aligned}
\frac{F_{B_V}}{F_{A_V}} &= 100^{(M_{A_V} - M_{B_V})/5} && (10)\\
&= 100^{(4.4+5.7)/5} && (11)\\
&\sim 11,000 && (12)
\end{aligned}
$$

Our hypothetical galaxy still has a mass of 10^{12} solar masses. Of that mass, 1.1 x 10^{11} solar masses are accounted for by the 1.1 solar mass stars. That leaves $10^{12} - (1.1 \text{ x } 10^{11}) = 10^{12} * (1 - 0.11) = 0.89 \text{ x } 10^{12}$ solar masses that must be in the 60 solar mass stars. That means there are $(0.89 \text{ x } 10^{12})/60 \sim 1.5 \text{ x } 10^{10}$ such stars. Thus the total V band flux from the galaxy would be:

$$
\begin{aligned}
F_{gal_V} &= 10^{11} * F_{A_V} + 1.5 \text{ x } 10^{10} * F_{B_V} && (13)\\
&= 10^{11} * F_{A_V} + 1.5 \text{ x } 10^{10} * (11,000 * F_{A_V}) && (14)\\
&= 1.651 \text{ x } 10^{14} * F_{A_V} && (15)
\end{aligned}
$$

Where we can see that the the flux of the galaxy is *entirely dominated* by the flux from the 60 solar mass stars. Note that the contribution from the 1.1 solar mass stars to the total galactic flux enters only in the third decimal place (which we probably should have dropped anyway). Now that we know the total flux from the galaxy, we can find the absolute V band magnitude, M_{gal_V}, of the galaxy.

$$
\begin{aligned}
M_{gal_V} &= M_{A_V} + 2.5 \log \frac{F_{A_V}}{F_{gal_V}} && (16)\\
&= 4.4 + 2.5 \log \frac{1}{1.651 \text{ x } 10^{14}} && (17)\\
&= -31.1 && (18)
\end{aligned}
$$

and we're done! It is worthwhile to note that, as constructed, our ficticious galaxy is brighter than real galaxies that we observe because of the large number of very massive (and therefore very luminous) stars. The overall galactic flux is then larger than what is typically observed. The brightest galaxies in nearby (z$\sim$0.05-0.1) Abell clusters have $M_V \sim -23$.[4]

[4]See Hoessel, Gunn, and Thuan, "The Photometric Properties of Brightest Cluster Galax-

5 Measured Magnitude Depends on Wavelength

Because stars are approximately black body emitters, they output electromagnetic radiation continuously over a huge range of wavelengths. Our eyes are sensitive to a small fraction of these wavelengths (say 400-700 nanometers). Thus, if you look at a star, you are only measuring the brightness or magnitude of that star *in a small waveband* (the visible waveband in this case). Because the intensity of a blackbody emitter is a strong function of wavelength, the measured magnitudes in each waveband will certainly not be the same. Thus, when quoting the magnitude of a star, you *must* specify the waveband of the observation.

It is no different when modern day telescopes are used at wavelengths that our eyes cannot detect. Infrared detectors will measure the flux centered at e.g. 1.2, 1.6 and 2.2 microns (J, H and K bands, with bandwidths of 0.24, 0.30 and 0.41 μm, respectively). Likewise, x-ray telescopes measure flux at e.g. 1.24-3.72 Å (10-3.34 keV), 3.72-8.0 Å (3.34-1.55 keV) and 8.0-41 Å (1.55-0.3 keV) where we have used $\lambda(\text{Å}) = 12.42/E(\text{keV})$.

The standard wavebands for optical astronomy are U, B, and V and all are approximately 100 nanometers wide.[5] U stands for ultraviolet and is centered at 365 nm, B stands for blue and is centered at 440 nm and V stands for visual and is centered at 550 nm.

It turns out that the difference between magnitudes in a pair of wavebands can tell you about the temperature, or color, of the star. This difference is referred to as the color index.

6 Color Index ($B - V$)

One way to classify stars or galaxies is by the ratio of the flux at one wavelength to the flux at another wavelength. Thinking back to the Planck curve for a black body emitter (a fairly good model for a star), we can see that this ratio is a strong function of temperature. Thus by measuring the ratio of fluxes, you can learn about the temperature of the star or galaxy (there are empirical correction terms to account for the non-ideal black body behaviour of stars and galaxies).

In Figure 1 we see how $B - V$ changes with temperature. Hot objects have more B band flux than V band flux (see Section 5 for a description of UBV bands in astronomy). This means that the $B - V$ color index will be negative. To see why, we need the following definition

$$B - V \equiv \mathrm{m}_B - \mathrm{m}_V = \mathrm{M}_B - \mathrm{M}_V = 2.5 \log \frac{F_V}{F_B} \tag{19}$$

where we have only used the flux-magnitude relation (see Section 2 and Appendix A). For a perfect black body, it can be shown that

$$B - V = -0.72 + \frac{7090}{T} \tag{20}$$

where T is the temperature of the black body, in Kelvin. This can be reformu-

ies. I. Absolute Magnitudes in 116 Nearby Abell Clusters," ApJ, **241**, 486-492, October 15, 1980. Available online at:
http://cfa-www.harvard.edu/~jbattat/references/absMagsOfAbellClusterGals.pdf

[5] The Sloan Digital Sky Survey used a new set of bands called ugriz and this has become a new standard. For more info on SDSS bands, you can see: Fukugita et al. "The Sloan Digital Sky Survey Photometric System," AJ, Vol 111 Number 4, April 1996. Or at:
http://cfa-www.harvard.edu/~jbattat/references/sdssPhotmetricSystem.pdf

lated as:

$$T = \frac{7090}{(B-V)+0.71}. \tag{21}$$

Because stars are not ideal black bodies (for example, they can have emission and absorption lines), there is a slightly modified version of the temperature-color index relationship, determined empirically

$$B - V = -0.865 + \frac{8540}{T} \tag{22}$$

which is valid for $4,000 < T < 10,000$. Again, this can be written in another way as:

$$T = \frac{8540}{(B-V)+0.865} \tag{23}$$

So we can now answer the following questions:

Given that a 2 solar mass star has $M_V = 1.95$ and $M_B = 2.10$ (as seen in the "Basic Stellar Data" sheet), find:
a) $B - V$
b) F_B/F_V
c) T (in Kelvin, for ideal black body and actual star)

a) We can find the $B - V$ color index very simply from:

$$B - V = \mathrm{M}_B - \mathrm{M}_V = 2.10 - 1.95 = 0.15 \tag{24}$$

b) The flux ratio can be found from Equation 1:

$$\frac{F_B}{F_V} = 100^{(\mathrm{M}_V - \mathrm{M}_B)/5} = 0.871 \tag{25}$$

c) For an *ideal* black body

$$T_{ideal} = \frac{7090}{0.15+0.7} = 8,341 \text{ K} \tag{26}$$

And from the empirical relationship:

$$T_{empirical} = \frac{8540}{0.15+0.865} = 8,413 \text{ K} \tag{27}$$

which is close enough to the value in the "Basic Stellar Data" sheet (8,200 K).

7 The Color Index of Galaxies

We are now prepared to compute the $B - V$ magnitude of a galaxy. We must combine our ability to compute the $B-V$ color index, given the absolute B and V band magnitudes, with our ability to compute the total absolute magnitude of a galaxy.

First, we note that the color index of a galaxy made up of lots of the same kind of star is simply the color index of the single star. That is, if you group

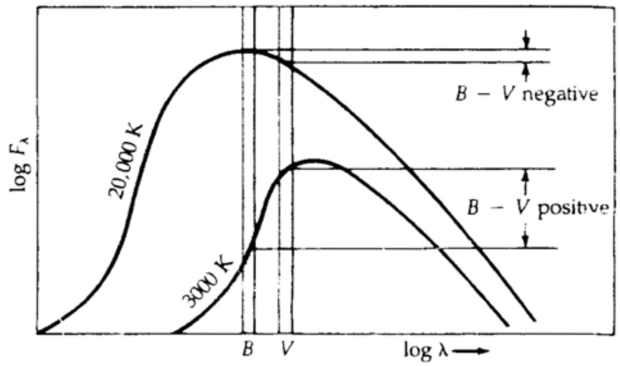

Figure 1: Two black body curves are shown, with $T_1 > T_2$. The hotter object will have a $B - V$ color index that is smaller (more negative) than the cooler object.

together a billion copies of a single star, the resultant color index will be unchanged. To see why this is true, consider the following: The $B - V$ color is a function of the ratio of fluxes, F_V/F_B. If you increase the number of stars from 1 to N, then the ratio of the fluxes is unchanged: $(NF_V)/(NF_B) = F_V/F_B$. Since the color index only depends on the ratio of the fluxes, and the ratio of the fluxes has not changed, then the color index must also still be the same.

Galaxies, however, are collections of many different star types. To begin to understand how to compute the color index of a galaxy, let's explore the case of the hypothetical galaxy from Section 4 and ask the question:

What is the $B - V$ color of the hypothetical galaxy?

If the B band magnitude of Star A is, $M_{A_B} = M_{A_V} + (B - V) = 4.98$ and the B band magnitude of Star B is, $M_{B_B} = M_{B_V} + (B - V) = -6.03$ then we can compute the ratio of the B band flux in Star B to the B band flux in Star A

$$\frac{F_{B_B}}{F_{A_B}} = 100^{(M_{A_B} - M_{B_B})/5} \tag{28}$$

$$= 100^{(4.98+6.03)/5} \tag{29}$$

$$\sim 25,000 \tag{30}$$

so the blue band flux is 25,000 times greater in Star B than it is in Star A. And note that this ratio is larger than it was in V band (see Equation 12).

Now, the total B band flux of the galaxy is given by:

$$F_{gal_B} = 10^{11} * F_{A_B} + 1.5 \text{ x } 10^{10} * F_{B_B} \tag{31}$$

$$= 10^{11} * F_{A_B} + 1.5 \text{ x } 10^{10} * (25,000 * F_{A_B}) \tag{32}$$

$$= 3.751 \text{ x } 10^{14} * F_{A_B} \tag{33}$$

again, the B band flux is dominated by the 60 solar mass stars with the contribution from the 1.1 solar mass stars coming in the third decimal place.

We can now compute the total absolute B band magnitude of the galaxy:

$$M_{gal_B} = M_{A_B} + 2.5 \log \frac{F_{A_B}}{F_{gal_B}} \tag{34}$$

$$= 4.98 + 2.5 \log \frac{1}{3.751 \text{ x } 10^{14}} \tag{35}$$

$$= -31.5 \tag{36}$$

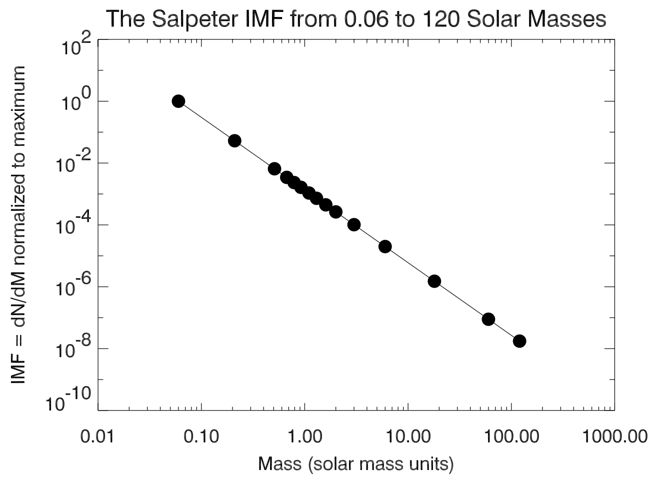

Figure 2: Salpeter IMF evaluated where we have B and V band magnitude information on the "Basic Stellar Data" sheet (i.e. for stellar masses between 0.06 and 120 M_{sun}. Note that both axes are logarithmic and the y-axis spans 8 orders of magnitude.

And we see that the galaxy is brighter in B band that it is in V band (see Equation 16 for the computation of the V band absolute magnitude). Thus we expect the $B - V$ color index to be negative, which it is.

$$(B - V)_{gal} = M_{gal_B} - M_{gal_V} = -31.5 + 31.1 = -0.4 \tag{37}$$

8 Color Index and Initial Mass Functions

Real galaxies do not only have two types of stars. Instead, stars form with a continuous distribution of masses. I will now use M to refer to the mass of the star (*not* the absolute magnitude). The initial mass function (IMF) quantifies the number of stars that form at each mass. It is assumed to have a power law shape, and is written as:

$$\text{IMF} = \frac{dN}{dM} = CM^{-(1+x)} \tag{38}$$

In this form it is clear that the IMF is the number of stars (dN) that have mass in the range M to $M + dM$. There is some unknown leading constant C and everyone's got their own favorite value of x. A popular version, called the Salpeter IMF, has $x = 1.35$, which gives $dN/dM \propto M^{-2.35}$. This fits observations in our solar neighborhood. A plot of the Salpeter IMF, normalized to its maximum value, is shown in Figure 2. The data points are plotted where we have absolute B and V magnitude data in the "Basic Stellar Data" sheet. Note, both axes are logarithmic and so the power law IMF is displayed as a straight line with slope $-(1 + x)$ (which is -2.35 in this case for the Salpeter choice of $x = 1.35$).

So how might you compute the $B - V$ index of a galaxy with a distribution of stars defined by the IMF? Well, you've already done this for an IMF that had only two values (1.1 and 60 M_{sun}). The only difference when moving to a power law IMF is that you have more terms in your sum when computing the total flux in each band (see for example, Equation 31).

In general, F_{gal_y}, which is the total flux, in some waveband, y (e.g. B or V), of a population of stars in a galaxy, is given by:

$$F_{gal_y} = \sum_i N_{stars}(M_i) F_y(M_i) \tag{39}$$

where $N_{stars}(M_i)$ is the number of stars with masses that fall into mass bin i, and $F_y(M_i)$ is the y band flux of a single star that has mass in the mass bin i and the sum is over all mass bins (for which you have absolute B and V band magnitude data, which in our case, if you look at the "Basic Stellar Data" sheet, is 0.06 to 120 solar masses).

The number of stars in each mass bin, ΔM can be found by integrating the IMF across each bin. Thus the total galactic flux in band y can be rewritten as:

$$F_{gal_y} = \sum_i \left[\int_{M_i}^{M_i+\Delta M} CM^{-(1+x)} \right] F_y(M_i) \tag{40}$$

For example, given a Salpeter IMF ($x = 1.35$), we can compute the number of stars that have mass between 10 and 10.1 solar masses ($\Delta M = 0.1M_{sun}$) by:

$$\begin{aligned} N_{stars}(10M_{sun} - 10.1M_{sun}) &= \int_{10M_{sun}}^{10.1M_{sun}} dN(M_i) & (41)\\ &= \int_{10M_{sun}}^{10.1M_{sun}} \frac{dN}{dM} dM & (42)\\ &= \int_{10M_{sun}}^{10.1M_{sun}} CM^{-2.35} dM & (43)\\ &= \frac{C}{1.35} M^{-1.35}\Big|_{10.1M_{sun}}^{10M_{sun}} & (44)\\ &= 0.000441 \frac{C}{M_{sun}^{1.35}} & (45) \end{aligned}$$

Despite the fact that you do not know the scaling factor, C, of the IMF, you can still compute the color index of the galaxy because that color index depends on the ratio of the total flux B to the total flux in V, both of which are proportional to C and so C will cancel out.

You must be careful, however, in how you choose to break up your range of masses into bins. It makes sense to select bins of equal width, but should they be equal width in linear space or logarithmic space?

9 Many more low mass than high mass stars

The IMF can be integrated to see how the number of stars in a fixed mass range changes with mass. For example, if we want to know how many stars there are in the mass range from $A * M_{sun}$ to $B * M_{sun}$ (where $B > A$), we can do the following integration

$$\begin{aligned} N_{stars}(AM_{sun} \text{ to } BM_{sun}) &= \int_{AM_{sun}}^{BM_{sun}} \frac{dN}{dM} dM & (46)\\ &= \int_{AM_{sun}}^{BM_{sun}} CM^{-(1+x)} dM & (47) \end{aligned}$$

$$= \frac{C}{xM_{sun}^x}\left[\frac{A^x - B^x}{(AB)^x}\right] \quad (48)$$

$$= \frac{C}{xM_{sun}^x}\gamma \quad (49)$$

where $\gamma \equiv \left[\frac{A^x - B^x}{(AB)^x}\right]$.

Let's then pick a fixed mass range (in linear space), say 10 solar masses, and compare the number of stars with masses from 0.06-10 M_{sun} to the number of stars with masses from 110-120 M_{sun} (for a Salpeter IMF, so $x = 1.35$).

Mass Range (M_{sun})	γ
0.06 - 10	44.6
110 - 120	0.00019

and we see that the ratio of stars in the lower mass range to the number of stars in the upper mass range is 235,000 (when taking the ratio, the factor of $C/(xM_{sun}^x)$ cancels). In other words there are 2.35 x 10^5 *times* more low mass stars than high mass stars. We can now see why the computed absolute magnitudes were so bright in our hypothetical galaxy (see Section 4).

10 Color Index changes with time

As the individual stars evolve (age) in the galaxy, the color index of that galaxy will also evolve. The primary reason for this is that the rate of evolution depends on the mass of the star, as does the color index. Thus the high mass stars, which tend to pull the $B-V$ index negative because of their high Planck temperatures, also are the first to run out of Hydrogen in their core. They are the first ones to evolve off of the main sequence (remember that the time a star spends on the main sequence, t_{MS}, often referred to as its lifetime, is proportional to the ratio of the mass to the luminosity, but the luminosity for main sequence stars is empirically found to be proportional to the mass to the 3.5 power, thus you have $t_{MS} \propto M/L \propto M/M^{3.5} \propto M^{-2.5}$ and thus high mass stars evolve off of the main sequence much faster than low mass stars). For example, as we can see in our "Basic Stellar Data" sheet, the main sequence lifetime of a 120 M_{sun} star is only 1 million years, whereas the main sequence liftetime of a 0.21 M_{sun} star is 3 *billion* years. Thus to a first approximation, you can consider that the high mass stars will be gone very rapidly.

If, in our hypothetical galaxy, we snap our fingers and form all the stars at the same time, and the mass distribution of those stars is given by the Salpeter IMF, then we can ask how the $B-V$ color index will change with time. Specifically, we can ask what $B-V$ will be in 5 billion years. According to the "Basic Stellar Data" sheet, by that time all stars with $M \gtrsim 1.3M_{sun}$ have evolved off of the main sequence, leaving only stars of mass $\lesssim 1.1M_{sun}$. With this knowledge, you can then repeat the exercise of finding the $B-V$ colors in exactly the same way as you did for the full IMF, except now you should only carry your integration up to $M \lesssim 1.1M_{sun}$. In other words, we will completely ignore the contribution to the B and V band magnitude from stars that have evolved off of the main sequence. This is a fine assumption. Remember that in Section 4 the $1.1M_{sun}$ stars (with $T \sim 6,000$ K) made a nearly negligible contribution to the total flux as compared with the $60M_{sun}$ stars? Well, the contribution by Red Giants (with $T \sim 3,000 - 5,000$ K) would be even more meager, so save your time and ignore them! We also assumed that the color index of the lower mass stars does not change over this time. Clearly the reality lies somewhere in-between, but you should start simple and elaborate later.

Appendix

A History of the Magnitude scale

Like most astronomical units or measures, the magnitude scale seems very strange at first. Larger numbers are assigned to dimmer stars, and it is not a linear scale (it's logarithmic). However, if one takes a moment to understand the origin of the beast, it becomes much more tame, and may even make quite a bit of sense. Here we go…

Imagine that you are shuttled back in time 2,000 years. And imagine that your task is to catalog the brightness of stars in the night sky. First of all, you'd see many more stars because you would not have light pollution from nearby cities, nor would you have smoggy, polluted skies. Anyway, you'd like to classify the stars by how bright they appear to you. Of course, your only tool is your naked eye. You do not have a telescope, nor do you have a camera with which to take images and compare stars. You certainly do not have a CCD attached to a computer to extract star brightnesses from digital photographs. But your eye is a remarkably valuable and sensitive tool, so all hope is not lost. You thus decide to continue on with your star brightness classification task. This is exactly what Hipparcos (120 B.C.) and Ptolemy (A.D. 140) did. They determined that they could group stars into 6 categories from brightest to dimmest. The brightest stars were called *first magnitude* (as in they win first place in the brightness contest) and the dimmest stars visible (to the naked eye) were *sixth magnitude*. This is a perfectly logical arrangement.

Then Galileo messed it all up. With the use of his new telescopes, he was able to see stars that were dimmer than those visible with the naked eye; they were dimmer than magnitude 6. But no category existed for such stars, so in his 1610 tract, *Sidereus Nuncius* he created one: magnitude 7, thereby opening the door for the existence of innumerable magnitude bins to which stars could be assigned (mag 8, 9, 10, ... as dimmer and dimmer stars were to be found).

Much later, in the mid-19th century, William Herschel used even better telescopes to make more precise measurements of the relative brightnesses of stars and determined that first magnitude stars were approximately 100 times brighter than sixth magnitude stars. This led Norman R. Pogson to suggest in 1856 that a difference of five magnitudes be *defined* to be a factor of *exactly* 100 in brightness. This meant that each step of one in magnitude corresponded to a factor of $100^{1/5} = 2.512$ increase in flux and was aptly named the Pogson ratio (big ups… better to be the namesake of a ratio than of a syndrome, I suppose). Thus a quantified magnitude scale was established in which the relationship between the fluxes, F, and magnitudes, m, of two objects (A and B) is given by:

$$\frac{F_A}{F_B} = 100^{(m_B - m_A)/5} \tag{50}$$

A final item of note is that current lists of the brightest stars visible in the sky have entries with negative apparent magnitudes. If the brightest stars used to have magnitude one, how did this happen? Well, the naked eye observations from 2,000 years ago, while commendable, were somewhat inaccurate (go figure). With good telescopes (in the 19th century) it became clear that the stars categorized as *first magnitude* actually exhibited a large range of brightnesses. The magnitude scale had to be extended once again, this time in the brighter direction (meaning toward smaller magnitudes). So magnitude zero stars were introduced. In fact negative magnitude stars exist. Of course, star magnitudes need not be integer values. They are simply computed from Equation 50. So, for example, Sirius, the brightest star in the sky apart from the sun, is magnitude -1.46, Arcturus is magnitude -0.04, Rigel is magnitude +0.12 and Deneb is magnitude +1.25. Our sun, which is also a star, appears exceptionally bright, due to its proximity to the earth. It's apparent magnitude, therefore, is very negative and is approximately -26.72. That means that the detected flux from the sun is larger than the flux from Sirius by a huge but calculable margin:

$$\frac{F_{sun}}{F_{Sirius}} = 100^{(-1.46+26.72)/5} = 1.27 \text{ x } 10^{10}. \tag{51}$$

In other words, the sun appears to be almost 13 billion times brighter than Sirius. Yowza.

By the way, with a pair of 50-millimeter binoculars, you can see stars down to 9th magnitude. A 6-inch amateur telescope will reach 13th magnitude and the Hubble Space Telescope can see objects as faint as 30th magnitude. Good god, man.