Ay 192 — FINAL EXAM — Fall 1999

Solutions

Problem 1. IR Detectors. (a) Calculate the limiting **magnitude** that you would reach at 2.2 μ in a 0.1 μ band, using an infrared detector system based on an InSb detector array on the 6.5-m MMT for an integration time of 100 seconds. For this problem, define 'limit' as reaching a Signal-to-Noise = 3. First do this assuming there is no sky background. Justify your assumptions.

(b) What is the background sky flux (ergs/cm^2/s) in a diffraction limited beam with the above bandpass and wavelength assuming that the sky is a 300 K blackbody with an emissivity of 10 percent? Hint: first figure out the size of a diffraction limited beam in solid angle, remember that surface brightness is conserved.

(c) Now what is the limiting magnitude you'll reach?

Answer: (a) The first thing to do is to characterize the detector. From a variety of sources (including some of the class discussion, the NOAO web page, etc.) a modern InSb array (e.g. Hughes SBRC Aladdins) has 27μ pixels, an RQE at 2.2 microns of nearly 85%, a read noise of 15 e$^-$ (for multiple reads) and a dark current of about 1 e$^-$/sec/pix. Note that the existing IR camera, PISCES, is an HgCdTe array with different characteristics (lower QE, etc.)

Now lets assume were going to do this right and observe with an InSb camera at the f/15 focus with a little bit of reimaging. The diffraction limit of the MMT at 2.2 microns is 1.22λ/D = 0.077". At f/15, a 27μ pixel is about 0.06", so it doesn't take a lot of reimaging to get 0.04" pixels to properly sample the diffraction limit. As an aside, with tip/tilt plus the naturally good seeing at 2 microns (0.3-0.5" at Mt. Hopkins), it should be possible to get near DL imaging fairly often. This then says that the source will fall on n = 4 pixels.

Now, assuming that the sky doesn't contribute, and assuming we can average the dark and read noise over a large area, we can write, with O = object, D = dark current, R = read noise, and n = the number of pixels:

$$\mathrm{S/N} \;=\; \frac{Ot}{\left[Ot + nDt + nR^2\right]^{1/2}} \;=\; 3$$

and for t = 100 seconds

$$10^4 \times O^2 \;=\; 9 \times [100O + 400 + 900]$$

which is a quadratic equation in O,

$$O^2 \;-\; 0.09O - 1.17 \;=\; 0.0$$

with solution

$$O \;=\; \frac{0.09 + \sqrt{0.0081 + 4.68}}{2}$$

$$= 1.13 \text{ photons per second}$$

Since our IR detector is pretty good, InSb has a QE $\sim$ 85%, the MMT mirrors, dewar and filter throughputs, etc. are $(0.98)^4(0.85)$, and we are neglecting atmospheric absorption, this limit must be raised by about 1/0.67 or

$$O \;\sim\; 1.68 \text{ counts per second}$$

From the calibration data handed out, at 2.2μ, 0th magnitude $= 4.2\times10^5$ photons/sec/cm$^2/\mu$. The MMT collecting area is $\pi/4\times$ [(650 cm)^2 - (50 cm)^2] = 3.3×10^5 cm^2 (the f/15 chopping secondary will be less than 20" across so the central obscuration is small). The bandwidth is approximately 0.1μ, so 0th magnitude at the MMT results in:

$$\text{0th magnitude at K at MMT} \;=\; 1.39 \times 10^{10} \text{ photons per sec}$$

so the limiting magnitude is

$$m_{lim} \;=\; -2.5\log\left[\frac{1.68}{1.39\times10^{10}}\right] \;=\; 24.8$$

Of course, S/N = 3 is a bit marginal unless you already know there's a source there, but this gives you an idea of the power the the 6.5-m and a good IR detector!

(b) To estimate the background, we need to calculate the incident flux through a diffraction limited beam from a 300°K blackbody sky.

$$P_\lambda \;=\; \frac{1}{\pi} F_\lambda \epsilon (A\Omega) T \;\Delta\lambda$$

where

$$\epsilon \;=\; \text{emissivity} \;\sim\; 0.1$$

$$\Delta\lambda \;= \text{bandpass} \;=\; 0.1\mu$$

$$T \;=\; \text{throughput} = 0.67 \text{ (from above),} \qquad \text{and}$$

$$(A\Omega) \;=\; \text{etendue for DLB} \;=\; 0.61\pi\lambda^2$$

From the Planck blackbody law

$$F_\lambda \;=\; F(2.2\mu, 300^\circ K) \;=\; \frac{2hc^2}{\lambda^5}\,\frac{1}{(e^{hc/\lambda KT}-1)}$$

$$= 2.315\times 10^{13}\,\frac{1}{3\times 10^9} \;=\; 7.67\times 10^3 \text{ergs/cm}^2\text{/sec/sterr/cm}$$

and

$$A\Omega \;=\; 0.61\pi(2.2\times 10^{-4})^2 \;=\; 9.28\times 10^{-8}\ \text{cm}^2\text{sterr}$$

thus

$$P_\lambda \;=\; \frac{1}{\pi}(7.7\times 10^3)(9.3\times 10^{-8})(0.1)(0.1\times 10^{-4})(0.67)$$

$$= \;1.53\times 10^{-10}\ \text{ergs/sec}$$

The collecting area of the telescope is already accounted for in the calculation of the etendue. (Here's one to think about: the flux from some uniform source like the sky in a diffraction limited beam is *independent* of the telescope aperture, all other things being fixed. Thats because the collecting area goes up as D^2 but the solid angle of the DLB goes down as D^2, so $A\Omega$, the etendue, is *constant.*)

Can you convert this into an equivalent magnitude? A 2.2μ photon has an energy of 9.0×10^{-13} ergs. The sky then produces about 170 photons per sec in a DLB on the 6.5-m. This photon rate can be converted into approximately 19th magnitude. (Note: If the aperture used normally for photometry is a 1" square instead of the 0.08" DLB, then the sky would be 5.5 magnitudes brighter, so our crude estimate of the thermal "sky" at K band corresponds to about 15th (!) magnitude per square arcsecond — one good reason for trying to get very small diffraction limited apertures in the IR.

In parallel to part (a), we have to add the Sky$\times$t term to the S/N equation.

$$\text{S/N} \;=\; \frac{Ot}{[Ot+St+nDt+nR^2]^{1/2}} \;=\; 3$$

and for $t = 100$ seconds

$$10^4\times O^2 \;=\; 9\times[100O+17000+400+900]$$

which is a quadratic equation in O,

$$O^2 \;-\; 0.09O - 16.47 \;=\; 0.0$$

with solution

$$O = \frac{0.09 + \sqrt{0.0081 + 65.9}}{2}$$

$$= 4.10 \text{ photons per second}$$

Applying the 1/0.67 factor for throughput, the limiting magnitude if there is a sky background is then

$$m_{lim} = -2.5\log\left[\frac{6.12}{1.39 \times 10^{10}}\right] = 23.4$$

So the sky loses us about 1.5 magnitudes of sensitivity at the detection limit even in a diffraction limited beam.

Note that in a real telescope, the telescope itself emits at 2 microns, and it does so with an emissivity near 1.0. The only way to beat this is to very carefully baffle your IR system so something cold inside the dewar is along the line of sight from the detector to any part of the telescope (like the vanes that hold up the secondary).

Problem 2. Sample Comparison/Parameter Estimation. (a) I've made available two tables conta ining two samples of numbers (ftp anonymously to fang-ftp.harvard.edu, cd /pub/class, get e.dat, get s.dat or go to the class website, http://cfa-www.harvard.edu/~huchra/ay192, and look in PROBLEMS). Test their consistency (ie. whether or not they represent samples that could have been drawn from the same parent populations using both the Rank-Sum Test and the K-S test. These are the redshifts of the early and late type galaxies in the first CfA slice.

(b) Calculate a crude luminosity function (log N_{gal}/Mpc^3/magnitude interval) for the combined sample using just the binning technique discussed in class. Remember to calculate the proper sample area and selection function and don't worry about galactic extinction.

Answer: (b) The first thing you have to do is decide what properties you are going to test. For each galaxy there are 4 properties listed, RA, DEC, apparent magnitude and velocity. The most ovbious choice to test is velocity, the second most obvious is magnitude. There are several ways to testing the consistency of the distributions, but I've asked you to use the K-S and Rank-Sum tests.

The E galaxy sample has 457 objects, the S 613. The results I get are:
For Magnitudes:

```
SAMPLE 1 RANK SUM=328096          SAMPLE 2 RANK SUM=244889
EXPECT 1 RANK SUM=328261.5        EXPECT 2 RANK SUM=244723.5
```

```
    VARIANCE =   5000.3
SET 1 DIFFERS FROM SET 2 BY :    -0.033  SIGMA
```

Max difference in cumulative distribution $D_n = 0.042$ at mag 14.1. More than 20% of random samples this size that are the same will have differences that large, so the two magnitude distributions are the same as judged by both statistics.
For velocities:

```
SAMPLE 1 RANK SUM=259563          SAMPLE 2 RANK SUM=313422
EXPECT 1 RANK SUM=244723.5        EXPECT 2 RANK SUM=328261.5
    VARIANCE =   5000.3
SET 1 DIFFERS FROM SET 2 BY :     2.968  SIGMA
```

Max difference in cumulative distribution $D_n = 0.171$ around 5700 km/s. Much less than 1% of random samples will have differences this great, so both tests clearly show that the vleocity distributions are **not** drawn from the same popularion. The attached figures show the cumulative distributions in magnitude and in velocity.

(b) The luminosity function I calculate via binning is relatively straightforward. I assume H=75 km/s/Mpc. Count the galaxies in 1/2 magnitude bins and divide by the mean limiting distance for that bin to get #/unit volume. You must get the solid angle right, which for the 1st CfA strip is $(135 \times \cos(30°) \times 6°) = 701$ sq deg, or about 1/59th of the sky. By definition, the bins with less than 5 or so galaxies are very uncertain. My computation (using the most recent velocities, so it might be slightly different than that in the dataset) is given below and in the attached figure. There are several published versions that estimate the LF using a variety of different techniques that you could have checked your result against (de Lapparent etal 1988 ApJ 332, 44, but with a Hubble constant of 100.)

BIN	NGAL	PHIBIN	Log Phi	ERRORS	
-11.5 TO -12.0	1	0.125E+01	0.099	0.30	1.00
-12.0 TO -12.5	0	0.000E+00	0.000	0.00	0.00
-12.5 TO -13.0	1	0.315E+00	-0.501	0.30	1.00
-13.0 TO -13.5	0	0.000E+00	0.000	0.00	0.00
-13.5 TO -14.0	3	0.238E+00	-0.624	0.20	0.37
-14.0 TO -14.5	3	0.119E+00	-0.924	0.20	0.37
-14.5 TO -15.0	2	0.398E-01	-1.400	0.23	0.53
-15.0 TO -15.5	5	0.498E-01	-1.302	0.16	0.26
-15.5 TO -16.0	6	0.300E-01	-1.523	0.15	0.23

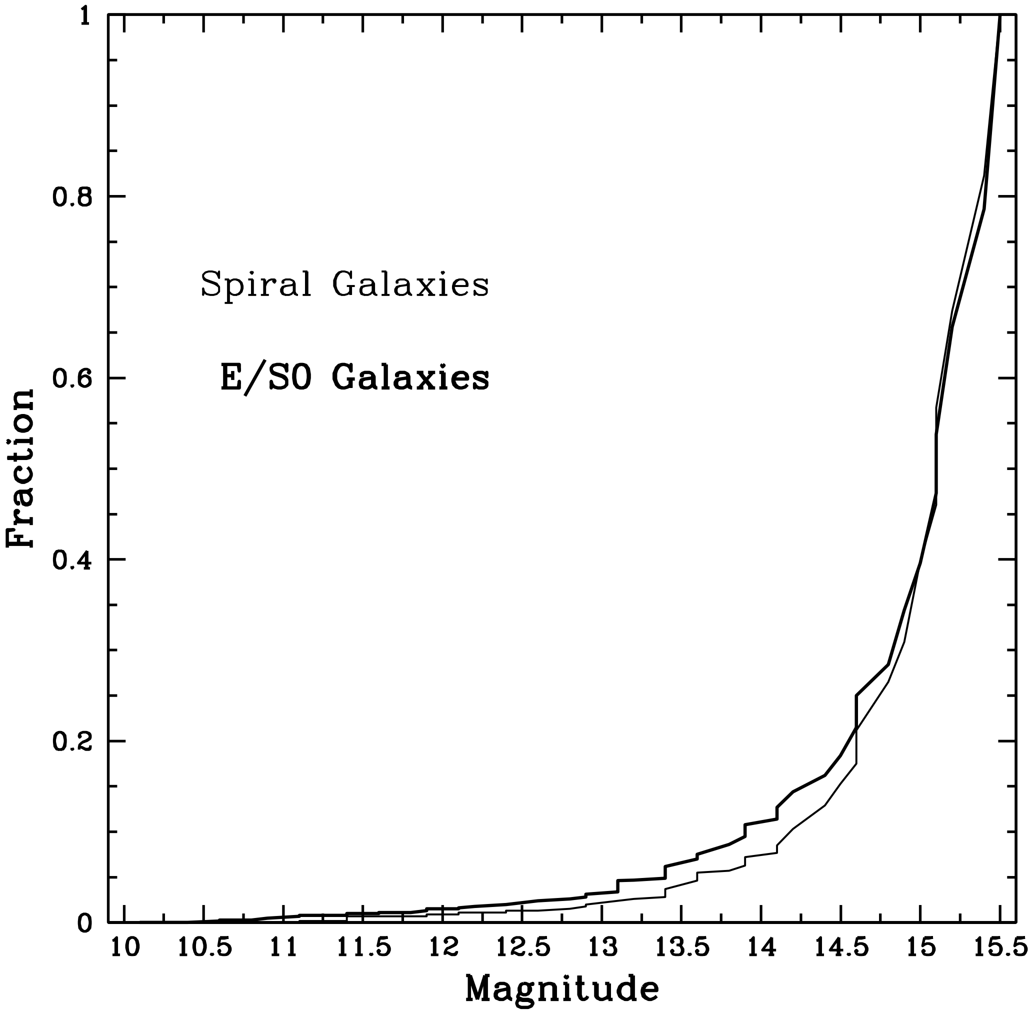

Figure 1: K-S test results for magnitudes. Plot shows the cumulative distributions (fractions) as a function of magnitude for early type galaxies versus late type galaxies.

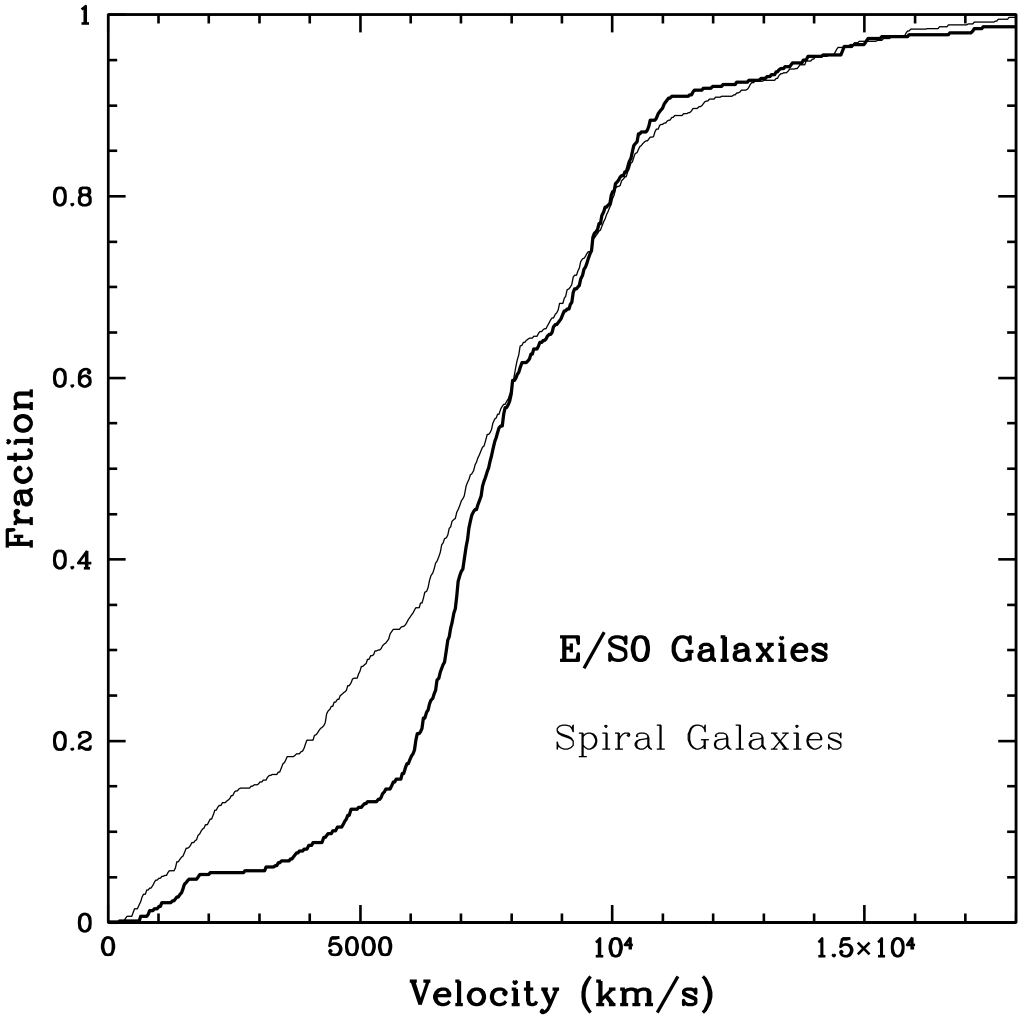

Figure 2: K-S test results for velocities. Plot shows the cumulative distributions (fractions) as a function of apparent velocity for early type galaxies versus late type galaxies.

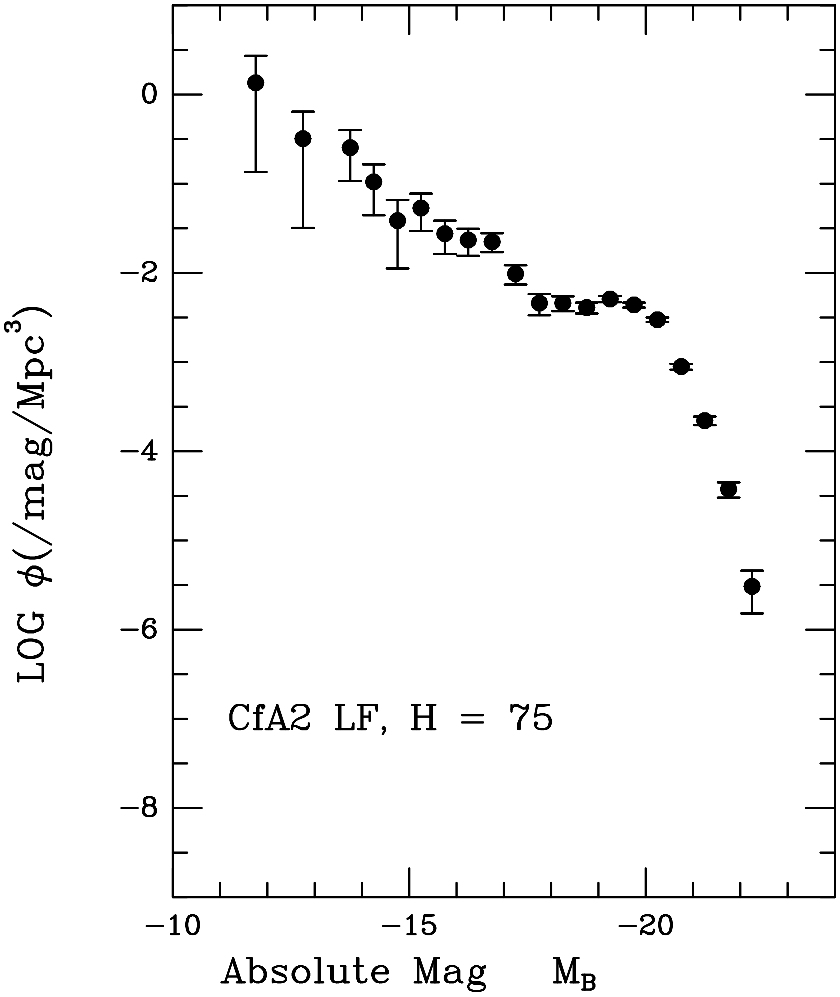

Figure 3: Luminosity Function (log of the number of galaxies per magnitude interval per unit volume) for the ∼1100 galaxies in the first CfA slice.

-16.0 TO -16.5	9	0.225E-01	-1.647	0.12	0.18
-16.5 TO -17.0	18	0.226E-01	-1.646	0.09	0.12
-17.0 TO -17.5	16	0.101E-01	-1.997	0.10	0.12
-17.5 TO -18.0	14	0.441E-02	-2.355	0.10	0.14
-18.0 TO -18.5	28	0.442E-02	-2.354	0.08	0.09
-18.5 TO -19.0	51	0.404E-02	-2.394	0.06	0.07
-19.0 TO -19.5	134	0.532E-02	-2.274	0.04	0.04
-19.5 TO -20.0	220	0.438E-02	-2.359	0.03	0.03
-20.0 TO -20.5	296	0.295E-02	-2.530	0.02	0.03
-20.5 TO -21.0	172	0.859E-03	-3.066	0.03	0.03
-21.0 TO -21.5	82	0.205E-03	-3.688	0.05	0.05
-21.5 TO -22.0	26	0.326E-04	-4.486	0.08	0.09
-22.0 TO -22.5	4	0.252E-05	-5.599	0.18	0.30

```
 INTEGRAL LUMINOSITY DENSITY ABOVE -12.5  =  0.161E+09 L(SOLAR)
  PHI IS IN UNITS OF GAL/MAG/MPC3
  VOLUME CORR FACTOR   58.840
  COMPLETENESS FACTOR 1.0000
  V/VM CORR FACTOR    1.000
  GALAXIES WITHIN 8D OF VIRGO ARE ASSUMED TO BE AT 16.10 MPC
  TOTAL NUMBER OF GALAXIES WITH VELOCITIES  1094
```

Many of the other techniques do a slightly better job on handling density inhomogeneities, too.

Problem 3. (a) What is an H-D curve? Describe its use in astronomy. (b) What image characteristics can you think of that might allow you to separate galaxies from stars using imaging data from either CCD's or photography? (c) Estimate the dynamic range of a Kodak 103aJ plate.

Answer: (a) The H-D curve or the Hurter and Driffield Curve is a plot of logarithmic density (or log 1/transmission) versus exposure for photographic plates. It is a measure of the sensitivity, dymanic range and speed of a plate. The curve is characterized by a fog level, a linear section, and a saturation level. These are joined by the "toe" and "shoulder." The range between the toe and shoulder is the dynamic range of the emulsion; the slope of the straight line section, γ, is a measure of the speed or "contrast" of the emuslion - ie. how fast it gets dark. This calibration curve is absolutely necessary if one wants to do any *quantitative* work with photographic emulsions, such as surface photometry. The dynamic range is given by the range in exposure between the toe and the shoulder, i.e the exposure range over which the response of the plate is roughly linear.

(b) The classic work in the field was done by Tyson and Jarvis (e.g. 1978, Ap. J. 230, L155; 1981, A.J. 86, 480) and later Valdez on a routine called FOCAS, now found in some versions IRAF. Galaxies are generally larger than stars and are often asymmetric. FOCAS relies on image shape parameters including ellipticity, central peak versus total brightness and a variety of other image moments as well as the object magnitude(!). This last is interesting because it works on the principle that galaxies are more numerous at faint magnitudes than stars or quasars, while stars are definitely more numerous at bright magnitudes (and low galactic latitudes).

At brighter magnitudes, one can simply describe most such classification procedures as a comparisons of the image properties to the stellar point spread function. At very faint magnitudes, where the S/N is poor and galaxies are not significantly larger than the stellar seeing disk, classification by moments and other image properties can be more robust.

(c) What is dynamic range? It is the range of intensity levels over which the response of a detector is more or less linear. For a plate that means the exposure levels between fog and saturation. All 103a emulsions have essentially the same graininess, speed, thickness, etc., so the dynamic range of the 103aJ is best estimated by finding in the Kodak manual *Plates and Films for Scientific Photography*, the curves for the 103a emulsion closest to the J in spectral response. On page 24d there are curves for both 103a) and 103aF emulsions/ Both show that for reasonable development times ($\sim$ 2-4 min) you get densities in the range 0.2 to 3.5 or so for log exposures ranging over 0 to 2. This implies a dynamic range of $\sim$ 100 for the 103a emulsions. Note that the actual dynamic range depends on development time on the type of developer, on how the plate has been treated pre and post exposure, etc. Usually the dynamic range for the 103a series is taken as about 80.

Problem 4. Interference Filters. For an interference filter made with two materials of indices of refraction $\eta = 1.3$ and 2.0, with the spacers low what are the (a) wavelength shift and (b) change in bandpass if the filter is used in a f/5.4 beam (which is what we'll have at the 6.5-m in the future)? Assume the filter is a 10 Å FWHM, Hα filter when in a collimated beam. What wavelength and bandpass would you have to specify for a filter to really get Hα at 10Å at zero redshift?

Answer: In the small angle limit, the formulae I gave in class are:

$$\Delta\lambda \sim \frac{1}{2}\Big[\frac{(n^{*2} - sin^2\theta/2)^{1/2}}{n^*} - 1\Big]\lambda_0$$

and for $n_L = 1.3$ and $n_H = 2.0$,

$$n^* = \frac{n_L}{[1 - n_L/n_H + (n_L/n_H)^2]^{1/2}} = 1.48$$

and

$$\theta = tan^{-1}(2 \times 1/f)^{-1} = 5.^\circ 3$$

given the focal ratio, $1/f = 5.4$. By the above formula, the shift is -1.6 Å at Hα = 6562.817 Å, a shift to the *blue*.

Unfortunately, an f/5.4 beam is not quite the small angle limit, so the full, proper formula for the wavelength shift, $\delta\lambda$, must be used. You can find it in the Handbook of Optics (or it can be derived from first principles by doing the integral over I(θ) in rings):

$$\delta\lambda/\lambda_0 = \frac{-\alpha^2}{4n^{*2}}$$

where α is the half-angle = $cot^{-1}f \times 2 = 0.049$, and the changed bandpass, $(\Delta\lambda)_\alpha$, is

$$\frac{(\Delta\lambda)_\alpha}{\Delta\lambda_0} = \left[1 + \left(\frac{\alpha^2\lambda_0}{2n^{*2}\Delta\lambda_0}\right)^2\right]^{1/2}$$

so in the full approximation

$$\delta\lambda = -1.8\text{Å} \quad \rightarrow \quad \lambda_\alpha = 6561.02\text{Å}$$

and the new bandpass is

$$\Delta\lambda_\alpha = 10.63\text{Å}$$

To specify a real 10 Å filter in this beam, you'd ask for a filter centered at 6564.62 Å, with a bandpass of 9.4 Å.

Problem 5. Spectroscopy. What signal-to-noise would you achieve in a one hour integration with the 6.5-m MMT Blue Channel spectrograph with the 300 line/mm grating (see class notes) on a 23th magnitude galaxy? How big will the galaxy appear (in pixels) on the spectrograph camera? Calculate this at 5500 Å(ie. the V band). Calculate the S/N per pixel on the 3kx1k CCD detector (you'll have to look up the dispersion). Use reasonable estimates for the throughput of the telescope and spectrograph. The sky background is approximately 22.0 mag/square (except at λ5577Å!) arcsecond, and the observations are made through a 1" wide, long slit. (You can also assume that the seeing is very, very good!)

(Note that you'll have to do some research on the spectrograph — what's the camera and collimator focal ratio's, etc. Don't put this off till the last minute.)

Answer: First, from the handout, 0th V magnitude, unextincted, is 948 photons per cm^2/sec/Å. The the throughput estimates are

$$\text{extinction} \ = \ 0.15 \text{ mag}, \quad t_A \ = \ 0.87$$

the telescope has 2 reflective surfaces at ~ 0.9 (at 5500 Å) and an obscuration of 2.0% at the f/9 focus, thus the

$$\text{telescope throughput}, \quad t_T \ = \ 0.79$$

and the spectrograph has 3 reflecting surfaces (collimator plus 2 mirrors in the Schmidt), one grating at 65%, and 4 transmissive surfaces at 98% (Schmidt window, coupling lens to detector), and a detector RQE of $\sim$ 80%, since its a thinned CCD. Thus

$$\text{detector} \ + \ \text{spectrograph throughput}, \quad t_S \ = \ 0.35$$

Only the last two are applied to the atmosphere, whereas all three are applied to the objects. Note that the MMT spectrograph is used at the f/9 and not the f/5.4 Cassegrain focus.

The dispersion of the blue spectrograph and 300 l/mm grating is 1.9Å/pixel, and the collecting area of the MMT is 331,800 cm^2, so the count rates per Å for a 1 hour integration on the objects and sky are respectively (applying the proper throughputs $t = t_A + t_T + t_S$ or $t = t_T + t_S$)

$$\text{sky} \ = \ 948 \times 10^{-22/2.5} \times 0.79 \times 0.35 \times 3.3318 \times 10^5 \times 3600$$

$$\text{23th maggal} \ = \ 948 \times 10^{-23/2.5} \times 0.87 \times 0.79 \times 0.35 \times 3.3318 \times 10^5 \times 3600$$

and thus

$$\text{sky} \ = \ 496/\mathring{A}rmsq\ arcsec \ = \ 942/pixel/\text{sq arcsec}$$

$$\text{23th maggal} \ = \ 172/\mathring{A} \ = \ 330/\text{pixelinwavelength}$$

Next, the spectrograph + 300 l/mm grating parameters are described in the manual. The spectral resolution is about 6.2 Å (or 3.2 pixels) at $\sim$ 5000 Å. Using the signal-to-noise formula from the first problem set and assuming "two channel" operation for sky subtraction (although one can do a bit better than this with optimal extraction techniques), we have:

$$(S/N)_{tall\ pixel} \ = \ \frac{N}{(N + 2S + 2D + 2R^2)^{1/2}}$$

The measured dark rate for the CCD is 6 e$^-$/pixel in 1 hour, and a read noise of 9 e$^-$ RMS per pixel, but the notation "tall pixel" above is because we also need to estimate the width of the spectrum on the detector.

The geometry of the situation on an unbinned detector goes something like this. The spectrograph focal plane scale is 0.3"/15 μ pixel, which efectively gives our 3.2 pixels per slit width. In the other direction, I expect our 23 mag galaxy to be a little under 2" in size. Then lets take the height of the spectrum to be $\sim$ 5 rows of the CCD, since the scale of the spectrograph focal plane is about 7 times that of the MMT or 25 arcsec/mm. Now we'll collapse the spectrum to 1-D to get pixels versus wavelength, so our spectrograph "tall pixel" is actually 1 unbinned detector pixel wide and 5 high. Thus our "effective" $D = 30$ and $nR^2 = 405$ per "tall" pixel, or pixel of the extracted spectrum. This, by the way, also means we should double the sky counts.

Thus we have, per pixel in a 1 hour exposure

$$S/N_{23} = \frac{171}{(171 + 2 \times 992 + 2 \times 30 + 2 \times 405)^{1/2}}$$

$$S/N_{23} = 3.1$$

Per resolution element the S/N will go up as the squareroot of the increase in the bandpass, or $\sqrt{3.2}$, so

$$S/N_{23} = 5.6$$

which is respectable for a 1 hour spectroscopic integration.

You can also get back a little S/N by binning the CCD perpendicular to the dispersion. (Generally, unless we're grossly oversamples, you'd never bin in the dispersion direction; binning also increases problems with cosmic rays.) If you bin$\times$3, you reduce the read noise above by $\times$3 — but the dark and sky stay the same, dark counts are a property of the silicon and not the amplifier. In that case, the S/N per resolution element is increased to 6.1, not a big gain since we're background limited.

In reality, our 23rd magnitude galaxy is physically extended. To estimate how much light is lost off the slit, we need to calculate the size (the brightness profile) of the galaxy. If we assume that this is a bright elliptical galaxy like M87 (not a terrible assumption for a problem like this), we can say we are observing a -21 magnitude galaxy ($H_0 = 100$ km/sec/Mpc) with a core radius, a, of 1 kiloparsec and a Hubble law ($I = I_0/(r+a)^2$) surface brightness profile. Depending on the details of ones assumptions about the cosmology and the K correction below, our 23rd magnitude galaxy will lie somewhere between a redshift of 0.25 and 0.4. The correction for K dimming includes the combination of stretching the lambda scale with redshift, the actual shift of the rest frame spectrum into the observed band (if z=0.5, our 5500Å band corresponds to 3670Å in the galaxy's rest frame), the lowering of the photon energy by a factor of 1+z, and changes in the galaxy's surface brightness due to coordinate stretching. Fortunately for us, 23rd magnitude galaxies are indeed small and, again, depending on your exact assumptions, between 40% and 70% of the *total* flux from the galaxy will pass through our 1"×2" slit, reducing the S/N to 1/2 to 2/3 of the values above (the object does *not* contribute much to the noise!).

Extra Credit: In addition to the above factors, there are other effects that will lower the S/N. Not all the light from the galaxy, because of seeing, will make it down the slit. Secondly, the extinction will increase above its value at one airmass that we assumed above. Thirdly, we've been very optimistic about throughput — if the mirrors and other optics haven't been cleaned lately, their reflectance and transmission will fall well below the book values. Lastly, when I'm observing, it's likely to be cloudy!

Problem 6. Properties of CCDs.

I've placed, also in the ftp site, some calibration images from one of our CCD's at Mt. Hopkins. These include Bias frames, Dark frames and Flat fields. Estimate

(1) The read noise

(2) The Dark rate in cts/pixel/sec. Watch out for Cosmic Rays!

(4) and possibly the chip gain.

The read noise can be estimated from the fluctuations in the bias level between reads, so, for example, one quick estimate can be obtained by subtracting two adjacent bias frames (adjacent in time, so that long time scale drifts in the bias level are removed), computing the average sigma for the pixels. There is a routine in IRAF that will do this called imstat. Then, if the gain is known, the read noise is

$$R = \sigma G/\sqrt{(2)},$$

where G is the gain, R is the read noise and σ is the dispersion or standard deviation of the mean difference, i.e.

$$\sigma = (\sum_n x^2)^{1/2}$$

where x^2 is the square of the pixel values in the difference frame, and the sum is the sum over all reasonable pixels in the image (clip the cosmic rays, bad chip edges, etc.)

An estimate of the gain of the CCD can also be obtained from two flat field images and two bias frames via

$$G = \frac{(f1_{mean} + f2_{mean} - b1_{mean} - b2_{mean})}{(\sigma^2_{f12} - \sigma^2_{b12})}$$

where $f1_{mean}$, etc. are the mean count rates of the flats and biases, and σ_{f12}, etc. are the dispersions, as above, in the difference images, f1-f2 and b1-b2.

Answer: This is plug and chug in IRAF. I chose to look at the set of biases and flats and darks near the end of this (not photometric) night. Using the formulae given in the problem and applying imstat to the images with rejection levels set to remove CR's and bad pixels, and comparing several nearby pairs of images to get the gain and read noise, I got gains in the range 3.2-3.8 and read noise R around 10 e^-. These are moderately close to the numbers in the headers (G=2.45, R=9). Below are example outputs from imstat.

IMAGE	NPIX	MEAN	STDDEV
a0084.BIAS	1062088	332.1	3.216
a0085.BIAS	1062100	332.3	3.217
a0086.BIAS	1062058	332.6	3.206
a0087.BIAS	1062031	332.9	3.191
a0088.BIAS	1062029	333.1	3.200
a0098.FLATI	1046381	11203.	293.7
a0099.FLATI	1046355	11271.	295.7
a0100.FLATI	1046186	11440.	300.5
a0101.FLATI	1046371	11237.	294.7
a0102.FLATI	1046247	11419.	300.0
a0103.FLATV	1044669	8128.	157.0
a0104.FLATV	1044689	8400.	163.3
a0105.FLATV	1044686	8357.	162.0
a0106.FLATV	1044709	8702.	170.1
a0107.FLATV	1044708	8724.	170.8
8485.diff	1065000	-0.2142	4.473
8586.diff	1065000	-0.2888	4.466
8687.diff	1065000	-0.2582	4.464
8788.diff	1064995	-0.2526	4.456
9899.diff	994476	-54.75	76.75
99100.diff	681089	-114.6	61.20
100101.diff	525456	126.9	56.71
101102.diff	626727	-119.4	59.44
103104.diff	198519	-146.8	54.95
104105.diff	1045359	38.31	70.94
105106.diff	48790	-112.8	83.41
106107.diff	1052170	-20.00	73.12

The dark rate was a little harder to calculate since the bias level was varying relatively strongly during the night. The only way to estimate the dark rate under these circumstances is to use the observer's trick of taking the overscan region to provide a local estimate of the bias level. The active area of the chip is 1024^2 pixels. The extra columns to the right are the overscan. I used the "imcopy" routine to extract the overscans and then imstat again to do statistics on them. Here are my cuts on the darks and on the trimmed bias section (columns 1028-1032 for all rows):

IMAGE	NPIX	MEAN	STDDEV
a0124.DARK	1061037	332.6	3.291
a0125.DARK	1060971	331.8	3.310
a0126.DARK	1061107	331.6	3.318
a0127.DARK	1061230	331.7	3.300
a0128.DARK	1061035	332.0	3.314
a0124.darksec	5160	332.3	3.088
a0125.darksec	5160	331.5	3.046
a0126.darksec	5160	331.4	3.031
a0127.darksec	5160	331.6	3.073
a0128.darksec	5160	331.9	3.069

With these numbers, the excess in each frame (per pixel) is 0.2 counts per pixel in 900 seconds, so the dark rate is 0.0002 cts/pix/sec. Pretty darn good! The dark rate is usally not a problem when imaging with a CCD.

Problem 7. Suppose you had a radio interferometer composed of two 25-m diameter antennas, placed 55° apart on the equator observing an unpolarized point source at zero declination at a frequency of 5 GHz. Assume that the atomic clock at each site keeps perfect time and that the two clocks are perfectly synchronized. Assume, too, that the Earth's rotation is strictly uniform about a spin axis that is (rigidly) fixed to the (rigid) Earth's crust with its direction also fixed in space. The (total) source has a flux density, S, of 1 Jy at 10 GHz and a spectrum of $S \propto F^{-1}$.

a) What is the brightness temperature of the source that would be inferred from total-flux-density measurements at 5 GHz made by one of these two antennas?

b) Describe quantitatively the fringe pattern on the plane of the sky for this inferometer from the time of rising above the horizon of the source at the western antenna to the time of setting of the source of the eastern antenna.

c) What is the normalized visibility function to be expected from (noise-free) observations at 5 GHz of this source with this inferometer for the total period of (mutual) visibility of the source at the two antennas?

d) Given that (i) the origin of right ascension is defined to be zero at a point directly over the midpoint of the of the inferometer baseline at t=0; (ii) the inferometer baseline vector is, a priori, known exactly; (iii) ten observations of the source are made, uniformly spaced over a period of mutual visibility; and (iv) each such observation is of (unambiguous) phase delay, with a standard deviation equivalent to 10° of phase, calculate by weighted least squares the resultant standard deviation in the estimate of the source right ascension from these observations under the above conditions. If any additional assumptions are needed to complete this calculation indicate the ones you made explicitly.

e) Given that no errors affect the pointing of each telescope (i.e., the pointing is exact), determine approximately the uncertainty in an estimate of the declination of the source from a total-flux-density measurement with the above equipment. State explicitly all subsidiary assumptions you made in arriving at your determination.

Problem 8. A two-element interferometer, with elements on the equator, separated along the surface by 5000 km and observing a distant quasar (right ascension = 0°; declination = 30°) at $\lambda = 10$ cm, obtained a visibility amplitude as follows:

```
          |
      0.6 |-           +
          |          +   +
|V|   0.4 |-        +     +
          |  +     +       +     +
      0.2 |-  +   +         +   +
          |    + +           + +
      0.0 |-----+-------------+--------------
                  t  --->
```

over the period of mutual visibility (note that the shape is not intended to be accurate; only the "trends" and the maximum and minima are intended to be accurate. Note also that we are using "visibility" in different part of this question; context is everything!).

a) What is the period of mutual visibility?

b) What (simple) model of the source can you concoct that would yield a $|V|$ vs. t curve consistent with the above representation of the observations? Describe
the main features of the model and their connection to the $|V|$ vs t curve.

c) Suppose one year later the same source was observed bt the same interferometer over the same period of mutual visibility. Suppose, too, that the visibility amplitude had the same general shape, save for the first minimum ocuring one-half hour earlier and the second on-half hour later than in the original observations. What conclusions might you draw about the changes in the source between these observations? Are there any "uniqueness" issues to be concerned about, even within the context of your model?

Problem 9. You are contemplating observing another compact unpolarized radio source (total flux density at $\lambda = 10$ cm of 1 mJy) with the interferometer (partially) described in problem 7. Suppose that each antenna has a diameter of 25 m and an efficiency at $\lambda = 10$ cm of 0.7. Suppose, further, that the system temperature for each antenna is 25K.

a) What would be the minimum "on-source" observing time required to achieve a signal-to-noise ratio (SNR) of 7, if the recorded bandwidth were 100 MHz and half of the total flux density stemmed from an extensive circular, very diffuse halo about one arcsecond in diameter?

b) If this source had an (unrealistic) spectrum such that its flux density, S, as a function of frequency, f, obeyed $S = f^3$ between 3 GHz and 9 GHz, how would your answer to A) change for observations at $\lambda = 3.3$ cm? What additional key assumptions, if any, did you make answering this question?

Note: All problems will "count" approximately equally in determining your grade on the exam.

Note also, as a public service, I repeat here the data on flux calibrations in the OIR:

FLUX CALIBRATION AT 0.0 MAGNITUDE

Filter	λ_{eff}	$\Delta\lambda$	F_λ	F_ν	# Photons
	Å	Å	(a)	(b)	s^{-1} cm^{-2} Å^{-1}
U	3600	800	4.35×10^{-9}	1.88×10^{-23}	789
B	4400	1100	7.20×10^{-9}	4.44×10^{-23}	1595
V	5556	1200	3.39×10^{-9}	3.48×10^{-23}	948
R	7000	1400	1.76×10^{-9}	3.01×10^{-23}	620
I_P	8200	1600			474
I	9000	2000	8.30×10^{-10}	2.43×10^{-23}	376
I′	10000	2000			321
	μ	μ			
Z	1.03	0.16			
J	1.25	0.3	2.90×10^{-10}	1.51×10^{-23}	183
H	1.65	0.3	1.08×10^{-10}	9.81×10^{-24}	89.7
K	2.20	0.4	3.8×10^{-11}	6.14×10^{-24}	42.1
L	3.5	0.6	7.3×10^{-12}	2.8×10^{-24}	12.9
M	5.0	1.0	2.2×10^{-12}	1.8×10^{-24}	
N	10.2	5.0	1.0×10^{-13}	3.7×10^{-25}	
Q	21	11	6.5×10^{-16}		

(a) ergs cm^{-2} s^{-1} Å^{-1}

(b) W m^{-2} Hz^{-1}

I_P is the photographic I band formed with an N emulsion and a Wr 88A filter.

In X-rays:

1 Uhuru count = 1.7×10^{-11} ergs cm^{-2} s^{-1} in 2-6 kev,

1 IPC count = 1.7×10^{-11} ergs cm^{-2} s^{-1} in 0.5-4.5 kev.

Note that 1μJy = 1.9×10^{-11} ergs cm^{-2} s^{-1} in the band 2-10 kev, so 1 Uhuru count is approximately 1.78 μJy for a flat source.