Problems:

Problem 1. Luminosity Functions

A useful parametrization of the $\phi(L)$, derived by Schechter, is

$$\phi(L) = \left(\frac{\phi_0}{L^*}\right)\left(\frac{L}{L^*}\right)^\alpha \exp\left(-\frac{L}{L^*}\right).$$

$L^*$ is the characteristic luminosity near the knee, $\phi_0$ is the normalization and $\alpha$ is the slope at the faint end. The Schechter function has the interesting property that the integral luminosity density, useful in cosmology, is just given by

$$L_{\text{int}} = \phi_0 L^* \Gamma(\alpha + 2)$$

where $\Gamma$ is the incomplete gamma function.

The Schechter function is also useful for estimating the amount of luminosity you fail to measure when you work with flux or magnitude limited samples. For example, in a nearby galaxy cluster you might be able to sample the LF 4 magnitudes fainter than $M^*$, while in one 5 times further away, to the same apparent magnitude limit, you will only sample 0.5 magnitudes below $M^*$.

A second issue, still hotly debated, is how much luminosity is actually hiding in low luminosity / low surface brightness objects. Modern redshift surveys find flat faint end slopes but only for relatively luminous galaxies and high surface brightness. An alternative view is that the real faint end slope rises much more steeply (see the works of Mike Disney, Greg Bothun and Chris Impey).

1. For a faint end slope of -1.1, how much of the integrated luminosity of the above more distant cluster will be lost relative to the nearby one?

2. For Schechter functions of the same $M^*$, what fraction of the integrated luminosity density lies below $M^*$ for $\alpha$’s of -1.0, -1.25, -1.5 and -1.85?
Problem 2. Mass Functions

The route to the luminosity and mass functions usually starts with a galaxy sample and proceeds through the determination of the masses either dynamically or via other properties such as morphological type or color. An interesting old galaxy sample to play with is that from the 1st CfA survey, which is complete to a blue magnitude of 14.5 and also has complete morphological type information. The data file can be found on the website at

http://www.cfa.harvard.edu/~huchra/zcat/cfa1.dat

and the format is moderately self explanatory. Write a program to first calculate the luminosity function, and then using an estimate of the M/L versus morphological type, turn this into the mass function. The way to accomplish this second step is to use something like the Bell (2003) et al. calibration of g-r or B-V color versus M/L (their Figure 6 or Table 7) and a calibration of B-V versus type (can you find one!). Be sure to specify/use the right band — remember the CfA1 magnitudes are B.

Do you need to worry about K-corrections? What about dust? Galactic extinction? How do you estimate distances and thus luminosities? And what should you do about the galaxies with negative or near zero velocities?