Andromeda's Distance

from E. Opik's argument

ApJ 55, 406 (1922)

Opik derived an expression for the absolute distance of Andromeda (or any other rotating system where size and rotation can be observed) in terms of the

linear speed, v_0
angular distance from the center, θ
apparent luminosity l
energy per unit mass (Mass-to-Light ratio) Y

If v_0 is the motion along a circular orbit at a given radius r given in in astronomical units, then we can write for the mass M contained withinthat radius

$$\left(\frac{v_0}{w}\right)^2 = \frac{M}{r} \qquad (1)$$

where w is the orbital velocity of the Earth around the Sun.

Let the integrated absolute luminosity of the nebula be L, and the apparent luminosity l, again in solar units. Then

$$\frac{l}{L} = \frac{1}{D^2} \qquad (2)$$

where D is the distance to the nebula in AU. With θ the angular distance from the center of the nebula,

$$r = D\sin(\theta) \approx D\theta \quad \text{(the small angle approximation)}$$

Define $Y = M/L$ as the mass-to-light ratio, then combining (1) and (2) we have

$$D = \frac{\theta}{lY}\left(\frac{v_o}{w}\right)^2 \qquad (3)$$

v_0 can be measured, w is known (29.78 km/s on average, todays value), and l can be measured.

For Andromeda, Opik used Pease's measurement of the rotation velocity at 150" radius, 72 km/s, corrected for inclination (b/a = 0.79) to 157 km/s, a value essentially the same as todays best. For Y, he used the average value of the Light-to-Mass ratio (which he called $E = 1/Y$) of stars in the solar neighbohood from Kapteyn's studies of

≈ 0.38 (giving $Y = 2.6$). For l, Reynolds had estimated the extrapolated magnitude of Andromeda to be 5.0 and the magnitude inside 150" to be 3/8 of that, or 6.1. With an apparent magnitude of the Sun of -26.6, Opik calculated that

$$l \;=\; 2.512^{-32.7} \;\approx\; 10^{(-32.7/2.5)}$$

Note that 2.512 is the 5th root of 100, called Pogson's value, and the is the correct factor corresponding to a difference in one magnitude, not the 2.5 we normally assume!

With these values, and with $\theta = 150"/206265"/\text{radian} = 0.000727$, and $Y = 2.6$, we then have for the estimate of Andromeda's distance

$$\begin{aligned} D \;&=\; \frac{0.000727}{(2.6)(8.3\times10^{-14})}\left(\frac{157\ \text{km/s}}{29.78\ \text{km/s}}\right)^2 \\ &= 9.36\times10^{10}\text{AU} \;=\; 4.5\times10^{5}\ \text{pc} \end{aligned}$$

Todays best estimate of the distance to Andromeda is about 2.5 million light-years or, for astronomers, $\sim 0.77 \pm 0.04$ Megapasecs. This probably indicates that by using Kapteyn's value of Y from the Solar neighborhood, Opik *overestimated* the mass-to-light ratio of the inner regions of M31 by a little (less than a factor of two, though).

But it was not at all bad for 1922! And the argument teaches us a bit about how to think of mass, light and galactic kinematics.