Evolution of black holes in the galaxy


*Department of Physics & Astronomy, State University of New York, Stony Brook, NY 11794-3800, USA

Floyd R. Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853, USA

Abstract

In this article we consider the formation and evolution of black holes, especially those in binary stars where radiation from the matter falling on them can be seen. We consider a number of effects introduced by some of us, which are not traditionally included in binary evolution of massive stars. These are (i) hypercritical accretion, which allows neutron stars to accrete enough matter to collapse to a black hole during their spiral-in into another star. (ii) The strong mass loss of helium stars, which causes their evolution to differ from that of the helium core of a massive star. (iii) The direct formation of low-mass black holes ($M \sim 2M_\odot$) from single stars, a consequence of a significant strange-matter content of the nuclear-matter equation of state at high density. We discuss these processes here, and then review how they affect various populations of binaries with black holes and neutron stars.

We have found that hypercritical accretion changes the standard scenario for the evolution of binary neutron stars: it now usually gives a black-hole, neutron-star (BH-NS) binary, because the first-born neutron star collapses to a low-mass black hole in the course of the evolution. A less probable double helium star scenario has to be introduced in order to form neutron-star binaries. The result is that low-mass black-hole, neutron star (LBH-NS) binaries dominate the rate of detectable gravity-wave events, say, by LIGO, by a factor $\sim 20$ over the binary neutron stars.

The formation of high-mass black holes is suppressed somewhat due to the influence of mass loss on the cores of massive stars, raising the minimum mass for a star to form a massive BH to perhaps $80M_\odot$. Still, inclusion of high-mass black-hole, neutron-star (HBH-NS) binaries increases the predicted LIGO detection rate by another $\sim 30$%; lowering of the mass loss rates of Wolf-Rayet stars may lower the HBH mass limit, and thereby further increase the merger rate.

We predict that $\sim 33$ mergers per year will be observed with LIGO once the advanced detectors planned to begin in 2004 are in place.

Black holes are also considered as progenitors for gamma ray bursters (GRB). Due to their rapid spin, potentially high magnetic fields, and relatively clean environment, mergers of black-hole, neutron-star
binaries may be especially suitable. Combined with their 10 times greater formation rate than binary neutron stars this makes them attractive candidates for GRB progenitors, although the strong concentration of GRBs towards host galaxies may favor massive star progenitors or helium-star, black-hole mergers.

We also consider binaries with a low-mass companion, and study the evolution of the very large number of black-hole transients, consisting of a black hole of mass $\sim 7M_\odot$ accompanied by a K or M main-sequence star (except for two cases with a somewhat more massive subgiant donor). We show that common envelope evolution must take place in the supergiant stage of the massive progenitor of the black hole, giving an explanation of why the donor masses are so small. We predict that there are about 22 times more binaries than observed, in which the main-sequence star, somewhat more massive than a K- or M-star, sits quietly inside its Roche Lobe, and will only become an X-ray source when the companion evolves off the main sequence.

We briefly discuss the evolution of low-mass X-ray binaries into millisecond pulsars. We point out that in the usual scenario for forming millisecond pulsars with He white-dwarf companions, the long period of stable mass transfer will usually lead to the collapse of the neutron star into a black hole. We then discuss Van den Heuvel’s “Hercules X-1 scenario” for forming low-mass X-ray binaries, commenting on the differences in accretion onto the compact object by radiative or semiconvective donors, rather than the deeply convective donors used in the earlier part of our review.

In Appendix A we describe the evolution of Cyg X-3, finding the compact object to be a black hole of $\sim 3M_\odot$, together with an $\sim 10M_\odot$ He star. In Appendix B we do the accounting for gravitational mergers and in Appendix C we show low-mass black-hole, neutron-star binaries to be good progenitors for gamma ray bursters. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The fate of massive stellar cores, both in single and binary stars, has many observable consequences, both for what types of compact object may be found in what type of binary, and for the formation rates of all types of compact-object binary. We have discussed various aspects of this problem in previous works, and here give an overview of all these together, applying the same set of principles to all and obtaining a consistent picture of the evolution of massive stars and binaries.

The best-known compact-object (i.e., neutron star or black hole) binaries are the binary neutron stars. They are key testing grounds of general relativity, and the usually favored gravity-wave source for LIGO. Until recently the theoretical formation rate of binary neutron stars gave at least one order-of-magnitude higher rate than was arrived at empirically by extrapolation from observed binary neutron stars. Because there are few binary neutron stars, and even fewer dominate the empirical estimates, the latter are frequently revised. The recent doubling of the estimated distance to PSR 1534 + 12 [2] has lowered the empirical birth rate significantly, widening the gap.

A solution to this discrepancy comes from combining the strange-matter equation of state, which results in a relatively low maximum mass for neutron stars, with hypercritical accretion [1]. In the standard scenario the first neutron star formed spirals into the other star, in a phase of
common-envelope evolution. Bethe and Brown [1] argued that when a neutron star spirals into
a red giant with a deeply convective envelope, it accretes matter at a very high rate of up to
$1 M_\odot \text{yr}^{-1}$. Photons are trapped in the flow and carried adiabatically inwards to the surface of
the neutron star [3]. The latter is heated to $T \sim 1 \text{MeV}$, temperatures at which neutrino emission can
carry off the thermal energy. Hence, the Eddington limit of $\dot{M}_{\text{Edd}} \sim 1.5 \times 10^{-8} M_\odot \text{yr}^{-1}$ does not apply. As a result, the neutron star accretes about a solar mass of material and collapses to a
low-mass black hole. Only if the two stars are initially so close in mass that at the time of spiral-in
the first supernova has not yet exploded (i.e. the object that spirals in is still a helium star) a binary
neutron star is formed. The sum total of binary neutron stars and black-hole, neutron-star binaries
is almost the same as what was found for binary neutron stars in previous estimates, but now the
binary neutron stars are only a small fraction of the total. The result is that an order of magnitude
more black-hole, neutron-star binaries than binary neutron stars are formed. Together with the
fact that the black holes are somewhat more massive than neutron stars, this implies that binaries
with black holes should play an important part in mergers producing gravitation waves. They may
also be good candidates for producing gamma-ray bursts.

No low-mass black-hole, neutron-star binaries have been observed. This is due to the fact that
the one neutron star in them is unrecycled, hence is observable for only a short time. The rarer
binary neutron stars, like PSR 1913 + 16, do have a long-lived recycled pulsar, which more than
offsets their lower formation rate and makes them dominate the observed population.

We do observe high-mass black holes in Cyg X-1 and in soft X-ray transients. In the former, the
black hole is of $\gtrsim 10 M_\odot$ [4]. The companion O-star is near its Roche Lobe, and its wind is
continuously feeding the black hole, which shines through X-ray emission. In addition to Cyg X-1,
high-mass black holes are seen in the LMC in LMC X-3 and perhaps LMC X-1. Much more
copious are the transient sources, with black holes of mass $M_{\text{BH}} \sim 7 M_\odot$, most of which flare up
only occasionally with long quiescent times between flare ups. Wijers [5] estimated $\sim 3000$ of
these in the Galaxy. That is, these are the numbers that are presently operative. Remarkable about
the transient sources with unevolved donors is that the main sequence star is K- or M-star, less
massive than our sun. Brown et al. [6] explain this in terms such that higher-mass donors can also
participate in the formation of such binaries containing a high-mass black hole, but will end up in
the evolution further away from the black hole so that they can pour matter on the latter only when
they evolve in subgiant or giant stage. Thus, there are a large factor estimated to be $\sim 22$ more of
those binaries which will not be seen until the main sequence star evolves [6]. The mechanism
describing the evolution of the transient sources required the massive progenitor of the black hole
to carry out core helium burning as if it were a single star, i.e., before having its H envelope removed
in RLOF by its main sequence companion. An interval of $\sim 20-35 M_\odot$ ZAMS was estimated for
the progenitors of the high-mass black hole. Consequently, this same interval of single stars, not in
binary, would be expected to end up as high-mass black holes. In the formation of these high-mass
black holes, most of the helium envelope of the progenitor must drop into the black hole in order to
form their high mass, so little matter is returned to the Galaxy.

This brings us to the intriguing matter of SN 1987A which we believe did go into a black hole,
but after the supernova explosion which did return matter to the Galaxy. The progenitor of SN
1987A was known to have ZAMS mass $\sim 18 M_\odot$. This leads us to the interesting concept of
low-mass black holes with delayed explosion, which result from the ZAMS mass range
$\sim 18-20 M_\odot$, although the precise interval is uncertain. The delayed explosion mechanism has
been elucidated by Prakash et al. [7]. The range of ZAMS masses of single stars in which neutron stars are formed is thus only $\sim 10 - 18 M_\odot$. The absence of matter being returned to the Galaxy in the ZAMS mass range $\sim 20 - 35 M_\odot$ impacts on nucleosynthesis, especially in the amount of oxygen produced. Bethe and Brown [8] suggested that matter was again returned to the Galaxy by stars in the ZAMS range $\sim 35 - 80 M_\odot$. In this case, the progenitor was stripped of H envelope in an LBV phase, and the naked He star was suggested to evolve to a low-mass black hole, with return of matter to the galaxy before its formation in a delayed explosion, or to a neutron star. Thus, elements like oxygen were produced in a bimodal distribution of ZAMS masses $M \leq 20 M_\odot$ and $35 M_\odot \leq M \leq 80 M_\odot$.

The Bethe and Brown [8] suggestion was based on naked He stars evolved by Woosley et al. [9], who used a too-large wind loss rate for He stars. Wellstein and Langer [10] have evolved naked He stars with lower rates, in which case the final He envelope is somewhat larger. However, the central carbon abundance following core He burning is high $\sim 33\%$. With this abundance, the stars will not skip the convective carbon burning stage in their evolution, and according to the arguments of Brown et al. [6] would still be expected to end up as low-mass compact objects, in which case matter would be returned to the Galaxy. This matter will not, however, be settled until the CO cores evolved with lowered He-star wind loss rates by Wellstein and Langer have been burned further up to the Fe core stage, so the Bethe and Brown [8] bimodal mass region for nucleosynthesis should be viewed as provisional.

In Section 2, we discuss the maximum mass of neutron stars and the processes that determine which range of initial stellar masses gives rise to what compact object, and how mass loss in naked helium stars changes those ranges. Then we describe the Bethe and Brown [1] scenario for the evolution of massive binary stars, and especially their treatment of common-envelope evolution and hypercritical accretion (Section 3). We then discuss a few specific objects separately, first binary neutron stars (Section 4), then Cyg X-1 and its ilk (Section 5) and the black-hole transients (Section 6). Then we comment briefly on how our results would affect the evolution of low-mass X-ray binaries with neutron stars (Section 7) and summarize our conclusions (Section 8). The discussion of Cyg X-3 and the possible implications of neutron-star, black-hole binaries for gravity waves and gamma-ray bursts are in Appendices A–C.

2. The compact star

Thorsson et al. [11] and Brown and Bethe [12] have studied the compact core after the collapse of a supernova, assuming reasonable interactions between hadrons. Initially, the core consists of neutrons, protons and electrons and a few neutrinos. It has been called a proto-neutron star. It is stabilized against gravity by the pressure of the Fermi gases of nucleons and leptons, provided its mass is less than a limiting mass $M_{\text{PC}}$ (proto-compact) of $\sim 1.8 M_\odot$.

If the assembled core mass is greater than $M_{\text{PC}}$ there is no stability and no bounce; the core collapses immediately into a black hole. It is reasonable to take the core mass to be equal to the mass of the Fe core in the pre-supernova, and we shall make this assumption, although small corrections for fallback in the later supernova explosion can be made as in Brown et al. [13]. If the center collapses into a black hole, the outer part of the star has no support (other than centrifugal force from angular momentum) and will also collapse.
If the mass of the core is less than $M_{PC}$, the electrons will be captured by protons
\[ p + e^- \rightarrow n + \nu \]  
and the neutrinos will diffuse out of the core. This process takes of order of 10 s, as has been shown by the duration of the neutrino signal from SN 1987A. The result is a neutron star, with a small concentration of protons and electrons. The Fermi pressures of the core are chiefly from the nucleons, with small correction from the electrons. On the other hand, the nucleon energy is increased by the symmetry energy, i.e., by the fact that we now have nearly pure neutrons instead of an approximately equal number of neutrons and protons. Thorsson et al. [11] have calculated that the maximum mass of the neutron star $M_{NS}$ is still about 1.8$M_\odot$, i.e., the symmetry energy compensates the loss of the Fermi energy of the leptons. Corrections for thermal pressure are small [14].

The important fact is that the 10 s of neutrino diffusion from the core give ample time for the development of a shock which expels most of the mass of the progenitor star.

But this is not the end of the story. The neutrons can convert into protons plus $K^-$ mesons,
\[ n \rightarrow p + K^- . \]  
This is short-hand for the more complicated interaction $N + e^- \rightarrow N' + K^- + \nu$ where $N$ is a nucleon. The neutrinos leave the star. The times are sufficiently long that chemical equilibrium is assured. Since the density at the center of the neutron star is very high, the energy of the $K^-$ is very low, as confirmed by Li et al. [15] using experimental data. By this conversion the nucleons can again become half neutrons and half protons, thereby saving the symmetry energy needed for pure neutron matter. The $K^-$, which are bosons, will condense, saving the kinetic energy of the electrons they replace. The reaction equation (2) will be slow, since it is preceded by
\[ e^- \rightarrow K^- + \nu \]  
(with the reaction equation (2) following) as it becomes energetically advantageous to replace the fermionic electrons by the bosonic $K^-$'s at higher densities. Initially, the neutrino states in the neutron star are filled up to the neutrino chemical potential with trapped neutrinos, and it takes some seconds for them to leave the star. These must leave before new neutrinos can be formed from the process equation (3). Thorsson et al. [11] have calculated that the maximum mass of a star in which reaction equation (2) has gone to completion is
\[ M_{NP} \simeq 1.5M_\odot , \]  
where the lower suffix NP denotes their nearly equal content of neutrons and protons, although we continue to use the usual name “neutron star”. This is the maximum mass of neutron stars, which is to be compared with the masses determined in binaries. The masses of 19 neutron star masses determined in radio pulsars [16] are consistent with this maximum mass.

The core mass $M_C$ formed by the collapse of a supernova must therefore be compared to the two limiting masses, $M_{PC}$ and $M_{NP}$. If
\[ (I) \quad M_C > M_{PC} , \]  
\[ (II) \quad M_C < M_{NP} , \]
we get a high mass black hole. If

\[ M_{PC} > M_C > M_{NP} , \]  

(6)

we get a low-mass black hole, of mass \( M_C \). Only if

\[ M_C < M_{NP} \]  

(7)
do we get a neutron (more precisely, “nucleon”) star from the SN. Only in this case can we observe a pulsar. In cases (II) and (III) we can see a supernova display. In case (I) we receive only initial neutrinos from electrons captured in the collapse before \( M_C \) becomes greater than \( M_{PC} \) but no light would reach us. (Except perhaps if the new black hole rotates rapidly enough to power an explosion, a mechanism proposed by MacFadyen and Woosley [93] for gamma-ray bursts.)

Woosley et al. [17] evolve massive stars with mass loss. For stars in the ZAMS mass range \( \sim 20-30M_\odot \), mass loss is relatively unimportant and since \( M_{PC} \gtrsim 1.8M_\odot \) for this range, we find from the earlier calculation of Woosley and Weaver [18] that most of the single stars in this range will go into high-mass black holes. Evolution of these stars in binaries is another matter. Timmes et al. [19], Brown et al. [13], and Wellstein and Langer [10] find that substantially smaller core masses result if the hydrogen envelope is taken off in RLOF so that the helium star is naked when it burns. Thus, stars of ZAMS masses \( \sim 20-35M_\odot \) in such binaries evolve into low-mass compact cores, black hole or neutron star. Woosley et al. [17] used helium-star wind loss rates which were too high by a factor \( \sim 2-3 \), but lower wind losses give only slightly larger He cores in the ZAMS mass range \( \sim 20-35M_\odot \) [10] so our above conclusion is unlikely to be reversed.

On the other hand, the fate of single stars in the ZAMS mass range \( \sim 35-80M_\odot \) is uncertain. In the published Woosley et al. [17] work with too high mass loss rate, so much matter is blown away, first in LBV stage and later in W-R stage that low-mass compact objects, black-hole or neutron-star, result [13]. Bethe and Brown [8] attribute this to the fact that convective carbon burning is not skipped in these stars. In this stage a lot of entropy can be removed by \( v\bar{v} \) emission, so that a low entropy, and therefore small, core results. In this range, Wellstein and Langer [10] find central \( ^{12}\text{C} \) abundances of 33-35% following He core burning, more than double the \( \sim 15\% \) required for convective carbon core burning. Therefore, we believe that this range of stars will still go into low-mass compact objects, even though their final He cores are substantially larger because of the lower, more correct, He-star wind mass loss rates used by Wellstein and Langer [10]. However, this problem cannot be considered as settled until the Wellstein and Langer CO cores are burned up to the Fe core stage. We will therefore not discuss the evolution of Cyg X-1 like objects, high-mass black holes accompanied by sufficiently massive giant companion so that they shine continuously in X-rays. It is not clear to us whether LMC X-3, with a high-mass black hole and a B-star companion of roughly equal mass, has a history more like Cyg X-1 or like the transient black-hole binaries which we discuss below.

Bethe and Brown [8] took \( 80M_\odot \) as the lower mass limit for high-mass black-hole formation in binaries which experience RLOF, i.e., in those for which helium core burning proceeds in a naked helium star. Because of our above discussion, we believe this mass limit may be too high, so that the contributions from high-mass black-hole, neutron-star binaries were, if anything, underestimated in their work. However, we will not know until the CO cores obtained with better He-star mass loss rates are evolved further.
3. Evolution of binary compact objects

We summarize the Bethe and Brown [1] evolution of binary compact objects, paying special attention to their common envelope evolution. In particular, we shall show that their schematic evolution should be applicable to donors with deeply convective envelopes, whereas for non-convective or semiconvective envelopes, such as encountered in the evolution of low-mass X-ray binaries, their common envelope evolution would not be expected to apply.

We call the star that is initially heavier star A, the other star B. We denote initial masses by subscript i, so we have masses $M_{A,i}$, $M_{B,i}$. We denote their ratio by $q$; thus

$$ q = M_{B,i}/M_{A,i} \leq 1 . $$(8)

Following Portegies Zwart and Yungelson [20], we assume that $q$ is distributed uniformly between 0 and 1. Likewise, we also assume that $\ln a$ is uniformly distributed, where $a$ is the semi-major axis of their orbit.

However, we assume different limits for $a$ than Portegies Zwart and Yungelson [20]. Initially, both stars are massive main sequence stars, with radius at least $3R_\odot$, so $a > 6R_\odot = 4 \times 10^6$ km. At the other end of the scale, we require $a < 4 \times 10^9$ km. We assume that 50% of all stars are binaries with separations in this range (stars in wider binaries would evolve as if they were single). Then the fraction of binaries in a given interval of ln $a$ is

$$ d\phi = d(\ln a)/7 . $$(9)

We assume that a star needs an initial mass of

$$ M > M_s = 10M_\odot $$

in order to go supernova. Therefore, if $\alpha$ is the total rate of SNe, the rate of SNe in mass interval $dM$ is given by

$$ d\alpha = n \left( \frac{M}{10M_\odot} \right)^{-n} dM , $$

where we have used a power-law initial mass function with $n = 1.5$ (close to the Salpeter value $n = 1.35$). The birth rate of supernova systems was taken to be

$$ \alpha = 0.02 \text{ yr}^{-1} $$

in the Galaxy. By a supernova system we mean a single star that goes supernova (i.e., has $M_{ZAMS} > 10M_\odot$) or a close binary containing at least one such star (close here means within the separation range mentioned above). Bethe and Brown [1] find that if the primary is massive enough to go supernova, then there is an $\sim 50\%$ chance for the secondary to also go supernova. This was calculated for a distributions flat in $q = M_{B,i}/M_{A,i}$. Therefore, the supernova rate in our notation would be $1.25\alpha = 0.025 \text{ yr}^{-1}$.

Using the Cordes and Chernoff [21] distribution of kick velocities, 43% of the binaries were found to survive the first explosion. Thus, at this stage, we are left with a birth rate of

$$ R = 0.02 \times \frac{1}{2} \times \frac{1}{2} \times 0.43 \approx 2 \times 10^{-3} \text{ per yr} $$

(13)
for the formation of binaries consisting of a neutron star with a companion massive enough to go supernova ($M > 10M_\odot$). The lifetime of such systems is the companion lifetime of $\sim 10^7$ yr, but star A will be a pulsar for only $\sim 5 \times 10^6$ yr because it will spin down electromagnetically until it is no longer observable. From these numbers we estimate the number of such systems to be $\sim 10^4$ in the Galaxy.

Since the pulsar is unrecycled, the expected number should be compared with the detected population of active radio pulsars in the galaxy, about $10^3$. This number should be multiplied by a factor of 1/2 for binarity, a further factor of 1/2 for a binary in which both stars can go supernova and the 0.43 for survival of the first explosion. This would leave the large number $\sim 10^2$ if pulsars with massive companions were as easily detected as single pulsars. In fact, only 2 are observed; PSR 1259-63 with a Be-star companion and PSR 0045-73 with a B-star companion. Stellar winds interfere with the radio pulses from these binaries, obscuring the narrower ones. Doppler shifts also make these difficult to observe. Nevertheless, the factor necessary to reduce their observability is large. We return to the subject later.

At this stage we have an $\sim 1.4M_\odot$ neutron star with O or B-star companion. We take the latter to have mass $\sim 15M_\odot$. The giant has a He core containing some 30% of its mass, surrounded by an envelope consisting mainly of H. We take the envelope to be deeply convective, so the entropy is constant. The particles, nuclei and electrons, are nonrelativistic and thus have $\gamma = 5/3$. Therefore, the envelope forms a polytrope of index $n = 3/2$. Applegate [26] shows that the binding energy of the envelope is

$$E \approx 0.6GM_\odot^2 R^{-1},$$

where $R$ is the outer radius. In this formula the binding energy is decreased 50% by the kinetic energy, $E$, containing both effects.

The major difference of the Bethe and Brown calculations and of case H of Portegies Zwart and Yungelson [20] compared with other work is the use of hypercritical accretion. In a series of papers, Chevalier [27,28] showed that once $\dot{M}$ exceeded $\sim 10^4M_\odot\text{yr}^{-1}$, the photons were carried inwards in the adiabatic inflow, onto the neutron star. The surface of the latter was heated sufficiently that energy could be carried off by neutrino pairs. Brown [3] reproduced Chevalier's results in analytical form. The idea has a much longer history: Colgate [73] showed already in 1971 that if neutrinos carry off the bulk of the energy, accretion can proceed at a much greater rate than Eddington. In 1972 Zeldovich et al. [74], before the introduction of common envelope evolution, used hypercritical accretion of a cloud onto a neutron star. Bisnovatyi-Kogan and Lamzin [75] and Chevalier [27] pointed out that during the common envelope phase of binary evolution, photons would be trapped and accretion could occur at much higher rates, and that neutron stars that go through this phase generally will go into black holes.

We begin by considering the work done by the neutron star on the envelope matter that it accretes. This will turn out to be only a fraction of the total work, the rest coming from the

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1 The assumption that the envelope is deeply convective is essential for our later treatment of common envelope evolution with hypercritical accretion. Recent developments with nonconvective or semiconvective donors show that the accretion rate is also highly super-Eddington, but still significantly less [22–24]. For very massive donors the rate is always highly super-Eddington.
production of the wake, but it illustrates simply our procedure. Taking the neutron star to beat rest, the envelope matter is incident on it with the Keplerian velocity $v$. The rate of accretion is given by the Bondi–Hoyle–Lyttleton theory

$$\frac{dM_A}{dt} = \pi \rho v R_{ac}^2,$$

(15)

where $\rho$ is the density of the B material, $v$ is its velocity relative to the neutron star A, and $R_{ac}$ is the accretion radius

$$R_{ac} = 2GM_A v^{-2}.$$  

(16)

The rate of change of momentum $P$ is

$$\frac{dP}{dt} = v \frac{dM_A}{dt},$$

(17)

the matter being brought to rest on the neutron star, and this is equal to the force $F$. Consequently, the rate at which the neutron star does work in the material is

$$\dot{E} = Fv = v^2 \frac{dM_A}{dt}.$$  

(18)

Inclusion of the work done in creating the wake involves numerical calculations [29–31] with the result that the coefficient of the right-hand side of Eq. (18) is changed, i.e.,

$$\dot{E} = \left(\frac{c_d}{2}\right) v^2 \frac{dM_A}{dt},$$

(19)

with $c_d \sim 6–8$ for our supersonic flow. It is, in fact, very important that the wake plays such a large role, in that its the fact that $c_d/2 > 1$ (we consider $c_d/2$ to be $\gg 1$) that makes our later common envelope evolution strongly nonconservative, the proportion of the total H-envelope mass accreted onto the neutron star being relatively small.

In Eq. (19) $v^2$ is the velocity of the B (giant) material relative to A, the neutron star. This is given by

$$v^2 = G(M_A + M_B) a^{-1}.$$  

(20)

The interaction energy of A and B is

$$E = \frac{1}{2} GM_A M_B a^{-1}.$$  

(21)

Since we know $M_{B, i}$ and $M_{B, f}$, the initial mass of B and the mass of its He core, our unknown is $a_f$. We can obtain it by considering

$$Y = M_B a^{-1}$$  

(22)

as one variable, $M_A$ as the other. Differentiating Eq. (21) we have

$$\dot{E} = \frac{1}{2} G (\dot{M}_A Y + M_A \dot{Y}).$$  

(23)
whereas combining Eqs. (19) and (20) and neglecting $M_A$ with respect to $M_B$, we have

$$\dot{E} = G\left(\frac{c_d}{2}\right)YM_A.$$  \hspace{2cm} (24)

Thus, Eqs. (23) and (24) are equal, so we have

$$\frac{\dot{M}_A}{M_A} = \frac{1}{(c_d - 1) \dot{Y}},$$ \hspace{2cm} (25)

which can be integrated to give

$$M_A \propto Y^{1/(c_d - 1)} = Y^{1/5}$$ \hspace{2cm} (26)

where we have chosen $c_d = 6$ [29]. The final energy is then

$$E_f = \frac{1}{2} G M_{A,i} Y_f (Y_f / Y_i)^{6/5}.$$ \hspace{2cm} (27)

The binding energy $E_f$ of star A to star B serves to expel the envelope of star B, whose initial binding energy is given by Eq. (14). Mass transfer begins at the Roche Lobe which lies at ~ $0.6a_i$ for the masses involved. However, star B expands rapidly in red giant stage before the mass transfer can be completed. To keep the numbers easy to compare with Bethe and Brown [1], we use their approximation of starting spiral-in when the giant’s radius equals the orbital separation rather than the Roche-lobe radius. Since for the large mass ratios considered here, $R_L/a \sim 0.5$ for the giant, this implies we require $E_f$ of Eq. (27) to be about twice the binding energy (Eq. (14)), i.e.

$$E_f = \frac{0.6}{a} G \frac{M_{B,i}^2}{a_i} = 1.2 G \frac{M_{B,i}^2}{a_i}.$$ \hspace{2cm} (28)

(We set the common-envelope efficiency, $\alpha$, to 0.5.) The ejected material of B is, therefore, released with roughly the thermal energy it had in the envelope; in other words, the thermal energy content of the star is not used to help expel it. Inserting Eq. (28) into Eq. (27) yields

$$(Y_f / Y_i)^{1/2} = 2.4 M_{B,i}/M_{A,i}.$$ \hspace{2cm} (29)

Star A is initially a neutron star, $M_{A,i} = 1.4M_\odot$. For star B we assume $M_{B,i} = 15M_\odot$. Then Eq. (29) yields

$$Y_f / Y_i = 15.$$ \hspace{2cm} (30)

We use this to find the result of accretion, with the help of Eq. (26),

$$M_{A,f} / M_{A,i} = 1.73$$ \hspace{2cm} (31)

or

$$M_{A,f} = 2.4M_\odot.$$ \hspace{2cm} (32)

This is well above any of the modern limits for neutron star masses, so we find that the neutron star has gone into a black hole.

Our conclusion is, then, that in the standard scenario for evolving binary neutron stars, if the giant is deeply convective, accretion in the common envelope phase will convert the neutron star into a black hole.
Star B, by losing its envelope, becomes a He star. We estimate that
\[ \frac{M_{B,f}}{M_{B,i}} \simeq 0.3 . \tag{33} \]
The size of the orbit is determined by Eq. (22),
\[ \frac{a_f}{a_i} = \frac{M_{B,i}}{M_{B,f}} \frac{Y_f}{Y_i} = 50 . \tag{34} \]
The final distance between the stars \( a_f \) should not be less than about \( 10^{11} \) cm, so that the He star (mass \( M_{B,f} \)) fits within its Roche lobe next to the black hole of mass \( M_{A,f} \). Bethe and Brown [1] showed that if the black hole and the neutron star resulting from the explosion of star B are to merge in a Hubble time, then \( a_f < 3.8 \times 10^{11} \) (for circular orbits; correction for eccentricity will be given later). Therefore, the initial distance of the two stars, after the first mass exchange and the first supernova should be
\[ 0.5 \times 10^{13} \text{ cm} < a_i < 1.9 \times 10^{13} \text{ cm} . \tag{35} \]
If the initial distribution of distances is \( \text{da}/7a \), the probability of finding \( a \) between the limits of Eq. (35) is
\[ P = 18\% . \tag{36} \]
As noted earlier, 43% of the binaries survive the first explosion, so the combined probability is now
\[ P = 8\% \tag{37} \]
for the survivors falling in the logarithmic interval in which they survive coalescence, but are narrow enough to merge in a Hubble time. Our final result, following from a birth rate of \( 10^{-2} \) binaries per year in which one star goes supernova, half of which have both stars going supernova, is
\[ R = 10^{-2} \times 0.5 \times 0.08 \times 0.5 = 2 \times 10^{-4} \text{ yr}^{-1} \tag{38} \]
in the Galaxy. The final factor of 0.5 is the survival rate of the He-star, neutron star binary, calculated by Monte Carlo methods. Bethe and Brown [1] quoted \( 10^{-4} \text{ yr}^{-1} \), or half of this rate, in order to take into account some effects not considered by them in which the binary disappeared (e.g., Portegies Zwart and Verbunt [57]).
Our final rate is, then,
\[ R = 10^{-4} \text{ yr}^{-1} \text{ galaxy}^{-1} . \tag{39} \]
Using our supernova rate of 0.025 per year, which includes the case where both stars in the binary go supernova, we can convert this birth rate to 0.004 per supernova for comparison with other work. Portegies Zwart and Yungelson [20] in their case H, which included hypercritical accretion, got 0.0036 per supernova, within 10% of our value. Thus, the chief difference between our result in Eq. (39) and the \( R = 5.3 \times 10^{-5} \) of these authors is due to the different assumed SN rate.
In our above estimates we have assumed the second neutron star to be formed to have a circular orbit of the same \( a \) as its He-star progenitor. However, eccentricity in its orbit leads to a value of \( a_f \) substantially larger than the \( 3.8 \times 10^{11} \) cm used above as the maximum separation for merger. In general, most of the final binaries will have \( e > 0.5 \), with a heavy peak in the distribution close to \( e = 1 \). The rise occurs because preservation of the binary in the explosion is substantially greater if
the kick velocity is opposite to the orbital velocity before explosion. In this case the eccentricity $e$ is large. The most favorable situation is when the orbital and kick velocities are equal in magnitude. (See the figures in Wettig and Brown [32].) Eggleton [33] has kindly furnished us with a useful interpolation formula for the increase. The factor by which to multiply the time for merger in circular orbits, is

$$Z(e) \approx (1 - e^2)^{3.689 - 0.243e - 0.058e^2}.$$  \hfill (40)

This formula is accurate to about 1% for $e \leq 0.99$. Thus, if the initial eccentricity is 0.7, the time to shrink the orbit to zero is about 10% of the time required if the initial eccentricity were zero for the same initial period. The maximum $a_t = 3.8 \times 10^{11}$ cm for circular orbits would be increased by the fourth root of the decrease in time, i.e., up to $6.8 \times 10^{11}$ cm for this eccentricity. The maximum $a_t$ in Eq. (35) would go up to $3.4 \times 10^{13}$ cm, increasing the favorable logarithmic interval by $\sim 40\%$. We have not introduced this correction because it is of the same general size as the uncertainty in the supernova rate. However, this correction gives us some comfort that our final numbers are not unreasonably large.

If we produce an order of magnitude more low-mass black-hole, neutron-star binaries than binary neutron stars, the obvious question is why we have not seen any. The neutron star in this object is “fresh” (unrecycled) so it would spin down into the graveyard of neutron stars in $\sim 5 \times 10^6$ yr. The two relativistic binary pulsars we do see 1913 + 16 and 1534 + 12 have been recycled, have magnetic fields $B \sim 10^{10}$ G, two orders of magnitude less than a fresh pulsar, and will therefore be seen for about 100 times longer than an unrecycled neutron star. So even with a 10 times higher birth rate, we should see 10 times fewer LBH-NS binaries than NS–NS binaries. Furthermore, the binary with black hole will have a somewhat higher mass, therefore greater Doppler shift, and therefore be harder to detect. In view of the above, it is reasonable that our low-mass black-hole, neutron-star binaries have not been observed, but they should be actively looked for.

We should also calculate the rate of coalescences of the black hole with the He star. These have been suggested by Fryer and Woosley [34] as candidate progenitors for the long time gamma-ray bursters. Note that they will occur for a range of $0.04 \times 10^{13}$ cm < $a_t$ < $0.5 \times 10^{13}$ cm, a logarithmic interval double that of Eq. (35). Thus, the black-hole, He-star coalescence has a probability

$$P = 36\%.$$  \hfill (41)

Furthermore, this situation does not have the 50% disruption in the final explosion, so the black-hole, He-star coalescences occur with a total rate of 4 times that of the black-hole, neutron-star mergers.

There has been much discussion in the literature of the difficulties in common envelope evolution. We believe our model of deeply convective giants and hypercritical accretion offers an ideal case. Of course, the initiation of the common envelope evolution requires some attention, but it can be modeled in a realistic way [35]. As the giant evolves across its Roche lobe, the compact object creates a tidal bulge in the giant envelope, which follows the compact object, torquing it in. As the convective giant loses mass, the envelope expands in order to keep entropy constant. In Bondi–Hoyle–Lyttleton accretion, a density $\rho_x \sim 10^{-13}$ g cm$^{-3}$ is sufficient with wind velocities $\sim 1000$ km s$^{-1}$ in order to give accretion at the Eddington rate. Thus to achieve $\dot{M} \sim 10^8 M_{\text{Edd}} \sim 1 M_\odot$ yr$^{-1}$ we need $\rho \sim 10^{-5}$ g cm$^{-3}$ which is found at 0.9 $R$, where $R$ is the
radius of the giant. At this rate of accretion, angular momentum, etc., are hardly able to impede it appreciably. The total mass accreted onto the compact object is $\sim 1M_\odot$, so the common envelope evolution has dynamical time of years. As noted earlier, it is nonconservative.

4. Evolution of binary neutron stars

Since the standard scenario of evolution of binary compact objects ends up with low-mass black-hole, neutron-star binaries, another way must be found to evolve neutron star binaries. In the double He-star scenario was suggested by Brown [3] and developed further by Wettig and Brown [32] the neutron star avoids going through common envelope with a companion star. In this way the neutron star can avoid being converted into a black hole by accretion. For two giants to burn He at the same time, they must be within $\sim 5\%$ of each other in mass, the helium burning time being $\sim 10\%$ of the main sequence lifetime, and stellar evolution time going roughly with the inverse square of the mass. With a flat mass ratio distribution, this happens in $5\%$ of all cases, making the ratio of NS–NS to NS-LBH binaries 1:20. However, when the primary becomes an LBH, only half the secondaries will be massive enough to form a NS, whereas for the very close mass values of the double-He scenario this factor 2 loss does not occur. Thus, binary neutron stars should be formed $10\%$ as often as low-mass black-hole, neutron-star binaries. This $10\%$ is nearly model independent because everything else roughly scales.

The scenario goes as in Wettig and Brown [32]. The primary O-star evolves transferring its H-envelope to the companion. Often, this would lead to ‘rejuvenation’ of the secondary, i.e. its evolution would restart also from the ZAMS with the now higher total mass, and it would make a much heavier core. However, here the core of the secondary has evolved almost as far as the primary’s core, so the core molecular weight is much higher than that of the envelope. This prevents convection in the core from extending into the new envelope to make the bigger core, so no rejuvenation takes place [36]. Since $q \sim 1$, the first mass transfer is nearly conservative. The second is not, so the two He-cores then share a common H envelope, which they expel, while dropping to a lower final separation $a_f$.

Following the explosion of the first He star, the companion He-star pours wind matter onto the pulsar, bringing the magnetic field down and spinning it up [3,32]. The end result is two neutron stars of very nearly equal mass, although wind accretion can change the mass 2-3%.

The above scenario ends for He-star masses greater than 4 or $5M_\odot$, corresponding to ZAMS masses greater than $\sim 16$ or $18M_\odot$. However, less massive He stars evolve in the He shell-burning stage, and a further mass transfer (Case C) can take place. The transfer of He to the pulsar can again bring about a black hole, which Brown [37] very roughly estimates to occur in $\sim 50\%$ of the double-neutron star binaries. This is roughly consistent with results of Fryer and Kalogera [38].

Taking a rate of $R = 10^{-4}$ per year per galaxy for the low-mass black-hole, neutron-star binaries, we thus arrive at a birth rate of

$$R \approx 5 \times 10^{-6} \text{ per year per galaxy}$$

for binary neutron-star formation. However, the black holes formed in the He shell burning evolution will not have accreted much mass and will have about the same chirp mass as binary neutron stars (see below) for gravitational merging.
Our best guess values, Eqs. (39) and (42), thus give an $\sim 20$ to 1 ratio for formation of low-mass black-hole, neutron-star binaries to binary neutron stars. The former are better progenitors for gravitational waves from mergers because of their higher masses and they have many advantages as progenitors of gamma-ray bursters [39]. Note that our estimated rate of $R = 5 \times 10^{-6}$ per galaxy per year for binary neutron star formation is consistent with the empirical rates discussed in our introduction.

5. High-mass black-hole O/B-star binaries

We will be brief in our review of these, because we believe the evolution of these objects such as Cyg X-1, LMCX-1 and LMCX-3 to be less well understood than the low-mass black-hole, neutron-star binaries. Evolutionary calculations now proceeding by Alexander Heger, using the CO cores evolved by Wellstein and Langer [10] should clarify this situation substantially.

Bethe and Brown [8] arrived at a limit of ZAMS mass $80M_\odot$ for stars in binaries to go into high-mass black holes (unless Case C mass transfer takes place as we discuss in our next section). This limiting mass is much higher than other workers have used. It was based on calculations of Woosley et al. [17] and was so high because of very high mass loss rates used by these authors. With more correct lower rates the limiting mass may come down, so the Bethe and Brown evolution should be viewed as giving a lower limits to the number of high-mass black-hole, O/B-star binaries. Their estimated birth rate of about $3 \times 10^{-5}$ per galaxy per year does agree reasonably well with the fact that only one such system is known in the Galaxy. However, since even with a twice larger separation the accretion rate of the black hole from the fast wind of the O star becomes small, it is possible that substantially more systems with somewhat wider orbits exist undetected, and that Cyg X-1 is the only one presently in the (very short) phase of incipient Roche lobe overflow when it is bright. Bethe and Brown [8] found this narrowness of the Cyg X-1 orbit ($40R_\odot$ according to Herrero et al. [4]) to be puzzling: the massive stars in the progenitor binary initially had to fit within their Roche Lobes, therefore a separation of at least double the current $17R_\odot$ was needed. And most evolutionary effects from then on, such as wind mass loss or supernova-like mass loss, would tend to widen the orbit. Of course, the orbit could be narrowed in Case A mass transfer (i.e. during the main sequence) since the progenitor of the black hole was more massive than the present donor, but it could not become so narrow that the present donor filled its Roche lobe, and would widen again once the mass ratio became reversed and widen further due to wind loss after the whole primary envelope was lost.

In any case, a binary as narrow as Cyg X-1 would coalesce in the common envelope evolution once the O-star companion of the massive black-hole goes into red giant phase, according to the Bethe and Brown [8] estimates. Since the black hole in Cyg X-1 has mass $\gtrsim 10M_\odot$ and is probably the most massive black hole in a binary observed in the Galaxy, in the Fryer and Woosley [34] model where the black hole “eats” the W.-R. companion, such a coalescence should produce the most energetic long-lasting gamma-ray burster. We are unable to evaluate the probability of Cyg X-1 like objects merging following common envelope evolution because we have been unable to understand why Cyg X-1, before common envelope evolution, is so narrow. The LBV, RSG, and WNL stages of W.-R. development are not quantitatively understood.
After the main sequence star in a Cyg X-1-like object explodes and becomes a neutron star, according to Bethe and Brown [8] the binary will eventually merge. They estimated the contribution to the merger rate of these systems to be \((4-6) \times 10^{-6} \text{ yr}^{-1} \text{ galaxy}^{-1}\), however with considerable uncertainty due to the fact that the evolution of Cyg X-1 itself is uncertain. Lowering the mass limit for black-hole formation by having lower mass loss rates would increase this number (e.g. a limit of \(40 M_\odot\) would increase the merger rate by a factor 5).

6. The formation of high-mass black holes in low-mass X-ray binaries

6.1. General

Crucial to our discussion here is the fact that single stars evolve very differently from stars in binaries that lose their H-envelope [6,10,13,19] either on the main sequence (Case A) or in the giant phase (Case B). However, stars that transfer mass or lose mass after core He burning (Case C) evolve, for our purposes, as single stars, because the He core is then exposed too close to its death for wind mass loss to significantly alter its fate. Single stars above a ZAMS mass of about \(20 M_\odot\) skip convective carbon burning following core He burning, with the result, as we shall explain, that their Fe cores are substantially more massive than stars in binaries, in which H-envelope has been transferred or lifted off before He core burning. These latter “naked” He stars burn \(^{12}\text{C}\) convective-ly, and end up with relatively small Fe cores. The reason that they do this has to do chiefly with the large mass loss rates of the “naked” He cores, which behave like W.-R.’s. Unfortunately, in calculation until recently, substantially too large mass loss rates were used, so we cannot pin limits down quantitatively. In this section we will deal with the ZAMS mass range \(\sim 20-35 M_\odot\), in which it is clear that many, if not most, of the single stars go into high-mass black holes, whereas stars in binaries which burn “naked” He cores go into low-mass compact objects. In this region of ZAMS masses the use of too-high He-star mass loss rates does not cause large effects [6].

The convective carbon burning phase (when it occurs) is extremely important in pre-supernova evolution, because this is the first phase in which a large amount of entropy can be carried off in \(\nu\bar{\nu}\)-pair emission, especially if this phase is of long duration. The reaction in which carbon burns is \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) (other reactions like \(\text{C} + \text{C}\) would require excessive temperatures). The cross section of \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) is still not accurately determined; the lower this cross section the higher the temperature of the \(^{12}\text{C}\) burning, and therefore the more intense the \(\nu\bar{\nu}\) emission. With the relatively low \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) rates determined both directly from nuclear reactions and from nucleosynthesis by Weaver and Woosley [42], the entropy carried off during \(^{12}\text{C}\) burning in the stars of ZAMS mass \(\sim 10-20 M_\odot\) is substantial. The result is rather low-mass Fe cores for these stars, which can evolve into neutron stars. Note that in the literature earlier than Weaver and Woosley [42] often large \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) rates were used, so that the \(^{12}\text{C}\) was converted into oxygen and the convective burning did not have time to be effective. Thus its role was not widely appreciated.

Of particular importance is the ZAMS mass at which the convective carbon burning is skipped. In the Woosley and Weaver [18] calculations this occurs at ZAMS mass \(19 M_\odot\) but with a slightly lower \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) rate it might come at \(20 M_\odot\) or higher [37]. As the progenitor mass increases, it follows from general polytropic arguments that the entropy at a given burning stage increases. At the higher entropies of the more massive stars the density at which burning occurs is lower, because
Fig. 1. Compact core masses resulting from the evolution of single stars, Case B of solar metallicity of Woosley and Weaver [18]. The horizontal dashed lines indicate the mass of the heaviest known well-measured pulsar [16], the maximum mass of a neutron star, and our estimate of $M_{PC}$ (proto-compact), the maximum compact core mass for which matter can be returned to the galaxy.

the temperature is almost fixed for a given fuel. Lower densities decrease the rate of the triple-$\alpha$ process which produces $^{12}\text{C}$ relative to the two-body $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ which produces oxygen. Therefore, at the higher entropies in the more massive stars the ratio of $^{12}\text{C}$ to $^{16}\text{O}$ at the end of He burning is lower. The star skips the long convective carbon burning and goes on to the much shorter oxygen burning. Oxygen burning goes via $^{16}\text{O} + ^{16}\text{O}$ giving various products, at very much higher temperature than $C(\alpha,\gamma)$ and much faster. Since neutrino cooling during the long carbon-burning phase gets rid of a lot of entropy of the core, skipping this phase leaves the core entropy higher and the final Chandrasekhar core fatter.

In Fig. 1 the large jump in compact object mass in single stars at ZAMS mass $\sim 19M_\odot$ is clearly seen. From our discussion in Section 2 we see that this is just at the point where the Fe core mass goes above the proto-compact mass of $\sim 1.8M_\odot$ and, therefore, above this mass one would expect single stars to go into high-mass black holes. Arguments have been given that SN 1987A with progenitor ZAMS mass of $\sim 18M_\odot$ evolved into a low-mass black hole [12]. We believe from our above arguments and Fig. 1 that just above the ZAMS mass of $\sim 20M_\odot$, single stars go into high-mass black holes without return of matter to the Galaxy. Thus, the region of masses for low-mass black hole formation in single stars is narrow, say $\sim 18$–$20M_\odot$, although we believe it to be much larger in binaries.

Thus far our discussion has been chiefly about single stars, in which the He burns “clothed” by a hydrogen envelope. In this case the convective helium core grows in stars as time passes. In the “naked” He cores, in which the H envelope has been lifted off in RLOF or driven off by wind either before or early in the He burning the temperature and the entropy will be slightly lower, because the insulating layer is gone, so it is not surprising that their carbon abundance is large. Furthermore, the core mass continually decreases because of mass loss by wind. In fact, even for the naked $20M_\odot$ He core, corresponding to ZAMS mass $45M_\odot$, the central carbon abundance was $\sim 33\%$ at the end of He core burning [18] whereas only $\sim 15\%$ is necessary for convective carbon burning.
For lower mass He stars the $^{12}\text{C}$ abundance was, of course, larger. Even with He-star wind mass loss rates reduced by half, Wellstein and Langer [10] find a central carbon abundance of $\gtrsim 1/3$ at the end of He core burning all the way up through $60M_\odot$ stars, so it is clear that convective carbon burning will take place. Unfortunately, the cores have not yet been evolved past the CO stage. Thus, in the range of ZAMS masses up to $\geq 60M_\odot$, if the H envelope is lifted off early in the core He burning phase, the convective carbon burning will take place after the He burning.

By ZAMS mass $\sim 40M_\odot$, where stars evolve into WR stars almost independent of whether they have a companion, the ultimate fate of the compact core is uncertain: Brown et al. [13] suggest that 1700-37, with a progenitor of about $40M_\odot$ went into a low-mass black hole. This would seem to indicate that the H-envelope of such massive stars is blown off in an LBV phase rapidly enough that the He core again burns as “naked”. In any case, $^{12}\text{C}$ is burned convectively following He core burning, so the resulting Fe core should be small.

We believe that our discussion earlier in this section indicates that single stars in the region of ZAMS masses $\sim 20-35M_\odot$ end up as high-mass black holes. We can obtain the high-mass black holes, according to our above discussion, if we make the He-stars burn with “clothing”, i.e., lift their H-envelope off only following He core burning. Thus, the evolving massive star should meet the companion main sequence star only following He core burning (in the supergiant stage). By then its radius $R$ is several hundred $R_\odot$, and its binding energy $0.6GM^2/R$, very small because of the large $R$. In order to see effects of matter stripped off from the main sequence companion in the transient sources, we want it to end up close to the black hole. Because of its low binding energy the supergiant envelope will be expelled by a relatively small binding energy of the companion, $\frac{1}{2}M_\star M_{B,i}/a_i$ where $a_i$ is the distance between black hole and companion. In order to make $a_i$ small the mass $M_\star$ of the companion must be small. (More massive main sequence stars will spiral in less far, hence end up further from the black hole, and not fill their Roche Lobes. However, when they evolve in subgiant or giant phase they will fill it.) Both Portegies Zwart et al. [43] and Ergma and Van den Heuvel [44] have suggested that roughly the above region of ZAMS masses must be responsible for the $\sim 7M_\odot$ black holes in the transient X-ray sources in order to form enough such sources. Our scenario is essentially the same as that of de Kool et al. [45] for the black hole binary A0620-00. We refer to this work for the properties of the K-star companion, stressing here the evolutionary aspects of the massive black hole progenitor.

6.2. Calculation

We now calculate the common envelope evolution following the formalism of Section 3. Here $M_A$ is the mass of the main sequence companion, $M_B$ that of the massive black hole progenitor. The ratio

$$q = \frac{M_{\star,i}}{M_{B,i}}$$  \hspace{1cm} (43)

is very small and there is great uncertainty in the initial number of binaries for such a small $q \sim 1/25$. We again take the distribution as $dq$, and again assume $\ln a$ to be uniformly distributed over a logarithmic interval of 7. Again, the fraction of binaries in a given interval is

$$d\phi = d(ln a)/7.$$  \hspace{1cm} (44)
We evolve as typical a $25M_\odot$ star (B) with a companion $\sim 1M_\odot$ main sequence star (star A) as the progenitor of the transient X-ray sources. The common envelope evolution can be done as in Section 3. With $M_{B,i} = 25M_\odot$ and neglect of the accretion onto the main sequence mass $M_A$, we find from Bethe and Brown [1]

$$\left(\frac{Y_f}{Y_i}\right)^{1.2} = \frac{1.2}{\zeta_{ce}} \frac{M_{B,i}}{M_\odot},$$

(45)

where $Y = M_B/a$. Here the coefficient of dynamical friction $c_d$ was taken to be 6. The result is relatively insensitive to $c_d$, the exponent 1.2 resulting from $1 + 1/(c_d - 1)$.

Thus, in our case

$$\frac{Y_f}{Y_i} = 17 \left(\frac{\zeta_{ce} M_A}{M_\odot}\right)^{-0.83} = 30 \left(\frac{0.5 M_\odot}{\zeta_{ce} M_A}\right)^{0.83}.$$  

(46)

We expect $\zeta_{ce} \approx 0.5$, under the assumption that the thermal energy of the expelled envelope is equal to that it originally possessed in the massive star (i.e. that it is not used as extra energy to help remove the envelope), but it could be smaller. From this we obtain

$$\frac{a_i}{a_t} = \frac{M_{B,i} Y_f}{M_{B,f} Y_i} = 90 \left(\frac{0.5 M_\odot}{\zeta_{ce} M_A}\right)^{0.83},$$

(47)

where we have taken the He star mass $M_{B,t}$ to be 1/3 of $M_{B,i}$. In order to survive spiral-in, the final separation $a_t$ must be sufficient so that the main sequence star lies at or inside its Roche Lobe, about $0.2a_t$ if $M_A = M_\odot$. This sets $a_t \approx 5R_\odot = 3.5 \times 10^{11}$ cm and

$$a_i = 3.15 \left(\frac{0.5}{\zeta_{ce}}\right)^{0.83} \times 10^{13} \text{ cm},$$

(48)

which is about 2 AU. This exceeds the radius of the red giant tip in the more numerous lower mass stars in our interval, so the massive star must generally be in the supergiant phase when it meets the main sequence star, i.e., the massive star must be beyond He core burning. E.g., the red giant tip (before the He core burning) for a $20M_\odot$ star is at $0.96 \times 10^{13}$ cm, for a $25M_\odot$ star, $2.5 \times 10^{13}$ cm [46]. These numbers are, however, somewhat uncertain. Notice that decreasing $\zeta_{ce}$ will increase $a_i$. Decreasing $M_A$ has little influence, because with the smaller stellar radius the minimum $a_t$ will decrease nearly proportionately. Note that neglect of accretion onto the main sequence star would change the exponent 0.83 to unity, so accretion is unimportant except in increasing the final mass.

Now a ZAMS $25M_\odot$ star ends up at radius $6.7 \times 10^{13}$ cm ($\sim 2a_i$) following He shell burning [47]. Thus, the interval between $a_t$ and $6.7 \times 10^{13}$ cm is available for spiral-in without merger so that a fraction

$$\frac{1}{7} \ln \left(\frac{6.7}{3.15(0.5/\zeta_{ce})^{0.83}}\right) \approx 0.11$$

(49)

of the binaries survive spiral-in, but are close enough so that the main sequence star is encountered by the evolving H envelope of the massive star. The He core burning will be completed before the supergiant has moved out to $\sim 2$ AU, so binaries which survive spiral-in will have He cores which burn as “clothed”, namely as in single stars.
Given our assumptions in Section 3, the fraction of supernovas which arise from ZAMS stars between 20 and $35M_\odot$ is

$$1/2^{3/2} - 1/3.5^{3/2} = 0.20 ,$$

(50)

where we have assumed the mass $10M_\odot$ is necessary for a star to go supernova. A Salpeter function with index $n = 1.5$ is assumed here. Our assumption that the binary distribution is as $dq$ is arbitrary, and gives us a factor 1/25 for a $1M_\odot$ companion. Thus, for supernova rate 2 per century, our birth rate for transient sources in the Galaxy is

$$2 \times 10^{-2} \times 0.5 \times 0.11 \times 0.20 \times 0.04 \approx 8.8 \times 10^{-6} \text{yr}^{-1}$$

(51)

where 0.5 is the assumed binarity, 0.11 comes from Eq. (49), and the final (most uncertain) factor 0.04 results from a distribution flat in $q$ and an assumed $1M_\odot$ companion star.

In order to estimate the number of transient sources with black holes in the Galaxy, we should know the time that a main sequence star of mass $\sim 1M_\odot$ transfers mass to a more massive companion. This depends on the angular-momentum loss rate that drives the mass transfer. A guaranteed loss mechanism for close binaries is gravitational radiation, which for a main-sequence donor gives a mass transfer rate of $10^{-10}M_\odot \text{yr}^{-1}$, almost independent of donor mass [48]. As mass is transferred, the mass of the donor decreases and with it the radius of the donor. Quite a few low-mass X-ray binaries have X-ray luminosities that imply accretion rates in excess of $10^{-6}M_\odot \text{yr}^{-1}$, leading to suggestions of additional mechanisms for loss of angular momentum from the binary, to increase mass transfer. Verbunt and Zwaan [49] estimate that magnetic braking can boost the transfer of mass in a low-mass binary. We somewhat arbitrarily adopt an effective mass transfer rate of $10^{-9}M_\odot \text{yr}^{-1}$ for main sequence stars. In order to estimate the number of high-mass black hole, main sequence star binaries in the Galaxy we should multiply the birth rate equation (51) by the $10^9 \text{yr}$ required, at the assumed mass loss rate, to strip the main sequence star, obtaining 8800 as our estimate. From the observed black-hole transient sources, Wijers, [5] arrives at 3000 low-mass black hole sources in the Galaxy, but regards this number as a lower limit. With the uncertainties in formation rate and lifetime, the agreement between the two numbers is as good as may be expected.

6.3. Observations

We believe that there are many main sequence stars more massive than the $\lesssim 1M_\odot$ we used in our schematic evolution, which end up further away from the black hole and will fill their Roche Lobe only during subgiant or giant stage. From our earlier discussion, we see that a $2M_\odot$ main sequence star will end up about twice as far from the black hole as the $1M_\odot$, a $3M_\odot$ star, three times as far, etc. Two of the 9 systems in our Table 1 have subgiant donors (V404 Cyg and XN Sco). These have the longest periods, 6.5 and 2.6 days and XN Sco is suggested to have a relatively massive donor of $\sim 2M_\odot$. It seems clear that these donors sat inside their Roche Lobes until they evolved off the main sequence, and then poured matter onto the black hole once they expanded and filled their Roche Lobe. For a $2M_\odot$ star, the evolutionary time is about a percent of the main-sequence time, so the fact that we see two subgiants out of nine transient sources means that
Table 1
Parameters of suspected black hole binaries with measured mass functions [5,50–54]. N means nova, XN means X-ray nova. Numbers in parentheses indicate errors in the last digits

<table>
<thead>
<tr>
<th>X-ray names</th>
<th>Other name(s)</th>
<th>Compan. type</th>
<th>$P_{\text{orb}}$ (d)</th>
<th>$f(M_X)$ ($M_\odot$)</th>
<th>$M_{\text{opt}}$ ($M_\odot$)</th>
<th>$(l,b)$</th>
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<tr>
<td></td>
<td></td>
<td>$q$ ($M_{\text{opt}}/M_X$)</td>
<td>$K_{\text{opt}}$ (km s$^{-1}$)</td>
<td>$i$ (deg)</td>
<td>$M_X$ ($M_\odot$)</td>
<td>$d$ (kpc)</td>
</tr>
<tr>
<td>Cyg X-1</td>
<td>V1357 Cyg</td>
<td>O9.7 Iab</td>
<td>5.5996</td>
<td>0.25(1)</td>
<td>33(9)</td>
<td>(73.1, + 3.1)</td>
</tr>
<tr>
<td>1956 + 350</td>
<td>HDE 226868</td>
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<td>74.7(10)</td>
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<td>16(5)</td>
<td>2.5</td>
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<tr>
<td>LMC X-3</td>
<td>B3 Ve</td>
<td></td>
<td>1.70</td>
<td>2.3(3)</td>
<td>5.6–7.8</td>
<td>(273.6, – 32.1)</td>
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<tr>
<td>0538-641</td>
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<td>235(11)</td>
<td></td>
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<tr>
<td>LMC X-1</td>
<td>O7–9 III</td>
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<td>0.14(5)</td>
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<td>(280.2, – 31.5)</td>
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<td>68(8)</td>
<td></td>
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<td>55</td>
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<tr>
<td>XN Mon 75</td>
<td>V616 Mon</td>
<td>K4 V</td>
<td>0.3230</td>
<td>2.83–2.99</td>
<td>0.53–1.22</td>
<td>(210.0, – 6.5)</td>
</tr>
<tr>
<td>A0620-003</td>
<td>N Mon 1917</td>
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<td>0.057–0.077</td>
<td>37–44a</td>
<td>9.4–15.9</td>
<td>0.66–1.45</td>
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<tr>
<td>XN Oph 77</td>
<td>V2107 Oph</td>
<td>K3 V</td>
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<td>4.44–4.86</td>
<td>0.3–0.6</td>
<td>(358.6, + 9.1)</td>
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<td>420(30)</td>
<td>60–80</td>
<td>5.2–8.6</td>
<td>5.5:</td>
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<td>XN Vul 88</td>
<td>QZ Vul</td>
<td>K5 V</td>
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<td>0.17–0.97</td>
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<td>43–74</td>
<td>5.8–18.0</td>
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<tr>
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<td>V404 Cyg</td>
<td>K0 IV</td>
<td>6.4714</td>
<td>6.02–6.12</td>
<td>0.57–0.92</td>
<td>(73.2, – 2.2)</td>
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<td>0.055–0.065</td>
<td>208.5(7)</td>
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<td>10.3–14.2</td>
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<td>V15 Cyg</td>
<td>K5 V</td>
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<td>2.86–3.16</td>
<td>0.41–1.4</td>
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<td>GS1124-683</td>
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<td>406(7)</td>
<td>54–65</td>
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<td></td>
<td>0.2127(7)</td>
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<td>0.10–0.97</td>
<td>(197.3, – 11.9)</td>
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<td></td>
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<td>0.029–0.069</td>
<td>380.6(65)</td>
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<td>3.4–14.0</td>
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<td>XN Sco 94</td>
<td>F5-G2</td>
<td></td>
<td>2.6127(8)</td>
<td>2.64–2.82</td>
<td>1.8–2.5</td>
<td>(345.0, + 2.2)</td>
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<tr>
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<td></td>
<td></td>
<td>0.33–0.37</td>
<td>227(2)</td>
<td>67–71</td>
<td>5.5–6.8</td>
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<tr>
<td>XN 4U1543-47</td>
<td>MX1543-475</td>
<td>A2 V</td>
<td>1.123(8)</td>
<td>0.20–0.24</td>
<td>1.3–2.6</td>
<td>(330.9, + 5.4)</td>
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<tr>
<td>XN Vel 93</td>
<td>K6-M0</td>
<td></td>
<td>0.2852</td>
<td>3.05–3.29</td>
<td>0.50–0.65</td>
<td>9.1(11)</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>0.137 ± 0.015</td>
<td>475.4(59)</td>
<td>~ 78</td>
<td>3.64–4.74</td>
</tr>
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</table>

*A much higher inclination for A0620 has been claimed by Haswell et al. [55] of up to $i = 70$. In this case, the lower limits on the component masses would be $M_X > 3.8$ and $M_{\text{opt}} > 0.22.$
many more of these massive donors are sitting quietly well within their Roche Lobes. Indeed, we could estimate from the relative time, that there are \( \frac{2}{9} \times 100 = 22 \) times more of these latter quiet main sequence stars in binaries.

Amazingly, this factor 22 almost cancels the 1/25 we had for the interval in \( q \) over which the donors contribute. This is not coincidental. Essentially any mass donor, at least almost up to the \( 25M_\odot \) progenitor of the black hole, can give rise to a common envelope phase. The BH progenitor crosses the Herzsprung gap very quickly, in a time in which the companion can hardly accept its mass. (The ratio of \( q \lesssim 1/4 \) for common envelope evolution was determined by Kippenhahn and Meyer-Hofmeister [56] for case A mass transfer.) Thus, one can expect essentially all companions, up to \( q \lesssim 1 \), to go into common envelope evolution and contribute. Beginning from Wijers’ empirical estimate we would thus have \( (2/9) \times 100 \times 3000 = 6.7 \times 10^4 \) binaries with high-mass black holes and main-sequence companions. This number is determined, as shown above, chiefly by the number of observed systems with subgiant donors.

If we assume that ZAMS masses \( \sim 10–18M_\odot \) evolve into a neutron star, we should have \( \sim 3 \) times more neutron stars than high-mass black holes (see Eq. (50)). The range follows from our belief that SN 1987A with progenitor \( \sim 18M_\odot \) ZAMS went into a low-mass black hole, following the scenario of Brown and Bethe [12]. On the basis of a Monte Carlo calculation using the kick velocities of Cordes and Chernoff [21] we find that \( \sim 1/2 \) of the binaries containing He-star, low-mass main sequence companion (with \( M \approx 1M_\odot \)) will be disrupted in the explosion. Thus, we find only a slightly higher birth rate for LMXBs (low mass X-ray binaries) with neutron stars than with black holes, although the numbers could be equal to within our accuracy. With comparable lifetimes (since the donor masses and mass transfer rates are comparable), this would give us one to a few thousand LMXBs with neutron stars, much above the total number of observed LMXBs (\( \sim 130 \)). Indeed, from Table 6 of Portegies Zwart and Verbunt [57] one sees that their estimated empirical birth rate for low-mass X-ray binaries is \( 2 \times 10^{-7} \) yr\(^{-1} \), whereas in either theoretical evolution including kick velocities they obtain \( 4 \times 10^{-6} \) yr\(^{-1} \). This factor of 20 discrepancy is by far the greatest between theoretical and empirical rates in their table, and supports our point that many of the neutron stars must have disappeared along the way. Alternatively, a large number of LMXBs with neutron stars could be transients as well (like e.g. Aql X-1). Just at the present there are new developments in the evolution of low-mass X-ray binaries, which we shall shortly summarize in Section 7.

As we showed below Eq. (48), the He core of the massive star will in general be uncovered only after He core burning is completed. The remaining time for He burning (in a shell) will be short, e.g., for a \( 20M_\odot \) ZAMS star it is only \( 1.4 \times 10^4 \) yr [46]. Therefore the mass loss by wind after uncovering the He core will not be large, and when the star finally becomes a supernova, its mass will be almost equal to the He core of the original star. The latter can be calculated from

\[
M_{\text{He}} \approx 0.10 \ (M_{\text{ZAMS}})^{1.4}
\]  

so for ZAMS masses \( 20–35M_\odot \) \( M_{\text{He}} \) will lie in the interval \( \sim 7–14M_\odot \).

Bailyn et al. [51] find the black-hole masses in transient sources to be clustered about \( \sim 7M_\odot \), except for V404 Cyg which has a higher mass. This is in general agreement with our scenario, because most of the black holes will come from the more numerous stars of ZAMS mass not far
from our lower limit of $\sim 20M_\odot$. Two points are important to note:

(1) Not much mass can have been lost by wind. Naked He stars have rapid wind loss. However, in our scenario the He star is made naked only during He shell burning and therefore does not have much time ($\lesssim 10^4$ yr) to lose mass by wind.

(2) There are good reasons to believe that the initial He core will be rotating [58]. The way in which the initial angular momentum affects the accretion process has been studied by Mineshige et al. [59] for black-hole accretion in supernovae. In general, accretion discs which are optically thick and advection dominated are formed. The disc is hot and the produced energy and photons are advected inward rather than being radiated away. The disc material accretes into the black hole at a rate of $>10^6 \dot{M}_{\rm Edd}$ for the first several tens of days. Angular momentum is advected outwards. Our results show that little mass is lost, because the final $\sim 7M_\odot$ black hole masses are not much less massive than the He core masses of the progenitors, and some mass is lost by wind before the core collapses. The latter loss will not, however, be great, because there is not much time from the removal of the He envelope until the collapse.

Accretion of the He into the black hole will differ quantitatively from the above, but we believe it will be qualitatively similar. The fact that the helium must be advected inwards and that little mass is lost as the angular momentum is advected outwards is extremely important to establish. This is because angular momentum, essentially centrifugal force, has been suggested by Chevalier [28] to hold up hypercritical accretion onto neutron stars in common envelope evolution. (Chevalier [27] had first proposed the hypercritical accretion during this evolutionary phase to turn the neutron stars into black holes, the work followed up by Brown [3] and Bethe and Brown [1].) However, once matter is advected onto a neutron star, temperatures $\gtrsim 1$ MeV are reached so that neutrinos can carry off the energy. The accreted matter simply adds to the neutron star mass, evolving into an equilibrium configuration. Thus, this accretion does not differ essentially from that into a black hole. In either case of neutron star or black hole an accretion disc or accretion shock, depending on amount of angular momentum, but both of radius $\sim 10^{11}$ cm, is first formed, giving essentially the same boundary condition for the hypercritical accretion for either black hole or neutron star. Thus, the masses of the black holes in transient sources argue against substantial inhibition of hypercritical accretion by jets, one of the Chevalier suggestions [28].

Measured mass functions, which give a lower limit on the black hole mass are given in Table 1. Only GRO J0422 + 32 and 4U 1543-47 have a measured mass function $\lesssim 3M_\odot$. Results of Callanan et al. [60] indicate that the angle $i$ between the orbital plane and the plane of the sky for GRO J0422 + 32 is $i < 45^\circ$, and recent analysis [52] indicate that the angle $i$ for 4U 1543-47 is $20^\circ < i < 40^\circ$. So both GRO J0422 + 32 and 4U 1543-47 also contain high-mass black holes.

Based on the observations of Kaper et al. [61] that the companion is a hypergiant, Ergma and Van den Heuvel [44] argue that the progenitor of the neutron star in 4U1223-62 must have a ZAMS mass $\gtrsim 50M_\odot$. Brown et al. [13], by similar argumentation, arrived at $\sim 45M_\odot$, but then had the difficulty that 4U1700-37, which they suggested contains a low-mass black hole, appeared to evolve from a lower mass star than the neutron star in 1223. Wellstein and Langer [10] suggest the alternative that in 1223 the mass occurs in the main-sequence phase (Case A mass transfer), which would be expected to be quasi conservative. They find that the progenitor of the neutron star in 1223 could then come from a mass as low as $26M_\odot$. This is in agreement with Brown et al. [13]
for conservative mass transfer (their Table 1), but these authors discarded this possibility, considering only Case B mass transfer in which case considerable mass would be lost.

Wellstein and Langer [10] are in agreement with Brown et al. [13] that 4U1700-37 should come from a quite massive progenitor. Conservative evolution here is not possible because of the short period of 3.4 days [62]. The compact object mass is here $1.8 \pm 0.4 M_\odot$ [63]. Brown et al. [13] suggest that the compact object is a low-mass black hole. The upper mass limit for these was found by Brown and Bethe [12] to be $\sim 1.8 M_\odot$, as compared with an upper limit for neutron star masses of $\sim 1.5 M_\odot$. Thus, there seems to be evidence for some ZAMS masses of $\sim 40-50 M_\odot$ ending up as low-mass compact objects, whereas we found that lower mass stars in the interval from $\sim 20$ to $35 M_\odot$ ended up as high-mass black holes. In this sense we agree with Ergma and Van den Heuvel [44] that low-mass compact object formation “is connected with other stellar parameters than the initial stellar mass alone”. We suggest, however, following Brown et al. [13] that stars in binaries evolve differently from single stars because of the different evolution of the He core in binaries resulting from RLOF in their evolution. Namely, “naked” He cores evolve to smaller final compact objects than “clothed” ones.

In fact, this different evolution of binaries was found by Timmes et al. [19]. They pointed out that stars denuded of their hydrogen envelope in early RLOF in binaries would explode as Type Ib supernovae. They found the resulting remnant gravitational mass following explosion to be in the interval of $1.2-1.4 M_\odot$, whereas in exploding stars of all masses with hydrogen envelope (Type II supernova explosion) they found a peak at about $1.28 M_\odot$, chiefly from stars of low masses and another peak at $1.73 M_\odot$ more from massive stars. Our Fe core masses in Fig. 1 come from essentially the same calculations, but the “Remnant” masses of Woosley and Weaver [18] are somewhat greater than those used by Timmes et al. [19]. In fact, the differences between the masses we plot and those of Timmes et al. come in the region $\sim 1.7-1.8 M_\odot$ (gravitational). This is just in the Brown and Bethe [12] range for low-mass black holes. It may be that some of the stars with low-mass companions evolve into low-mass black holes. Presumably, these would give lower luminosities than the high-mass black holes, although at upper end of the mass range we discuss 4U1700-37 seems to be an example of such a system. Of course, here the high luminosity results from the high mass loss rate of the giant companion. There are substantial ambiguities in fallback, etc., from the explosion. Our point in this paper is that most of the higher mass single stars $20-35 M_\odot$ go into high-mass black holes. (The Brown and Bethe [12] limit for low-mass black-hole formation is $\sim 1.5-1.8 M_\odot$ gravitational, but there is some give and take in both lower and upper limit. Also the stars are not all the same. In particular, different metallicities will give different wind losses.)

7. Evolution of low-mass X-ray binaries

We shall briefly point out new developments in the evolution of low-mass X-ray binaries. These were foreseen in the excellent review by Van den Heuvel [64], and there has been substantial development in this field lately.

Low-mass X-ray binaries are considered to be progenitors of recycled pulsars with helium white dwarf companions. In order to bring the magnetic fields of the latter down to $\sim 10^8$ G and to speed them up to their final period, Van den Heuvel and Bitzaraki [65] had the neutron star accreting $\sim 0.5 M_\odot$ from the main-sequence donor. More detailed recent calculations by Tauris and
Savonije [66] find that if the initial orbital period is below ~30 days with a main sequence donor of ~1M\(\odot\) which undergoes stable mass transfer with the neutron star, the mass of the latter is increased up to ~2M\(\odot\) if the amount of material ejected as a result of propeller effect or disk instabilities is insignificant. This presents a problem for us because the Brown and Bethe [12] mass limit for neutron stars is 1.5M\(\odot\). From this limit, we would say that these neutron stars in low-mass X-ray binaries would have gone into black holes.

A way out of this problem was suggested by Van den Heuvel [64], which is called the evolution of Her X-1 type X-ray binaries (see especially the Appendix of Van den Heuvel [64]). In this case a radiative donor more massive than the neutron star pours matter in unstable mass transfer across the Roche Lobe onto the neutron star. This mass transfer can occur onto the accretion disc by as much as \(\sim 10^4\dot{M}_{\text{Edd}}\), if \(\dot{M}_{\text{Edd}} \sim 1.5 \times 10^{-8} M_\odot\,\text{yr}^{-1}\) is accreted onto the neutron star, since the Eddington limit goes linearly with \(R\) and the radius of the disc can be \(\sim 10^{10}\) cm. The advection-dominated inflow–outflow solution (ADIOS) of Blandford and Begelman [23] suggests that the binding energy of the matter released at the neutron star can carry away mass, angular momentum and energy from the gas accreting onto the edge of the accretion disc provided the latter does not cool too much. In this way, the binding energy of a gram of gas at the neutron star can carry off \(\sim 10^3\) g of gas at the edge of the accretion disc. Such radiatively-driven outflows are suggested by King and Begelman [22] to enable common envelope evolution to be avoided. Tauris and Savonije [66] have carried out a detailed evolution of low-mass X-ray binaries with \(P_{\text{orb}} > 2\) days using computer programs based on Eggleton’s, which for radiative and semiconvective donors follow, in at least a general way, the above ideas. For a deeply convective donor a short phase of rapid mass loss may reach a rate as large as \(10^4\dot{M}_{\text{Edd}}\) while the mass of the donor drops to well below the neutron star mass. Although rates \(> 10^4\dot{M}_{\text{Edd}}\) would be hypercritical for spherical accretion, somewhat higher rates survive hypercritical accretion provided angular momentum is taken into account [28]. The important point is that the donor mass can be brought down sufficiently far before stable mass transfer at a rate \(\lesssim \dot{M}_{\text{Edd}}\) sets in, so that the neutron star can avoid accreting sufficient mass to send it into a black hole. It is not clear what percentage of the neutron stars will survive black-hole fate. Our rough estimates in Section 6 indicate that only a small fraction need to do so.

For even more massive donors (2–6M\(\odot\)) which are either radiative or semiconvective, work by Tauris et al. [24] indicates that the low-mass X-ray binaries with C/O white-dwarf (CO) companions can be made in much the same way. In an earlier paper, Van den Heuvel [67] had suggested that these binaries would originate from donor stars on the asymptotic giant branch. In order to evolve these, he needed an efficiency \(\alpha > 1\), i.e., sources additional to those included in our earlier common envelope evolution, such as mass loss by instabilities in the AGB, dissociation energy, etc., have to participate in helping to remove the envelope of the donor star.

King and Ritter [25] have computed a scenario for Cyg X-2 with an initial donor mass of \(\sim 3.6M_\odot\). Currently, the donor has a mass of 0.5–0.7M\(\odot\) and a large radius, about 7R\(\odot\). About 2M\(\odot\) must have been lost in super-Eddington accretion, roughly along the lines sketched above. More massive donors can lead to relatively more massive white-dwarf companions, which will be C/O white dwarfs.

In fact, the present situation is that no circular NS-CO\(^2\) binaries which went through common envelope evolution seem to be observed, the alternative Tauris et al. [24] evolution which avoids

\(^2\) The lower suffix c (e) denotes the circular (eccentric) binaries.
common envelope evolution being preferred. This presents a real dilemma for the standard scenario of common envelope evolution. It seems clear [68] that in the binary B2303 + 46 the companion to the pulsar is a C/O white dwarf. B2303 + 46 is an eccentric binary NS-CO_e, indicating that the neutron star was formed last. This is confirmed by the unrecycled field strength of the pulsar $B = 8 \times 10^{11}$ G. Cases have made that the recently discovered J1141-6545 [69] and B1820-11 [70] are also NS-CO_e binaries.

On the other hand, evolutionary calculations show that formation probability of NS-CO_e binaries through common envelope evolution is $\gtrsim 50\%$ as probable as of NS-CO_e binaries [71]. In this evolution the pulsar magnetic moment will be recycled, brought down at least a factor of 100 [3] and possibly even further, down to the empirical values of $\sim 5 \times 10^8$ G found in the NS-CO_e binaries. The lowering of the magnetic fields increases the time of observation by a factor of $\sim 100$ or of 2000, depending on whether the theoretical or empirical magnetic field is used. Since we fairly certainly observe at least one NS-CO_e binary, we should see either 100 or 2000 NS-CO_e binaries which have gone through common envelope evolution. We certainly do not see anything like this, at most the 5 that had earlier been attributed to common envelope evolution, and probably none. Brown et al. [71] remove at least most of this discrepancy by showing that with the introduction of hypercritical accretion the neutron star in common envelope evolution with the evolving main sequence companion goes into a black hole.

8. Discussion and conclusion

Our chief new point in the evolution of binaries of compact objects is the use of hypercritical accretion in common envelope evolution, although the idea of hypercritical accretion is not new (Section 3). Chevalier [28] discussed the possibility that angular momentum might hinder hypercritical accretion. In his treatment of the accretion disc, he assumed gas pressure to dominate, in order to raise the temperature sufficiently for neutrinos to be emitted. This entailed a tiny viscosity, characterized by $\alpha \lesssim 10^{-6}$ in the $\alpha$-description. More reasonable values of $\alpha$ are $\sim 0.1$.

Bethe et al. [72] have shown that for larger $\alpha$'s, $\alpha \sim 0.01-1$, the disc pressure is radiation dominated, and they find a simple hypercritical advection-dominated accretion flow (HADAF) of matter onto the neutron star.

The Bethe et al. HADAF appears to reproduce the Armitage and Livio [77] numerical two-dimensional hydro solution. These latter authors suggest that jets will prevent hypercritical accretion by blowing off the accreting matter. At such high rates of accretion $\sim 1 M_\odot \text{ yr}^{-1}$ the Alfvén radius is, however, close to the neutron star surface, and we believe that this will effectively shut down any magnetically driven jets.

In Section 7 we discussed the advection of a rotating He envelope into a black hole. We believe that two possibilities exist. Phinney and Spruit suggest [80] that the magnetic turbulence is strong enough to keep the He envelope in corotation with the core of the star until shortly before it evolves into a black hole. Then not much angular momentum would have to be advected away in order to let the matter accrete. Alternatively, magnetic turbulence is strong enough so that angular momentum can be carried away from a rapidly rotating He core; then the matter can accrete. From the measured masses of $\sim 7 M_\odot$ we know that most of the He core must fall into the black hole, so
one of these scenarios should hold. Both favor high magnetic turbulence, lending credence to the Chevalier suggestion we quoted.

Note added in proof

The evolutionary calculations by Alexander Heger, referred to in Section 5: High-mass black-hole O/B-star binaries, evolving the CO cores of Wellstein and Langer [10] have now been completed. These CO cores were evolved with lower, more correct, He-star wind losses than had been used by Woosley, Langer, and Weaver [17]. A factor 2–3 reduction is now favored. The Heger calculations show that the scenario outlined in Section 5, particularly the limit of \( \sim 80M_\odot \) ZAMS mass for a star in a binary to go into a black hole, is unchanged by the factor of 2–3 lower He wind loss rates. A paper by Brown et al. [40] summarizing these results and suggesting a scenario for the evolution of high mass X-ray black hole binaries is now in preparation.

The discrepancy between eccentric and circular NS–CO binaries discussed at the end of Section 7 has been increased by details of observations on the eccentric PSR J1141-6545 [41]. This pulsar is found to have a characteristic age \( \tau_c = 1.4 \times 10^6 \) yr, inferred surface magnetic field strength \( B = 1.3 \times 10^{12} \) G. The total mass of the system is \( 2.300 \pm 0.012M_\odot \). Arguments are given that “the companion is probably a massive white dwarf, which formed prior to the birth of the pulsar. Since the companion to the pulsar has not yet been observed optically, there is a small chance that J1141-6545 is a double neutron star system of nearly equal masses. However, \( \sim 1.15M_\odot \) is substantially smaller than any other well measured neutron star masses. If J1141-6545 is confirmed as an eccentric (NS–CO\(_e\)) binary, then one would expect to see an additional \( \sim 70 \) circular (NS–CO\(_c\)) ones, because of the much longer time the latter can be observed due to their low fields.

Finally, we believe that there have been important developments in the theory of gamma ray bursters. First of all, a supernova origin for the black hole in Nova Sco 1994 (GRO J1655-40) [76] has been observed. The atmosphere of the companion F-star (see our Table 1) has a large excess of \( z \)-particle nuclei, especially \( ^{32}\text{S} \). In ordinary supernova explosions little of this element but in the highly-energetic explosions called hypernovae, which accompany gamma ray bursters, much of the latter is produced. Following the Israelian et al. suggestion that a hypernova explosion took place in the formation of the black hole, Brown et al. [78] suggested that Nova Sco 1994 was a relic of a gamma ray burster. This theme was then developed in Ref. [79] which showed that the binary progenitors of the transient black hole sources were also good progenitors of gamma ray bursters.

Acknowledgements

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Appendix A. Common envelope evolution of Cygnus X-3

The closeness of the compact object in Cyg X-3 to its \( \sim 10M_\odot \) companion helium star bears witness to an earlier stage of common envelope evolution. Although the mass of the He star has not been measured, the star is similar to V444 Cygni, the mass of which is \( 9.3 \pm 0.5M_\odot \) \[81\]. For example, from the period change its mass loss rate would be \( \dot{M}_{\text{dyn}} = 0.6 \times 10^{-5}(M_{\text{He}}/10M_\odot)M_\odot \text{ yr}^{-1} \) \[82\], whereas that of V444 Cygni is \( \dot{M}_{\text{dyn}} = 1 \times 10^{-5}M_\odot \text{ yr}^{-1} \) \[83\] indicating an \( M_{\text{He}} \sim 10M_\odot \). Mass loss rates cannot easily be obtained from W.-R. winds because of large nonlinear effects which necessitate corrections for “clumpiness”. However, polarization measurement of the Thomson scattering, which depend linearly on the wind, give a mass loss rate of \( \sim \dot{M} = 0.75 \times 10^{-5}M_\odot \text{ yr}^{-1} \) \[84\], roughly compatible with the period change. In agreement with many other authors we take \( M_{\text{He}} = 10M_\odot \) in Cyg X-3.

Here we evolve a massive O-star binary with initial ZAMS masses of \( 33M_\odot \) and \( 23M_\odot \) as possible progenitor for Cyg X-3. In red giant phase the \( 33M_\odot \) star will transfer its H envelope to the \( 23M_\odot \) companion, leaving a He star of 

\[
M_{\text{He}} = 0.1M_{\text{ZAMS}}^4 = 13M_\odot .
\]

With efficiency of mass transfer assumed to go as \( q^2 \), about half of the \( 20M_\odot \) H-envelope will be accepted by the companion, which then becomes a rejuvenated \( 33M_\odot \) star. The He core of the primary then explodes, going into a \( 1.5M_\odot \) compact object, neutron star or low-mass black hole. After the companion \( 33M_\odot \) star evolves, the binary will go into common envelope evolution. Eq. (29) can be written

\[
\left( \frac{Y_f}{Y_i} \right) = \left( \frac{2.4M_{\text{B,i}}}{M_{\text{A,i}}} \right)^{(c_d - 1)/c_d} (A.2)
\]

where we again take \( c_d = 6 \). With \( M_{\text{B,i}} = 33M_\odot \) and \( M_{\text{A,i}} = 1.5M_\odot \),

\[
Y_f/Y_i = 27 . \quad (A.3)
\]

The compact object mass scales as

\[
M_A \propto Y^{1/(c_d - 1)} = Y^{1/5} \quad (A.4)
\]

so that

\[
M_{A,f} = 2.9M_\odot \quad (A.5)
\]

and the final compact object is certainly a black hole, in agreement with Cherepaschhuk and Moffat \[85\] and Ergma and Yungelson \[86\]. We believe our evolution here to show that this \( \sim 3M_\odot \) black hole is about the most massive that can be formed in common envelope evolution by accretion onto a low-mass compact object, since our \( 33M_\odot \) companion is near to the ZAMS mass range that will lose mass in an LBV phase, unsuitable for common envelope evolution, so it cannot be made much more massive. We next find

\[
a_i/a_f = M_{\text{B,i}}/M_{\text{B,f}} Y_f/Y_1 \approx 70 . \quad (A.6)
\]

For an \( a_f \sim 3.5R_\odot \) this gives

\[
a_i \sim 250R_\odot \quad (A.7)
\]

comfortably within the red-giant range.
Following Ergma and Yungelson [86] we calculate the accretion rate as

\[
\dot{M}_{\text{acc}} = 0.14 \left( \frac{M_{\text{BH}}}{M_{\odot}} \right)^2 v_{1000}^4 P_{\text{hr}}^{-4/3} \left( \frac{M_{\odot}}{M_{\text{tot}}} \right)^{2/3} \dot{M}_{\text{wind}}.
\]  

(A.8)

Here \( v_{1000} \) is the wind velocity in units of 1000 km s\(^{-1} \) and \( P_{\text{hr}} \) is the period in hours.\(^3 \) For \( \dot{M}_{\text{wind}} \) we, as Ergma and Yungelson, take \( \dot{M}_{\text{dyn}} \). These authors take \( v_{1000} = 1.5 \), essentially the result of Van Kerkwijk et al. [82]. An earlier estimate by Van Kerkwijk et al. [87] was \( v_{1000} = 1 \). We believe that the \( v_{\text{wind}} \) to be used here may be different from the (uncertain) measured terminal wind velocities, because the velocity near the compact object is substantially less. Therefore, we take \( v_{1000} = 1 \). Taking \( \dot{M}_{\text{wind}} = \dot{M}_{\text{dyn}} \) we obtain

\[
\dot{M}_{\text{acc}} = 2.2 \times 10^{-7} M_{\odot} \text{ yr}^{-1}.
\]  

(A.9)

This is to be compared with

\[
\dot{M}_{\text{Edd}} = 4\pi c R / \kappa_{\text{es}} = 2.6 \times 10^{-8} \left( \frac{M_{\text{BH}}}{M_{\odot}} \right) M_{\odot} \text{ yr}^{-1},
\]  

(A.10)

where \( \kappa_{\text{es}} = 0.2 \text{ g/cm}^2 \) for He accretion. Our result is in fair agreement with Ergma and Yungelson [86], who find \( \dot{M}_{\text{Edd}} \sim 2.3 \times 10^{-7} M_{\odot} \text{ yr}^{-1} \) for a 10\( M_{\odot} \) black hole. The presence of jets in Cyg X-3 argues for super-Eddington rates of accretion, which we find.

Cherepaschchuk and Moffat [85] estimated the total luminosity of Cyg X-3 to be \( L_{\text{bol}} \sim 3 \times 10^{39} \text{ erg} \). The efficiency of black-hole accretion varies as

\[0.057 < \varepsilon < 0.42\]  

(A.11)

for a black hole at rest to a (maximally rotating) Kerr black hole. We expect the black hole to be spun up by accretion from the wind or accretion disc. Taking an intermediate \( \varepsilon = 0.2 \), we find

\[L = 2.5 \times 10^{39} \text{ erg s}^{-1}\]  

(A.12)

in rough agreement with the Cherepaschchuk and Moffat value.

Cyg X-3 is often discussed as the “missing link” in binary pulsar formation. In fact, because of its high He star mass, upon explosion of the latter, it most probably will break up. But it should be viewed as “tip of the iceberg” [64], in that there must be a great many more such objects with lower mass He stars which are not seen. We have shown, in Section 3 however, that these objects are more likely to contain a black hole than a neutron star.

In our evolutionary scenario, the He star progenitor has about the same ZAMS mass as that of the primary. Thus, the fate of the “naked” He star should be the same low-mass compact object, neutron star or low-mass black hole that resulted from the explosion of the primary.

\[^3\text{Through a slip, the two factors preceding } \dot{M}_{\text{wind}} \text{ appear in the denominator in [86], although we confirm that they carried out their calculations with the correct formula.}\]
Appendix B. Implications for LIGO

Our results that there are 10 times more\(^4\) black hole, neutron star binaries than binary neutron stars has important results for LIGO, the detection rates of which were based on the \(\sim 10^{-5}\) per year per galaxy rates of merging for the latter. The combination of masses which will be well determined by LIGO is the chirp mass

\[
M_{\text{chirp}} = \mu^{3/5} M^{2/5} = (M_1 M_2)^{3/5} (M_1 + M_2)^{-1/5},
\]

where \(M = M_1 + M_2\) is the total system mass. The chirp mass of a NS–NS binary, with both neutron stars of mass 1.4\(M_{\odot}\), is 1.2\(M_{\odot}\). A 10\(^{-5}\) birth rate implies a rate of 3 yr\(^{-1}\) out to 200 Mpc [88]. Kip Thorne informs us that LIGO’s first long gravitational-wave search in 2002–2003 is expected to see binaries with \(M_{\text{chirp}} = 1.2M_{\odot}\) out to 21 Mpc.

The chirp mass corresponding to the Bethe and Brown [1] LMBH-NS binary with masses 2.4\(M_{\odot}\) and 1.4\(M_{\odot}\) is 1.6\(M_{\odot}\). Including an \(\sim 30\%\) increase in the rate to allow for high-mass black-hole, neutron-star mergers (which should be regarded as a lower limit because of the high-mass limit of 80\(M_{\odot}\) used by Bethe and Brown for going into a HMBH) gives a 26 times higher rate than Phinney’s estimate for NS–NS mergers. There factors are calculated from the signal-to-noise ratio, which goes as \(M_5^{3/5}\), and then cubing it to obtain the volume of detectability. We then predict a ratio of 3 \(\times (21/200)^3 \times 26 = 0.09\) yr\(^{-1}\) for 2003, rather slim. The enhanced LIGO interferometer planned to begin in 2004 should reach out beyond 150 Mpc for \(M_{\text{chirp}} = 1.2M_{\odot}\), increasing the detection rate to 3 \(\times (150/200)^3 \times 26 = 33\) yr\(^{-1}\). We therefore predict that LIGO will see more mergers per month than NS–NS mergers per year.

Appendix C. Binary contributions to gamma-ray bursters

The sheer numbers of black-hole, neutron-star binaries should dominate the mergers for gravitational waves, which could be detected by LIGO [39]. For gamma-ray bursts, the presence of an event horizon eases the baryon pollution problem, because energy can be stored in the rotational energy of the black hole, and then released into a cleaner environment via the Blandford–Znajek magnetohydrodynamic process.

Binaries containing a black hole, or single black holes, have been suggested for some time as good progenitors for gamma-ray bursts [89–92,34,93]. Reasons for this include the fact that the rest mass of a stellar mass black hole is comparable to what is required to energize the strongest GRB. Also, the horizon of a black hole provides a way of quickly removing most of the material present in the cataclysmic event that formed it. This may be important because of the baryon pollution problem: we need the ejecta that give rise to the GRB to be accelerated to a Lorentz factor of 100 or more, whereas the natural energy scale for any particle near a black hole is less than its mass. Consequently, we have a distillation problem of taking all the energy released and putting

\(^4\)Actually about 20 times more if we include the binaries in which the pulsar goes into a black hole in the He shell burning evolution. However, these will have masses not very different from the binary neutron stars so we do not differentiate them.
it into a small fraction of the total mass. The use of a Poynting flux from a black hole in a magnetic field \[94\] does not require the presence of much mass, and uses the rotation energy of the black hole, so it provides naturally clean power.

As a neutron star in a binary moves nearer to a black hole companion, it is distorted into a torus around the latter. Most of the torus matter enters the black hole from the last stable Keplerian orbit of \( R = \frac{6GM_{BH}}{c^2} \), carrying considerable angular momentum. In the process the black hole is spun up until it rotates with some fraction of the speed of light. A magnetic field which originates from the neutron star, but which could have been enhanced by differential rotation is anchored in the remaining part of the torus, the accretion disc.

When a rapidly rotating black hole is immersed in a magnetic field, frame dragging twists the field lines near the hole, which causes a Poynting flux to be emitted from near the black hole. This is the Blandford–Znajek mechanism \[94\]. The source of energy for the flux is the rotation of the black hole. The source of the field is the surrounding accretion disk or debris torus. We showed \[95\] that at most 9% of the rest mass of a rotating black hole can be converted to a Poynting flux, making the available energy for powering a GRB

\[
E_{BZ} = 1.6 \times 10^{53} (M/M_\odot) \text{erg} .
\]  

(C.1)

The power depends on the applied magnetic field:

\[
P_{BZ} \sim 6.7 \times 10^{50} B_{15}^2 (M/M_\odot)^2 \text{erg s}^{-1}
\]  

(C.2)

(where \( B_{15} = B/10^{15} \text{G} \)). This shows that modest variations in the applied magnetic field may explain a wide range of GRB powers, and therefore of GRB durations. There has been some recent dispute in the literature whether this mechanism can indeed be efficient \[96\] and whether the power of the BH is ever significant relative to that from the disk \[97\]. The answer in both cases is yes, as discussed by Lee et al. \[95\].

The issue, therefore, in finding efficient GRB sources among black holes is to find those that spin rapidly. There are a variety of reasons why a black hole might have high angular momentum. It may have formed from a rapidly rotating star, so the angular momentum was there all along (‘original spin’, according to Blandford \[98\]); it may also have accreted angular momentum by interaction with a disk (‘venial spin’) or have formed by coalescence of a compact binary (‘mortal spin’). We shall review some of the specific situations that have been proposed in turn.

Neutron star mergers are among the oldest proposed cosmological GRB sources \[99–101\], and especially the neutrino flux is still actively studied as a GRB power source \[102\]. However, once the central mass has collapsed to a black hole it becomes a good source for BZ power, since it naturally spins rapidly due to inheritance of angular momentum from the binary \[103\]. Likewise BH–NS binaries \[104\] will rapidly transfer a large amount of mass once the NS fills its Roche lobe, giving a rapidly rotating BH \[105\]. The NS remnant may then be tidally destroyed, leading to a compact torus around the BH. It is unlikely that this would be long-lived enough to produce the longer GRB, but perhaps the short (\( t \leq 1 \text{s} \)) ones could be produced \[106\]. However, mass transfer could stabilize and lead to a widening binary in which the NS lives until its mass drops to the minimum mass of about 0.1\( M_\odot \), and then becomes a debris torus \[107\]. By then, it is far enough away that the resulting disk life time exceeds 1000 s, allowing even the longer GRB to be made. Thus BH–NS and NS–NS binaries are quite promising. They have the added advantage that their environment
is naturally reasonably clean, since there is no stellar envelope, and much of the initially present baryonic material vanishes into the horizon.

In addition to the mergers from compact objects, Fryer and Woosley [34] suggested that GRBs could originate from the coalescence of low-mass black hole and helium-star binaries in the Bethe and Brown [1] scenario. From Eq. (35) we see that binaries survived in the initial range of $0.5 \times 10^{13} \text{ cm} < a_i < 1.9 \times 10^{13} \text{ cm}$. Inside that range for $0.04 \times 10^{13} \text{ cm} < a_i < 0.5 \times 10^{13} \text{ cm}$ the low-mass black hole coalesces with the core. Hence, using a separation distribution flat in $\ln a$, coalescences are more common than low-mass black-hole, neutron-star binaries by a factor $\ln(0.5/0.04)/\ln(1.9/0.5) = 1.9$. In Bethe and Brown [1] the He star compact-object binary was disrupted ~ 50% of the time in the last explosion, which we do not have here. Thus, the rate of low-mass black-hole, He-star mergers is 3.8 times the formation rate of low-mass black-hole, neutron-star binaries which merge, or $R = 3.8 \times 10^{-4} \text{ yr}^{-1}$ in the Galaxy.

In Table 2 we summarize the formation rates of GRBs and gravity waves from the binaries considered in this review.

Because gamma-ray bursts have a median redshift of 1.5–2 [108]), and the supernova rate at that redshift was 10–20 times higher than now, the gamma-ray burst rate as observed is higher than one expects using the above rates. However, for ease of comparison with evolutionary scenarios we shall use the GRB rate at the present time (redshift 0) of about 0.1 GEM. (Wijers et al. [108] found a factor 3 lower rate, but had slightly underestimated it because they overestimated the mean GRB redshift; see Ref. [106] for more extensive discussions of the redshift dependence.) An important uncertainty is the beaming of gamma-ray bursts: the gamma rays may only be emitted in narrow cones around the spin axis of the black hole, and therefore most GRBs may not be seen by us. An upper limit to the ratio of undetected to detected GRB is 600 [109], so an upper limit to the total required formation rate would be 60 GEM. We may have seen beaming of about that factor or a bit less in GRB 990123 [110], but other bursts (e.g. 970228, 970508) show no evidence of beaming in the afterglows (which may not exclude beaming of their gamma rays). At present, therefore, any progenitor with a formation rate of 10 GEM or more should be considered consistent with the observed GRB rate.

An exciting possibility for the future will be to receive both gravitational-wave and gamma-ray burst signals from the same merger, with attendant detailed measurement, which would give witness to them arising from the same binary.

Because we dealt in this review with binaries, we did not explain one popular model of GRBs, the Woosley Collapsar model [92]. In this model a black hole is formed in the center of a rotating

Table 2
Summary of the formation rates of various sources of gamma-ray bursts (GRB) or gravity waves (GW) from the binaries considered in this review. L(H)BH means low- (high-)mass black hole

<table>
<thead>
<tr>
<th>Object</th>
<th>GRB</th>
<th>GW</th>
<th>Rate [GEM*$^*$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS–NS merger</td>
<td>X</td>
<td>X</td>
<td>10</td>
</tr>
<tr>
<td>NS–BH merger</td>
<td>X</td>
<td>X</td>
<td>100</td>
</tr>
<tr>
<td>WR star–LBH merger</td>
<td>X</td>
<td></td>
<td>380</td>
</tr>
</tbody>
</table>

*GEM means Galactic Events per Megayear; rates are quoted for redshift 0.
W.-R. star. The outer matter can then be accreted into the neutron star, spinning it up. If, however, magnetic turbulence is sufficient to keep the envelope of the progenitor in corotation with the core until a few days before collapse of the latter, as suggested by Phinney and Spruit [80] the He envelope could not furnish enough angular momentum to the black hole for the latter to drive the necessary jets (see the end of Section 8).

References


