Halo mass distribution from moving loops

I. MATTER ERA

The halo around a loop of mass $m \sim \alpha\mu t_i$ and radius $R \sim \beta t_i$, formed in the matter era at time $t_i > t_{eq}$ (scale factor $a_i > a_{eq}$) starts growing at

$$a_{min} \sim X a_i, \tag{1}$$

where

$$X \sim \left(\frac{R^2 v_i}{Gmt_i}\right)^{2/5} \sim \left(\frac{\beta^2 v_i}{\alpha G\mu}\right)^{2/5} \sim 40\mu_{-8}^{-2/5}, \tag{2}$$

and grows as

$$M(a; a_i) \sim m(a/a_{min})^{5/3} \tag{3}$$

until

$$a_{max} \sim X^{5/2} a_i \sim 10^4 \mu_{-8}^{-1} a_i. \tag{4}$$

The total growth during this period is by the factor

$$\left(\frac{a_{max}}{a_{min}}\right)^{5/3} \sim X^{5/2} \sim \frac{a_{max}}{a_i}. \tag{5}$$

At later times, the halo mass grows as

$$M \sim mX^{5/2}(a/a_{max}). \tag{6}$$

We first note that, assuming that $\mu_{-8} \lesssim 1$, for all loops formed in the matter era we have $a_{max} > 1$ (since $a_i > a_{eq} \sim 10^{-4}$). This indicates that all such loops have not yet reached the "normal" growth regime (6).

Using the above equations, we can then express the mass of the halo around a loop formed at redshift z_i as

$$M(z) \sim 10^{11} \mu_{-8}^{5/3} (1 + z_i)^{1/6} (1 + z)^{-5/3} M_\odot. \tag{7}$$

Note that the loops that form earlier end up having larger halos. Note also the very weak dependence on z_i: the halo masses of all loops formed in the matter era differ by less than an order of magnitude. Since the loop density is higher at earlier times, this means that the dominant contribution to the total stellar mass comes from the earliest loops, formed at $z_i \sim z_{eq}$.

To express this quantitatively, the number density of loops formed at time t_i is (at that time)

$$t_i \frac{dn}{dt_i}(t_i) \sim t_i^{-3}. \tag{8}$$

At later times,

$$t_i \frac{dn}{dt_i}(t) \sim \frac{1}{t_i t^2}. \tag{9}$$

From Eq.(7), $t_i \propto M^{-9}$, and thus the mass fraction is

$$M^2 \frac{dn}{dM} \propto M^{10}. \tag{10}$$

This distribution terminates at

$$M_{max}(z) \sim 4 \times 10^{11} \mu_{-8}^{5/3} (1+z)^{-5/3} M_\odot, \tag{11}$$

corresponding to $z_i \sim z_{eq}$.

The conclusion is that the total halo mass is strongly dominated by the earliest loops, formed at z_{eq}. This suggests that loops formed in the radiation era give an even greater contribution.

II. RADIATION ERA

The length of a loop formed at $t_i < t_{eq}$ is still given by $L \sim \alpha t_i$ with $\alpha \sim 0.1$, and the velocity of such a loop at t_{eq} is

$$v_{eq} \sim v_i (t_i/t_{eq})^{1/2}. \tag{12}$$

The length distribution of loops at t_{eq} is

$$L \frac{dn}{dL} \sim \frac{1}{t_{eq}^{3/2} L^{3/2}}. \tag{13}$$

We can study accretion onto such loops in the same way as before, except now we should use the loop velocity v_{eq} at time t_{eq} as initial conditions. The role of the parameter X for these loops is played by

$$\tilde{X}(t_i) \sim \frac{a_{min}}{a_{eq}} \sim \left(\frac{R^2 v_{eq}}{G m t_{eq}} \right)^{2/5} \sim X \left(\frac{t_i}{t_{eq}} \right)^{3/5}. \tag{14}$$

For some loops this gives $\tilde{X} < 1$, indicating that halo formation starts right at t_{eq}. Equating $\tilde{X} \sim 1$, we find the corresponding borderline value of t_i,

$$t_{i*} \sim X^{-5/3} t_{eq} \sim 2 \times 10^{-3} \mu_{-8}^{2/3} t_{eq}, \tag{15}$$

and of the loop mass,

$$m_* \sim \alpha \mu t_{i*} \sim 3 \times 10^5 \mu_{-8}^{5/3} M_\odot. \tag{16}$$

All loops with $m < m_*$ have $\tilde{X} < 1$ and $a_{min}, a_{max} < a_{eq}$, indicating that (i) these loops start growing halos right after t_{eq} and (ii) they skip the accelerated growth regime (3) and go straight to the normal regime,

$$M(a) \sim m(a/a_{eq}). \tag{17}$$

By redshift z these loops will have halos with masses in the range

$$M \lesssim M_*(z) \sim 10^9 (1+z)^{-1} \mu_{-8}^{5/3} M_\odot. \tag{18}$$

The mass distribution of the halos at $z < z_{eq}$ retains the initial form (13),

$$M \frac{dn}{dM} \propto M^{-3/2}. \tag{19}$$

Actually, at $z < z_{eq}$ this form of the distribution extends to larger values of t_i (and M); the borderline value $\tilde{t}_i$ can be found from the condition

$$\frac{a_{max}(\tilde{t}_i)}{a_{eq}} \sim \tilde{X}^{5/2} \sim \frac{1+z_{eq}}{1+z}, \tag{20}$$

which gives

$$\tilde{t}_i(z) \sim t_{i*} \left(\frac{1+z_{eq}}{1+z} \right)^{2/3}. \tag{21}$$

The corresponding mass threshold is

$$\tilde{M}(z) \sim M_*(z) \left(\frac{1+z_{eq}}{1+z} \right)^{2/3} \sim 3 \times 10^{11} \mu_{-8}^{5/3} (1+z)^{-5/3} M_\odot. \tag{22}$$

Loops with masses smaller than $\tilde{M}(z)$ are already in the "normal" accretion regime (17) at redshift z.

Loops with $M > \tilde{M}(z)$ are still in the accelerated accretion regime (3). The mass of halos around such loops is given by

$$M(z) \sim \alpha \mu t_i \left(\frac{a}{\tilde{X} a_{eq}} \right)^{5/3} \sim m_* \left(\frac{a}{a_{eq}} \right)^{5/3} \sim \tilde{M}(z). \tag{23}$$

Interestingly, the halo mass is independent of the loops size in this regime. Thus, the largest halos seeded by loops at redshift z have mass $\tilde{M}(z)$.

If we are interested only in the total stellar mass as a function of z, then much of the above analysis is unncessary. For a significant amount of star formation, we need a halo with a virial temperature above 10^4K to allow atomic hydrogen cooling. This requires

$$M \gtrsim M_{min}(z) \sim 10^9 (1+z)^{-3/2} M_\odot. \tag{24}$$

For μ_{-8} not too different from 1, we have $M_{min}(z) < \tilde{M}(z)$. Since the mass distribution (19) is dominated by smaller loops, the conclusion is that the dominant contribution to star formation at redshift z will come from the halos of mass $\sim M_{min}(z)$.