Accretion onto a moving loop

I. ACCRETION ONTO A MOVING POINT MASS

Here I reproduce Ed Bertschinger's analysis in [1].

Consider a point mass m moving in the z-direction with initial velocity v_i at time t_i in the matter era. Its trajectory in comoving coordinates is given by

$$z_m(t) = d(1 - a^{-1/2}), \tag{1}$$

where $d = 3v_i t_i$ and the scale factor

$$a(t) = (t/t_i)^{2/3} \tag{2}$$

is normalized so that $a(t_i) = 1$. Due to the symmetry, it will be sufficient to analyze accretion in the xz-plane.

Consider a matter particle at (x, z), with the point mass making its closest approach at $a(t) = a_0$. Using (1), this implies

$$\frac{d - z}{d} \sim a_0^{-1/2}. \tag{3}$$

In the impulse approximation, the velocity kick received by the particle is

$$v_x \sim \frac{Gm}{(a_0 x)^2}\Delta t \sim \frac{Gm}{v_i x}, \tag{4}$$

where

$$\Delta t \sim \frac{a_0 x}{v(t)} \sim \frac{a_0^2 x}{v_i} \tag{5}$$

and $v(t) = v_i/a(t)$ is the loop velocity. The displacement of the particle in a Hubble time $t_0 = t_i a_0^{3/2}$ is

$$a_0 \Delta x \sim v_x t_0 \sim v_x t_i a_0^{3/2}. \tag{6}$$

The corresponding density perturbation is

$$\delta_0 \sim \frac{\Delta x}{x} \sim \frac{Gm t_i a_0^{1/2}}{x^2 v_i}. \tag{7}$$

The density perturbation subsequently grows as

$$\delta(t) \sim \delta_0 (a/a_0), \tag{8}$$

and turnaround occurs when $\delta \sim 1$, that is

$$\frac{Gmt_i}{x^2 v_i} \frac{a}{a_0^{1/2}} \sim 1. \tag{9}$$

Expressing a_0 in terms of z from Eq.(3), we find the shape of the turnaround surface at time t,

$$x_{ta}^2 \sim \frac{Gm}{v_i^2}(d-z)a(t). \tag{10}$$

Bertschinger also found the turnaround surface using Zel'dovich approximation; it agrees with (10) up to a numerical factor.

The halo mass can be found as

$$M(t) \sim \rho(t_i) \int_0^{z_m(t)} x_{ta}^2 dz \sim ma(t), \tag{11}$$

where $\rho(t) \sim 1/Gt^2$ is the average matter density and $z_m(t)$ is from (1).

II. ACCRETION ON A LOOP OF FINITE RADIUS

Suppose now that the loop has a finite radius R. Then Eq.(7) for the initial density perturbation applies only at distances $a_0|x| > R$. Expressing a_0 in terms of z from (3), this gives the condition

$$|x| > R\left(\frac{d-z}{d}\right)^2. \tag{12}$$

In our approximation, the perturbation vanishes on smaller scales. Thus, the turnaround surface (10) should be cut off, keeping only the part satisfying Eq.(12).

With $x = x_{ta}$ and x_{ta} from Eq.(10), the condition (12) becomes

$$\frac{Gm}{v_i^2}(d-z)a(t) > R^2\left(\frac{d-z}{d}\right)^4. \tag{13}$$

This corresponds to a cutoff at $z = z_*(t)$, where z_* is found from

$$\frac{d-z_*}{d} \sim \left(\frac{d}{R}\right)^{2/3} [Qa(t)]^{1/3}, \tag{14}$$

where I have introduced the notation

$$Q \equiv \frac{Gm}{v_i^2 d}. \tag{15}$$

Only the part of the halo between $z_*(t)$ and $z_m(t)$ will actually collapse at time t.

The mass of the halo at time t is given by

$$M(t) \sim \rho(t_i) \int_{z_*(t)}^{z_m(t)} x_{ta}^2 dz. \tag{16}$$

This is nonzero only if $z_*(t) < z_m(t)$, which gives the condition

$$a > a_{min} \sim \left(\frac{R^2}{Qd^2}\right)^{2/5}. \tag{17}$$

Thus the halo formation for a moving loop begins only at $a \sim a_{min}$. At later times, the halo mass grows as

$$M(t) \sim \frac{Qd}{Gt_i^2}(d - z_*)^2 a(t) \sim m \left(\frac{a(t)}{a_{min}}\right)^{5/3}. \tag{18}$$

This rapid growth continues until the cutoff $z_*(t)$ reaches the edge of the halo at $z = 0$. This happens when $a \sim a_{max}$, where

$$a_{max} \sim \frac{R^2}{Qd^2}. \tag{19}$$

To summarize, the conclusion is that the halo mass starts growing at $a = a_{min}$ and grows like $a^{5/3}$ until a_{max}, at which point it matches with the spherical regime and continues to grow like $a(t)$.

For our values of the parameters, $Q \sim \alpha G\mu/v_i^3 \sim 10^{-6}$, $R/d \sim 0.1$, $a_{min} \sim 10^{8/5}$ and $a_{max} \sim 10^4$.

[1] E. Bertschinger, Ap.J. 316, 489 (1987).