## Cores in Dwarf Galaxies from Dark Matter with a Yukawa Potential

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(Dated: November 21, 2010)

We show that cold dark matter particles interacting through a Yukawa potential could naturally explain the recently observed cores in dwarf galaxies without affecting the dynamics of objects with a much larger velocity dispersion, such as clusters of galaxies. The velocity dependence of the associated cross-section as well as the possible exothermic nature of the interaction alleviates earlier concerns about strongly interacting dark matter. Dark matter evaporation in low-mass objects might explain the observed deficit of satellite galaxies in the Milky Way halo and have important implications for the first galaxies and reionization.

PACS numbers: 95.35+d,98.80-k,98.62-g

Introduction. The collisionless cold dark matter (CDM) model has been highly successful in accounting for the gravitational growth of density perturbations from their small observed amplitude at early cosmic times (as imprinted on the cosmic microwave background anisotropies [1]) to the present-day structure of the Universe on large scales. However, it is far from clear that the predictions of this model are valid on small scales.

New data on dwarf galaxies indicates that their dark matter distribution has a core in contrast to the cusped profile expected from collisionless CDM simulations [2]. The mean value of the logarithmic inner slope of the mass density profile in seven dwarf galaxies is observed to be  $-0.29 \pm 0.07$  [3], much shallower than the expected slope of  $\sim -1$  from pure CDM simulations. The latest data agree with earlier studies of the dynamics of individual cases, such as Fornax [4], Ursa-Minor [5], and Sculptor [6], in which a characteristic core density of  $\sim 0.1 \pm 0.05 M_{\odot} \ \mathrm{pc^{-3}} = (7 \pm 4) \times 10^{-24} \ \mathrm{g \ cm^{-3} \ was}$ inferred. Since these dwarf galaxies are dominated by dark matter throughout, it is challenging to explain the inferred cores by the gravitational interaction of the dark matter with the baryons [7]. Although it is conceivable that strong gas outflows from an early baryon-dominated nucleus would reduce the central dark matter density [8], Neal: please add the new paper by Oh et al., arXiv:1011.2777, to the above reference of 'Gov'. the formation of a massive baryonic nucleus would initially compress the CDM [9] and exacerbate the discrepancy that needs to be resolved, as well as overproduce the observed luminosities at higher redshifts [7].

To alleviate early signs of the above discrepancy, Spergel & Steinhardt [10] adopted the Strongly-Interacting Dark Matter (SIDM) model [11–13] in which the dark matter has a large cross-section for self interaction. It was expected that if dark matter scatters in

the cores of galaxies, then it might resemble a fluid with a flatter central density profile. The SIDM proposal fell out of favor because: (i) gravitational lensing and X-ray data indicate that the cores of clusters of galaxies are dense and ellipsoidal, whereas SIDM would predict they are shallow and spherical [14, 15]; (ii) relaxation of the core ultimately generates an even denser nucleus after many collision times due to the gravothermal catastrophe, familiar from core collapse of globular clusters [16], although this evolution might take more than the Hubble time; (iii) dwarf galaxies would be expected to evaporate when interacting with the higher velocity particles of the host halo; and (iv) theoretical biases suggested that the required cross section was incompatible with popular models of Weakly-Interacting Massive Particles (WIMPs).

Recently, there has been growing interest in the possibility that WIMPs exhibit "dark forces" as a means to address a wide range of anomalies [17]. In particular, it was realized theoretically that a new force carrier  $\phi$ (scalar or vector) might naturally mediate a long-range interaction on the scale of the de-Broglie wavelength of the WIMPs, leading to a self-interaction cross-section for scattering that is much greater than for WIMP annihilation. The studied forces have a variety of scales in them, from the screening scale set by the mass of the carrier particle  $m_{\phi}$  to the non-perturbative scale set by its coupling,  $\alpha_D m_\phi$ . Moreover, these forces are naturally accompanied by new energy states. Up- and down-scattering processes (which are endo- and exo-thermic, respectively) naturally introduces yet another scale into the problem, set by the energy splitting  $\delta$  between the states. The absence of dramatic departures from CDM predictions has allowed important constraints to be placed [18, 19].

In this *Letter*, we examine the possible existence of a dark force from a different perspective. Rather than limit

its allowed range of parameters based on observations, we show that it can *ameliorate* tensions in astrophysical data. In particular, we find that a Yukawa force in dark matter scattering would naturally produce cores in dwarf galaxies, while avoiding the myriad constraints on SIDM which arise in systems with a much larger velocity dispersion, such as clusters of galaxies. The specific velocity dependence of the interaction cross-section, as well as the possible exothermic nature of the interaction, alleviate earlier concerns about the SIDM model. To distinguish from previous approaches with a constant cross section or a simple power law velocity dependence, we label this scenario as Yukawa-Potential Interacting Dark Matter (YIDM).

Dark Forces. For our purposes, the mediator of the force  $\phi$  could be either a scalar or a vector, as magnetic-type interactions are negligible. Still, the simplest perturbative realizations of dark forces involve models with an additional gauge boson  $\phi_{\mu}$  of small mass  $m_{\phi} \stackrel{<}{\sim} \text{GeV}$ . We assume that the WIMPs are charged under a new  $U(1)_{dark}$  symmetry and allow the possibility of a small splitting between their Majorana states. This leads to the Lagrangian,

$$\mathcal{L} = \bar{\chi} \not \!\!\!D \chi + \frac{1}{4} F_{\mu\nu}^d F^{d\mu\nu} + \epsilon F_{\mu\nu} F^{d\mu\nu} + m^2 \phi_\mu \phi^\mu + M \bar{\chi} \chi + \delta \chi \chi. \tag{1}$$

This can be trivially generalized to a non-Abelian model with multiple excited states [17, 20], which induces splittings between the states radiatively at order  $\alpha_D m_\phi$ , where  $\alpha_D$  is the fine structure constant of the dark force. For light ( $\stackrel{<}{\sim}$  GeV) force carriers, the mixing is effectively with electromagnetism. Constraints on the presence of such a force come from a wide range of processes [21, 22], but ample parameter space remains for a small  $\epsilon \stackrel{<}{\sim} 10^{-3}$ . New searches are underway to find precisely such a force carrier at  $\sim$  GeV energy experiments [23].

Scattering through a massive mediator is equivalent to having a Yukawa potential. The elastic scattering problem is then analogous to the screened Coulomb scattering in a plasma [24], which is well fit by a cross-section [25],

$$\langle \sigma \rangle \approx \begin{cases} \frac{4\pi}{m_{\phi}^{2}} \beta^{2} \ln(1 + \beta^{-1}), & \beta < 0.1, \\ \frac{8\pi}{m_{\phi}^{2}} \beta^{2} / (1 + 1.5\beta^{1.65}), & 0.1 \le \beta \le 10^{3}, \\ \frac{\pi}{m_{\phi}^{2}} \left( \ln \beta + 1 - \frac{1}{2} \ln^{-1} \beta \right)^{2}, & \beta > 10^{3}, \end{cases}$$
(2)

where  $\beta = \pi v_{\sigma}^2/v^2 = 2\alpha_D m_{\phi}/(m_{\chi}v^2)$ , and v is the relative velocity of the particles. We use  $\langle \ \rangle$  to denote that this is the momentum-transfer weighted cross section. Here,  $v_{\sigma}$  is the velocity at which the momentrum-weighted scattering rate  $\langle \sigma v \rangle$  peaks at a cross section value of  $\sigma_{\rm max} = 22.7/m_{\phi}^2$ . The above expression can be generalized to the inelastic case by substituting  $m_{\phi} \rightarrow \sqrt{m_{\chi}\delta}$  for the characteristic minimum momentum transfer, when  $m_{\phi} < \sqrt{m_{\chi}\delta}$ . Figure 1 depicts the velocity de-

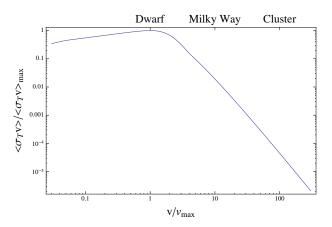


FIG. 1: Dependence of the self-interaction cross-section  $(\sigma)$  on the relative velocity (v) for dark matter interacting through a Yukawa potential. The normalizations of  $\sigma$  and v are set by the free parameters of the Lagrangian in Eq. (1), and we show two possible curves, one peaking at 10 km s<sup>-1</sup>, and another peaking at 100 km s<sup>-1</sup>.

pendence of the elastic cross-section in Eq. (2). Interestingly, the scattering rate is nearly constant at low velocities, peaks at a velocity  $v_{\sigma}$ , and declines sharply at v > $v_{\sigma}$ , allowing it to introduce cores in dwarf galaxies where the velocity dispersion is low ( $v \sim 10 \text{ km s}^{-1}$ ) but not in clusters of galaxies where the characteristic velocities are larger by two orders of magnitude ( $v \sim 10^3 \text{ km s}^{-1}$ ). The normalizations of the cross-section and velocity are set by the free parameters of the interaction Lagrangian in Eq. (1), with the Compton wavelength of the interaction setting the relevant spatial scale. We show two possible values of the peak velocity, one that would produce cores only in dwarf galaxies ( $v_{\sigma} = 10 \text{ km s}^{-1}$ ), and anothat that would produce cores in more massive galaxies  $v_{\sigma} = 10^2 \text{ km s}^{-1}$ ), as implied by data on low surface brightness galaxies [? ]. Neal: please add a reference here to Naray et al., arXiv:0912.3518. At any given halo mass, we expect scatter in the core properties of individual halos, due to variations in their age and assembly history.

Having one collision per Hubble time at the characteristic core density of dwarf galaxies  $\sim 0.1 M_{\odot}$  pc<sup>3</sup>, translates to the condition  $(m_{\chi}/1\text{GeV})(m_{\phi}/100\text{MeV})^2 \sim 1$ . An order of magnitude larger cross-sections are also allowed by the data. Figure 2 shows the allowed parameter ranges [19] that would naturally explain the dark matter distribution in observed astrophysical objects. We find that even though collisions shape the central profiles of dwarf galaxies, the standard collisionless treatment still provides an excellent approximation for the dark matter dynamics in X-ray clusters. NW: Gnedin and Ostriker discussed evaporation of dwarfs from strong interactions, and am looking into the precise quantitative constraint.

Neal: The normalization of the horizontal axis

in Fig. 1 needs to be changed. It can only apply to the solid curve but not the dashed curve. We should normalize 'v' by 10 km/s instead of  $\mathbf{v}_sigma.Thisway, the top labels will be correct as well.$ 

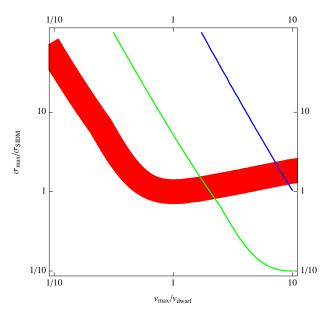


FIG. 2: Astrophysical constraints on the normalization of the self-interaction cross-section  $(\sigma_{\rm max})$  as a function of the velocity at which the peak collision rate is obtained  $(v_{\sigma})$  in Fig. 1. We show  $(red, solid)~\langle\sigma\rangle_{max}/m_{\chi}\approx 7\times 10^{-24}~{\rm cm}^2/{\rm GeV}$  and  $v_{\rm dwarf}\approx 10~{\rm km~s}^{-1},$  which should be regarded as the minimum interaction necessary to flatten the cores of dwarf galaxies. Additional lines indicate upper limits on the cross-section based on astrophysical considerations: X-ray cluster ellipticity (blue (dashed)), limiting  $(\sigma_{max}/m_{\chi})\stackrel{<}{\sim} 4\times 10^{-26}~{\rm cm}^2/{\rm GeV}$  at  $v\sim 10^3{\rm km~s}^{-1};$  and dwarf galaxy evaporation (green (dotted)), limiting  $(\sigma_{max}/m_{\chi})\stackrel{<}{\sim} 5\times 10^{-25}~{\rm cm}^2/{\rm GeV}$  [? ] at  $v\sim 10{\rm km~s}^{-1}.$ 

Neal: what is the velocity at which the above evaporation constraint applies? If it is at 10 km/s, then the limit mentioned in the caption of Fig. 2 is in conflict with the required cross-section value (also mentioned in the same caption). Neal: we should consider showing both cases of  $\mathbf{v}_maxinFigure2aswell$ .

Exothermic interaction. The presence of excited states related to a new force has important implications for the properties of dark matter. In particular, upscattering ("inelastic dark matter" (iDM) [?]) or downscattering [26???, 27] off nuclei can dramatically alter the properties of direct detection experiments. Dark matter self scattering into an excited state ("eXciting Dark Matter" (XDM) [28]) has been invoked to explain the excess 511 keV flux observed by INTEGRAL [26, 28–30, 30, 31].

We focus here on the response of dwarf galaxies to the presence of excited states, YIDM\*, which

we assume are copiously present from the early universe. In particular, the release of kinetic energy in YIDM\* collisions would help to evade the gravothermal catastrophe [16] on arbitrary timescales, in just the same way that the energy released by primordial stellar binaries weakens core collapse in globular clusters [32].

For Abelian gauge theories, the scattering always changes the number of excited states by a multiple of two, i.e.,  $11 \Longleftrightarrow 22$  and  $12 \Longleftrightarrow 21$ . This suggests that it will be difficult to inelastically scatter the 1 states. However, in the case of a non-Abelian theory, more states, and thus more scattering possibilities, are present. For instance,  $13 \Longrightarrow 22$  would allow an exothermic scattering, with all scatterings of comparable strength.

If excited states exist, then a major fraction of the CDM might be excited when the WIMPS decouple thermally in the early Universe. This excitation could be stable on cosmological times in models where the dark force mixes with electromagnetism [26, 27]. The scattering process can decouple in the early universe at temperatures above the splitting, leaving essentially equal abundances of all states of the dark matter. Hence, the early dynamics of CDM will be identical to the standard collisionless model until dwarf galaxies form and the crossing of dark matter streams at low v occurs in their cores, giving rise to self-interactions on a timescale shorter than the age of the Universe.

We denote the velocity imparted to an initially slowly-moving WIMPs ( $v < v_{\rm crit}$ ) upon scattering by  $v_{\rm crit} = \sqrt{\delta/m}$ . Since the gravitational potential of a dwarf galaxy halo is shallow, sufficiently exothermic collisions could eject colliding particles out of the halo if  $v_{\rm crit}$  exceeds the escape velocity. The halo core will lose particles until it reaches a density such that the interaction time is comparable to the age of the halo. Requiring that the final core particles will interact only once over the current age of the Universe yields a final mass density of dark matter,

$$\left(\frac{\rho_{\chi}}{0.1 M_{\odot} {\rm pc^{-3}}}\right) \left(\frac{\sigma/m_{\chi}}{6 \times 10^{-25} {\rm cm^2 GeV^{-1}}}\right) \left(\frac{v}{10~{\rm km s^{-1}}}\right) \sim 1.(3)$$

The profile of  $\rho_{\chi}$  will then be set by the velocity dependence of  $\langle \sigma v \rangle$  and the gravitational potential in a steady state. Below we show that for an exothermic interaction  $\langle \sigma v \rangle$  is constant, leading naturally to a constant density core in dwarf galaxies. This model predicts that dwarf galaxies of a similar age have a similar core density, in agreement with observations in the local Universe [33].

If the characteristic scattering velocity is much higher than  $v_{\rm crit}$ , the process is essentially elastic, and one can employ Eq. (2). On the other hand, if the process is highly exothermic then the scattering rate is given simply by  $\sigma v = 2\pi v_{\rm crit} \alpha_d^2/m_\phi^4$ , for  $m_\chi \delta < m_\phi^2$ . The resulting velocity-independent  $\sigma v$  would naturally produce cores with a flat density profile in dwarf galaxies.

The density flattening in dwarf galaxies does not imply an upper limit on the dark matter density in all its cusps. For massive halos, the release of excess kinetic energy by collisions has a marginal significance, since it only perturbs the low-velocity tail of the CDM distribution function and adds a negligible energy at high relative velocities where the majority of particles have a low interaction rate anyway.

The evaporation of exothermic YIDM\* from dwarf galaxies with a gravitational binding energy below the energy released in collisions could potentially account for the deficit in the observed abundance of dwarf galaxies relative to theoretical CDM expectations [34]. Numerical simulations are necessary to reliably quantify this important effect. Also, dark matter halos which accrete cold gas at early times but evaporate at late time might leave behind a star cluster without dark matter. If so, some old globular clusters [35] might be the sought-after remnants of the missing dwarf galaxies in the Milky Way halo.

Evolution with redshift. The primordial density perturbations are modified by WIMP scatterings, but for  $m_\chi \stackrel{>}{\sim} {\rm GeV}$  this modification ends well before observable modes enter the horizon, leaving the standard nearly scale invariant spectrum for cosmological structures. The imprint of collisions on the density profile of halos is expected to evolve with redshift, because at earlier cosmic times halos are denser and younger. A halo collapsing at a redshift  $z\gg 1$  has a characteristic virial radius,

$$r_{\rm vir} = 1.5 \left(\frac{M}{10^8 M_{\odot}}\right)^{1/3} \left(\frac{1+z}{10}\right)^{-1} \text{ kpc} , \qquad (4)$$

a corresponding circular velocity,

$$V_{\rm vir} = 17.0 \left(\frac{M}{10^8 M_{\odot}}\right)^{1/3} \left(\frac{1+z}{10}\right)^{1/2} \text{ km s}^{-1}$$
 (5)

and an age limit of  $\sim 0.5~{\rm Gyr}[(1+z)/10]^{-3/2}$ . It would be particularly interesting to explore the formation of the first galaxies using numerical simulations of the YIDM model. Deviations from the standard CDM predictions could be tested by upcoming galaxy surveys or 21-cm observations of the high-redshift Universe [36, 37]. For

example, the scaling  $\sigma \propto v_{\rm rel}^{-4}$  might not be allowed to continue to arbitrarily low velocities, as this would have delayed reionization beyond observational constraints [1]. Neal: please add a reference here after WMAP to Pritchard, J. R., Loeb, A., & Wyithe, J. S. B. 2010, Mon. Not. Roy. Astron. Soc., 408, 57.

Discussion. We have shown that a Yukawa potential interaction of dark matter can explain the recent data on dark matter cores in dwarf galaxies [3, 6], while evading the many constraints previously considered for SIDM. The new ingredients of the YIDM model involve the non-trivial velocity dependence of the scattering rate and the possibility of exothermic interactions.

The velocity dependence of the cross-section does not have a simple power-law form, as invoked previously [15?]. The presence of a plateau with a sharp cut-off in the scattering rate of YIDM allows the interaction to be effective for dwarf galaxies while being entirely suppressed at high velocities relevant for cluster cores and evaporation of small sub-halos within bigger halos.

Excited states naturally accompany these forces, and introduce a qualitatively new ingredient: the possibility of energy release. In deep gravitational potentials these scatterings will be similar to standard elastic scatterings, whereas in shallow gravitational potentials a single scatter can eject the colliding dark matter from the halo. Exothermic interactions drive dwarf halos to constant density cores. Moreover, for the earliest forming halos with the highest dark matter density, it is possible that the dark matter would evaporate at later times, leaving behind baryonically-dominated stellar clusters such as the oldest globular clusters.

Unlike SIDM with a single parameter of  $\sigma/m_\chi$ , YIDM has a broader parameter space. Out of the possible combinations of the underlying particle physics parameters  $(\alpha, \delta, m_\chi, m_\phi)$ , only three affect the properties of halos: the velocity  $v_\sigma$  at which the scattering rate  $\langle \sigma v \rangle$  peaks, and the velocity imparted to a particle undergoing an exothermic scattering  $v_{\rm crit}$ .

Since halos form hierarchically and the coarsegrained phase-space density can only decrease through their mergers (Liouville's theorem), the development of a core in dwarf galaxies would trim the central cusp in bigger halos as well. Numerical simulations are required to quantify the average behaviour as well as the scatter in the core properties of halos as a function of mass and redshift.

Finally, we note that the similarity between the required self-interaction cross-section per unit mass of the dark matter and baryons may indicate a deep underlying relationship between these components.

Acknowledgments. This work was supported in part by NSF grant AST-0907890 and NASA grants NNX08AL43G and NNA09DB30A for AL. NW is supported by DOE OJI grant #DE-FG02-06ER41417 and NSF grant #0947827, as well as by the Amborse Monell Foundation.

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