Partially Ionized Plasmas

Nick Murphy

Harvard-Smithsonian Center for Astrophysics

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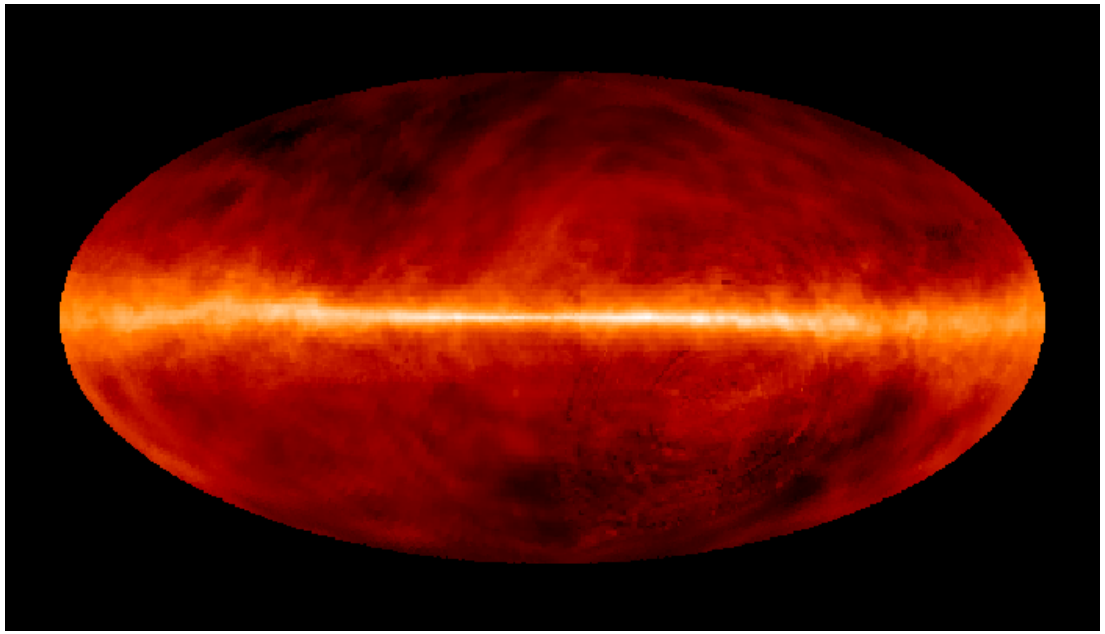
These lecture notes are based off of Leake et al. (2014), Meier & Shumlak (2012), Zweibel (1989), Zweibel et al. (2011), and other sources. The slides on non-equilibrium ionization are based off of discussions with John Raymond and Chengcai Shen.

Introduction

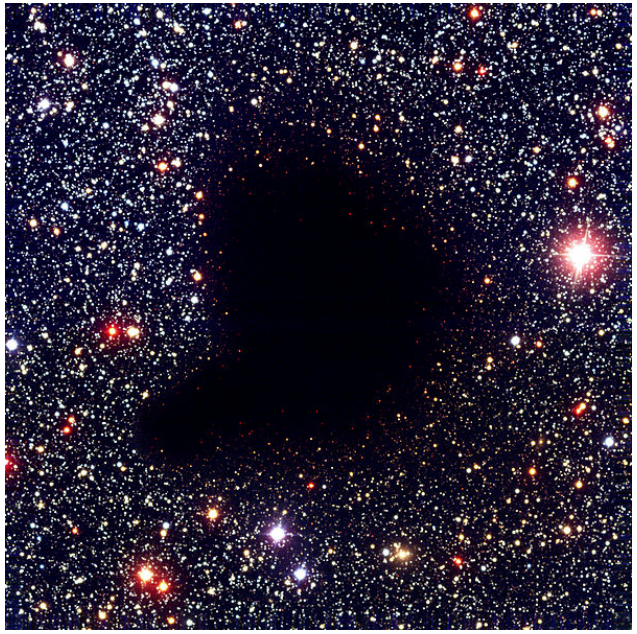
- MHD assumes that plasmas are fully ionized
- Plasmas below $\sim 10^4$ K are partially ionized (assuming: collisional)
- There exist many partially/weakly ionized plasmas in astrophysics and elsewhere
  - The multiphase interstellar medium
  - Molecular clouds
  - Protoplanetary disks
  - Stellar chromospheres
  - Earth's ionosphere
  - Some laboratory plasma experiments
- Need to modify equations to include neutrals

The multiphase interstellar medium



- Important tracer of neutral hydrogen: 21 cm line
- Cold neutral medium
  - $n \sim 10$ cm^{-3}
  - $T \sim 40$ to 200 K
- Warm neutral medium
  - $n \sim 1$ cm^{-3}
  - $T \sim 4000$ to 8000 K

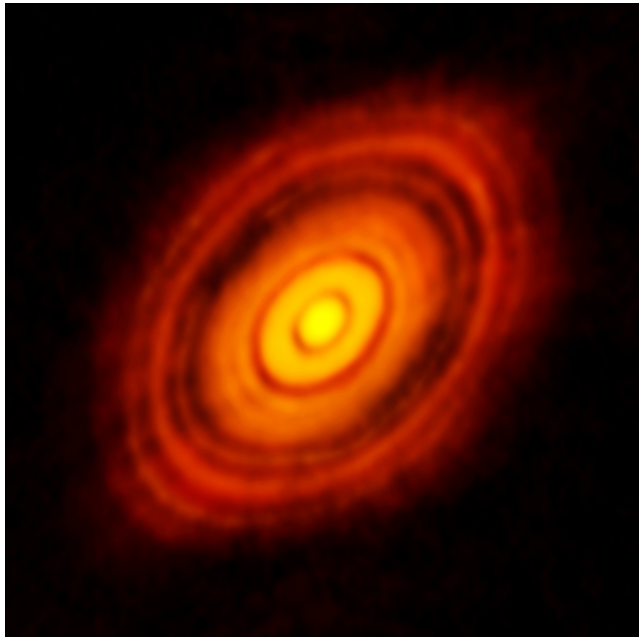
Molecular clouds



- Temperatures of $\sim$10–20 K; densities of $\sim 10^2$–10^6 cm^{-3}
- Ionization due to cosmic rays even for very cold temperatures
- Partial ionization effects important because of very long length scales
  - Coupling between ions and neutrals
- Problem: how is mass transported across field lines so stars can form?

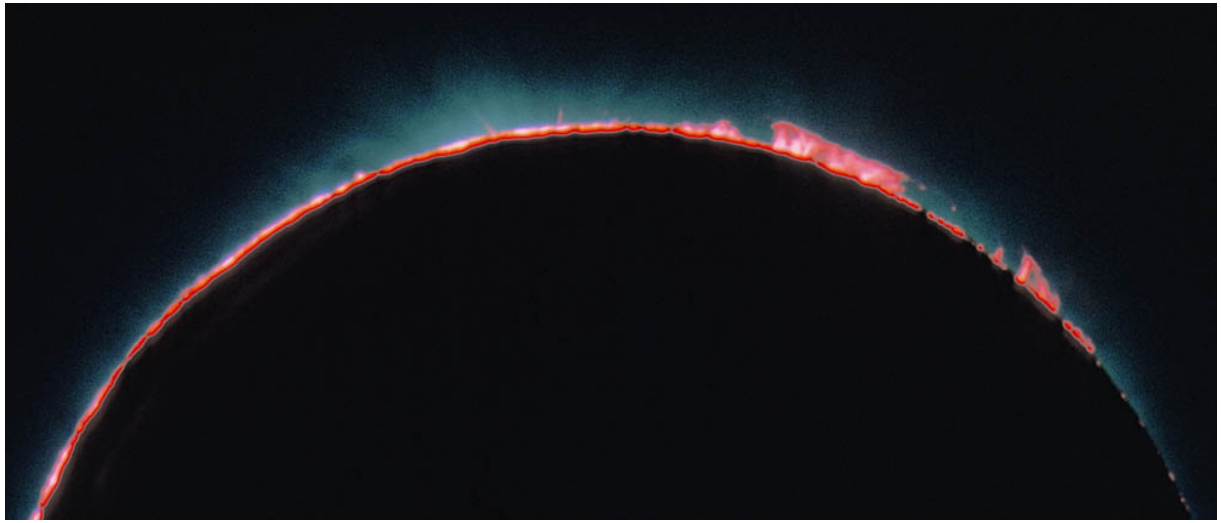
Protoplanetary disks

ALMA observation of HL Tau



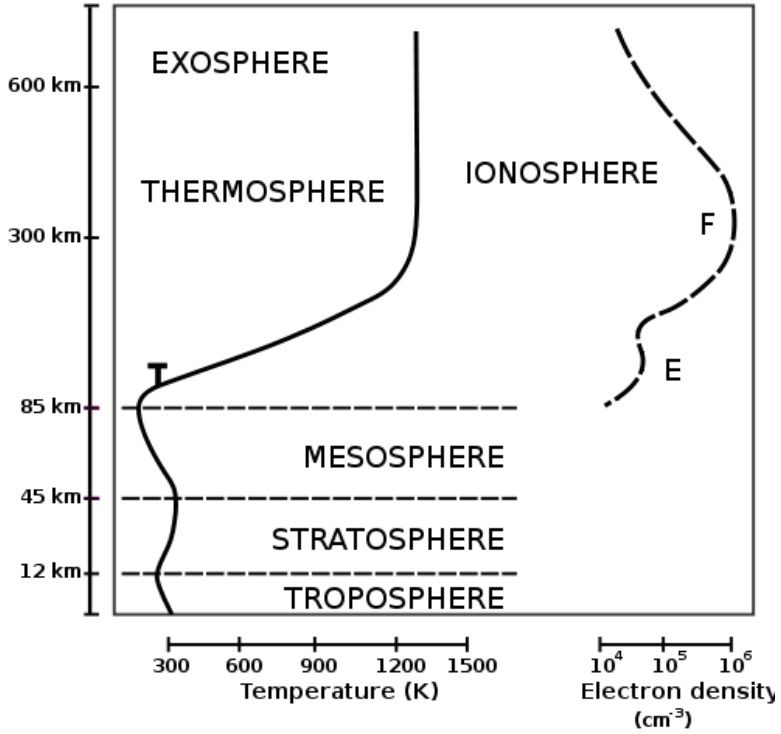
- Weakly ionized, dusty plasma
- Sometimes need additional terms in generalized Ohm's law

The solar chromosphere



- Ionization fraction ranges from $\sim$0.5% to $\sim$50%
- As we go outward from the photosphere to corona, there are transitions from
  - Pressure dominated ($\beta \gg 1$) to magnetically dominated ($\beta \ll 1$)
  - Weakly ionized to fully ionized
  - Optically thick to optically thin
- Modeling the chromosphere requires partial ionization effects and NLTE radiative transfer
- Very dynamic region with reconnection, instabilities, etc.

Earth's thermosphere and ionosphere



- Transition region between atmosphere and magnetosphere
  - Similarities to chromosphere (Leake et al. 2014)
- Driven from above and below
- Ionized by EUV and X-ray solar radiation
  - Variation over days, seasons, solar cycle
- Affects radio propagation

The continuity equations

- There are separate conservation of mass equations for ions, neutrals, and electrons

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{V}_i) = +\Gamma^{ion} - \Gamma^{rec} \quad (1)$$

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{V}_e) = +\Gamma^{ion} - \Gamma^{rec} \quad (2)$$

$$\frac{\partial n_n}{\partial t} + \nabla \cdot (n_n \mathbf{V}_n) = -\Gamma^{ion} + \Gamma^{rec} \quad (3)$$

  where we are considering only neutrals and singly charged ions
- Assuming $n_e = n_i$ yields identical ion and electron equations
- The ionization and recombination rates are Γ^{ion} and Γ^{rec}
  - Source and sink terms on the RHS

Let's look at two forms of the momentum equations

- Extended form
  - More realistic
  - Contains fewer approximations
  - Used in recent simulation efforts
- Simplified form
  - Makes use of approximations
  - Focus on the most important effects
  - Useful for deriving ambipolar diffusion

The momentum equations

- The ion, electron, and neutral momentum equations are

$$\frac{\partial}{\partial t}(\rho_i \mathbf{V}_i) + \nabla \cdot (\rho_i \mathbf{V}_i \mathbf{V}_i) = -\nabla \cdot \mathbf{P}_i + q_i n_i \left(\mathbf{E} + \frac{\mathbf{V}_i \times \mathbf{B}}{c} \right)$$
$$+\mathbf{R}_i^{ie} + \mathbf{R}_i^{in} + \Gamma^{ion} m_i \mathbf{V}_n - \Gamma^{rec} m_i \mathbf{V}_i \quad (4)$$

$$\frac{\partial}{\partial t}(\rho_e \mathbf{V}_e) + \nabla \cdot (\rho_e \mathbf{V}_e \mathbf{V}_e) = -\nabla \cdot \mathbf{P}_e - q_e n_e \left(\mathbf{E} + \frac{\mathbf{V}_e \times \mathbf{B}}{c} \right)$$
$$+\mathbf{R}_i^{ie} + \mathbf{R}_e^{en} + \Gamma^{ion} m_e \mathbf{V}_n - \Gamma^{rec} m_e \mathbf{V}_e \quad (5)$$

$$\frac{\partial}{\partial t}(\rho_n \mathbf{V}_n) + \nabla \cdot (\rho_n \mathbf{V}_n \mathbf{V}_n) = -\nabla \cdot \boldsymbol{P}_n - \mathbf{R}_i^{in} - \mathbf{R}_e^{en}$$
$$+\Gamma^{rec} (m_i \mathbf{V}_i + m_e \mathbf{V}_e) \quad (6)$$

- Okay . . . but what does this all mean?

Terms on the RHS of the momentum equation

- **Pressure forces** (e.g., $-\nabla \cdot \boldsymbol{P}_i$)
  - Acts within a species
  - Represented as a tensor, but will be a scalar if collisional
- **Lorentz force**
  - Acts only on electrons and ions
  - Does not directly act on neutrals
- Momentum transfer due to **drag forces** (e.g, $\mathbf{R}_i^{in}$)
  - Collisions allow momentum exchange between species
  - Allows coupling between the plasma & neutral components
  - Ions can drag along neutrals
- Momentum transfer due to **change of identity**
  - When an ion and electron recombine, each particle's momentum gets transferred to the neutral momentum equation
- One may also include **charge exchange** reactions
  - Electron from one atom gets transferred to another atom

A simpler form of the momentum equations

- Represent momentum transfer as a drag force

$$\rho_n \left(\frac{\partial}{\partial t} + \mathbf{V}_n \cdot \nabla \right) \mathbf{V}_n = -\nabla p_n - \rho_n \nu_{ni} \left(\mathbf{V}_n - \mathbf{V}_i \right) \tag{7}$$

$$\rho_i \left(\frac{\partial}{\partial t} + \mathbf{V}_i \cdot \nabla \right) \mathbf{V}_i = -\nabla p_i + \frac{\mathbf{J} \times \mathbf{B}}{c} - \rho_i \nu_{in} \left(\mathbf{V}_i - \mathbf{V}_n \right) \tag{8}$$

with

$$\rho_n \nu_{ni} = \rho_i \nu_{in} = \frac{\rho_i \rho_n \langle \sigma V \rangle_{in}}{m_i + m_n} \tag{9}$$

  - $\langle \sigma V \rangle_{in}$ is the rate coefficient (averaging over velocity distribution)
  - σ is the cross section
  - V is the relative velocity in the center of mass frame
  - $\nu_{\alpha\beta}$ is the collision frequency of a particle α with a particle β

For a weakly ionized plasma, we can neglect ion inertia and the ion pressure gradient

- The ion momentum equation becomes

$$\frac{\mathbf{J} \times \mathbf{B}}{c} = \rho_i \nu_{in} (\mathbf{V}_i - \mathbf{V}_n) \tag{10}$$

- The induction equation in the ion frame is

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V}_i \times \mathbf{B}) \tag{11}$$

- Use $\mathbf{V}_i \approx \mathbf{V} + (\mathbf{V}_i - \mathbf{V}_n)$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \nabla \times \left(\frac{\mathbf{J} \times \mathbf{B}}{\rho_i \nu_{in} c} \times \mathbf{B} \right) \tag{12}$$

The rightmost term corresponds to ambipolar drift

Ambipolar drift: simple geometry

- Suppose $\mathbf{B} = B_z(x,t)\,\hat{\mathbf{z}}$ with $\mathbf{V} = 0$. Then

$$\frac{\partial B_z}{\partial t} = \frac{\partial}{\partial x}\left(D_{AD}\frac{\partial B_z}{\partial x}\right) \tag{13}$$

  where

$$D_{AD} \equiv \frac{V_A^2}{\nu_{ni}} \tag{14}$$

  Ambipolar drift acts like nonlinear diffusion in slab geometry
- The magnetic field decouples from the bulk flow when

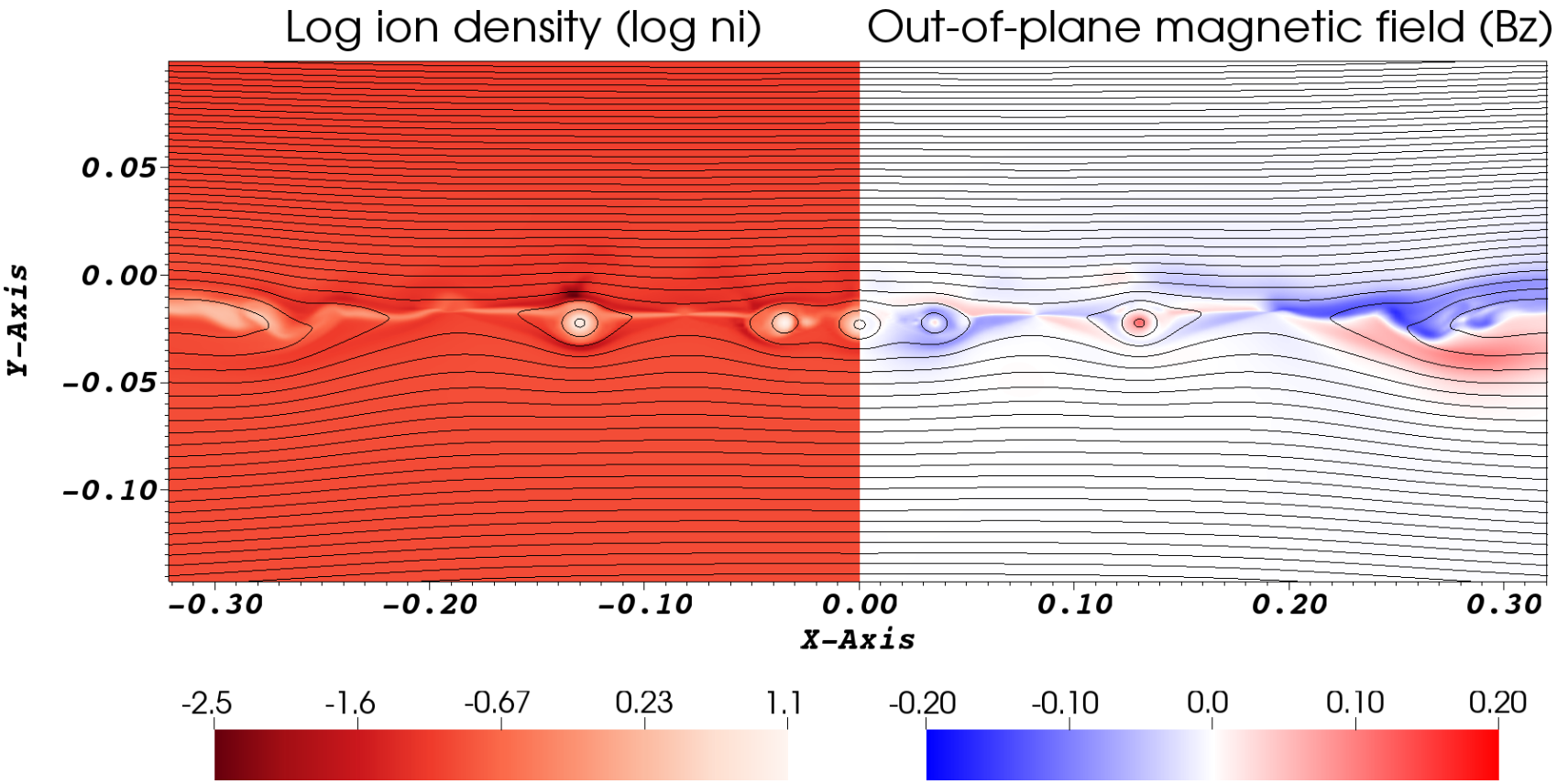
$$R_{AD} \equiv \frac{LV}{D_{AD}} \lesssim 1 \tag{15}$$

- Ambipolar diffusion can facilitate current sheet thinning and singularity formation, and can result in plasma heating

There are separate energy equations for different species (e.g., Meier & Shumlak 2012)

- Heating due to ion/neutral friction
- Thermal conduction
  - Isotropic for neutrals
  - Anisotropic for ions & electrons
- Heat transfer between species
  - Collisions
  - Charge exchange
- Ionization energy

Partial ionization impacts chromospheric reconnection dynamics (including during the plasmoid instability)



- The magnetic field sweeps excess ions into the current sheet
- The overabundant ions recombine to become neutrals which reduces the bottleneck associated with ion mass conservation

Non-equilibrium ionization (NEI)

- Ionization equilibrium occurs when the ionization and recombination time scales are much shorter than thermodynamic evolution time scales
  - This assumption is not met in many diffuse, quickly evolving plasmas!
- Examples of NEI plasma:
  - Solar wind & CMEs (outside of a few $R_\odot$)
  - Supernova remnants
- Relatively simple to model hydrogen, but we also care about heavier elements

Building up intuition for NEI processes

- Suppose you rapidly **heat** plasma (e.g., shocks)
  - Ionization takes time to catch up to temperature changes
  - The charge state distribution will imply that the plasma is cooler than it actually is
  - The plasma is **underionized**
- Suppose you rapidly **cool** plasma (e.g., adiabatic expansion)
  - Recombination takes time to catch up to temperature decrease
  - The charge state distribution will imply that the plasma is hotter than it actually is
  - The plasma is **overionized**

How do you model non-equilibrium ionization plasmas?

- Following a parcel of plasma, evolve the equation for every charge state of each element of interest

$$\begin{aligned}\frac{\mathrm{d}n_z}{\mathrm{d}t} &= n_e n_{z-1} q_i(Z, z-1, T) \\ &\quad - n_e n_z \left[q_i(Z, z, T) + \alpha_r(Z, z, T)\right] \\ &\quad + n_e n_{z+1} \alpha_r(Z, z+1, T)\end{aligned} \tag{16}$$

  where z is the charge, Z is the atomic number, q_i is the ionization rate, and α_r is the recombination rate. Assumes collisionally dominated.
- Beware: atomic data have errors!
  - $\sim$10–20% errors for best data
  - Higher errors for less well known data & theoretical calculations
  - Energetic particles can increase ionization rates
- The thermodynamic history of a plasma out of ionization equilibrium is encoded in the charge states

Summary

- Examples of partially ionized plasmas include stellar chromospheres, molecular clouds, and Earth's ionosphere
- Partially ionized plasmas are described using separate equations for the neutrals, ions, and electons
- These equations include momentum transport and energy transfer between species
- Ambipolar diffusion arises when the induction equation is written using the bulk velocity instead of the ion velocity
- Partial ionization impacts dispersion relations for waves & instabilities
- Non-equilibrium ionization is important in diffuse plasmas when temperature changes occur more quickly than ionization and recombination can keep up