These lecture notes are based off of Leake et al. (2014), Meier & Shumlak (2012), Zweibel (1989), Zweibel et al. (2011), and other sources. The slides on non-equilibrium ionization are based off of discussions with John Raymond and Chengcai Shen.
Introduction

- MHD assumes that plasmas are fully ionized
- Plasmas below $\sim 10^4$ K are partially ionized (assuming: collisional)
- There exist many partially/weakly ionized plasmas in astrophysics and elsewhere
  - The multiphase interstellar medium
  - Molecular clouds
  - Protoplanetary disks
  - Stellar chromospheres
  - Earth’s ionosphere
  - Some laboratory plasma experiments
- Need to modify equations to include neutrals
The multiphase interstellar medium

- Important tracer of neutral hydrogen: 21 cm line
- Cold neutral medium
  - $n \sim 10 \, \text{cm}^{-3}$
  - $T \sim 40$ to $200 \, \text{K}$
- Warm neutral medium
  - $n \sim 1 \, \text{cm}^{-3}$
  - $T \sim 4000$ to $8000 \, \text{K}$
Molecular clouds

- Temperatures of \(\sim 10–20 \text{ K}\); densities of \(\sim 10^2–10^6 \text{ cm}^{-3}\)
- Ionization due to cosmic rays even for very cold temperatures
- Partial ionization effects important because of very long length scales
  - Coupling between ions and neutrals
- Problem: how is mass transported across field lines so stars can form?
Protoplanetary disks

- Weakly ionized, dusty plasma
- Sometimes need additional terms in generalized Ohm’s law
The solar chromosphere

- Ionization fraction ranges from $\sim 0.5\%$ to $\sim 50\%$
- As we go outward from the photosphere to corona, there are transitions from
  - Pressure dominated ($\beta \gg 1$) to magnetically dominated ($\beta \ll 1$)
  - Weakly ionized to fully ionized
  - Optically thick to optically thin
- Modeling the chromosphere requires partial ionization effects and NLTE radiative transfer
- Very dynamic region with reconnection, instabilities, etc.
Earth’s thermosphere and ionosphere

- Transition region between atmosphere and magnetosphere
  - Similarities to chromosphere (Leake et al. 2014)
- Driven from above and below
- Ionized by EUV and X-ray solar radiation
  - Variation over days, seasons, solar cycle
- Affects radio propagation
The continuity equations

There are separate conservation of mass equations for ions, neutrals, and electrons

\[
\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{V}_i) = +\Gamma^{\text{ion}} - \Gamma^{\text{rec}} \tag{1}
\]

\[
\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{V}_e) = +\Gamma^{\text{ion}} - \Gamma^{\text{rec}} \tag{2}
\]

\[
\frac{\partial n_n}{\partial t} + \nabla \cdot (n_n \mathbf{V}_n) = -\Gamma^{\text{ion}} + \Gamma^{\text{rec}} \tag{3}
\]

where we are considering only neutrals and singly charged ions

Assuming \(n_e = n_i\) yields identical ion and electron equations

The ionization and recombination rates are \(\Gamma^{\text{ion}}\) and \(\Gamma^{\text{rec}}\)

Source and sink terms on the RHS
Let’s look at two forms of the momentum equations

- **Extended form**
  - More realistic
  - Contains fewer approximations
  - Used in recent simulation efforts

- **Simplified form**
  - Makes use of approximations
  - Focus on the most important effects
  - Useful for deriving ambipolar diffusion
The ion, electron, and neutral momentum equations are

\[
\frac{\partial}{\partial t} (\rho_i V_i) + \nabla \cdot (\rho_i V_i V_i) = -\nabla \cdot P_i + q_i n_i \left( E + \frac{V_i \times B}{c} \right) + R_{ie}^i + R_{in}^i - \Gamma_{rec} m_i V_i
\]

\[
\frac{\partial}{\partial t} (\rho_e V_e) + \nabla \cdot (\rho_e V_e V_e) = -\nabla \cdot P_e - q_e n_e \left( E + \frac{V_e \times B}{c} \right) + R_{ie}^e + R_{en}^e + \Gamma_{ion} m_e V_n - \Gamma_{rec} m_e V_e
\]

\[
\frac{\partial}{\partial t} (\rho_n V_n) + \nabla \cdot (\rho_n V_n V_n) = -\nabla \cdot P_n - R_{in}^i - R_{en}^e + \Gamma_{rec}^i (m_i V_i + m_e V_e)
\]

Okay . . . but what does this all mean?
Terms on the RHS of the momentum equation

- **Pressure forces** (e.g., $-\nabla \cdot P_i$)
  - Acts within a species
  - Represented as a tensor, but will be a scalar if collisional

- **Lorentz force**
  - Acts only on electrons and ions
  - Does not directly act on neutrals

- **Momentum transfer due to drag forces** (e.g., $R_{i}^{in}$)
  - Collisions allow momentum exchange between species
  - Allows coupling between the plasma & neutral components
  - Ions can drag along neutrals

- **Momentum transfer due to change of identity**
  - When an ion and electron recombine, each particle’s momentum gets transferred to the neutral momentum equation

- One may also include **charge exchange** reactions
  - Electron from one atom gets transferred to another atom
A simpler form of the momentum equations

- Represent momentum transfer as a drag force

\[
\begin{align*}
\rho_n \left( \frac{\partial}{\partial t} + V_n \cdot \nabla \right) V_n &= -\nabla p_n - \rho_n \nu_{ni} (V_n - V_i) \\
\rho_i \left( \frac{\partial}{\partial t} + V_i \cdot \nabla \right) V_i &= -\nabla p_i + \frac{\mathbf{J} \times \mathbf{B}}{c} - \rho_i \nu_{in} (V_i - V_n)
\end{align*}
\]

with

\[
\rho_n \nu_{ni} = \rho_i \nu_{in} = \frac{\rho_i \rho_n \langle \sigma V \rangle_{in}}{m_i + m_n}
\]

- \(\langle \sigma V \rangle_{in}\) is the rate coefficient (averaging over velocity distribution)
- \(\sigma\) is the cross section
- \(V\) is the relative velocity in the center of mass frame
- \(\nu_{\alpha\beta}\) is the collision frequency of a particle \(\alpha\) with a particle \(\beta\)
For a weakly ionized plasma, we can neglect ion inertia and the ion pressure gradient.

- The ion momentum equation becomes
  \[
  \frac{\mathbf{J} \times \mathbf{B}}{c} = \rho_i \nu_{in} (\mathbf{V}_i - \mathbf{V}_n) \quad (10)
  \]

- The induction equation in the ion frame is
  \[
  \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V}_i \times \mathbf{B}) \quad (11)
  \]

- Use \( \mathbf{V}_i \approx \mathbf{V} + (\mathbf{V}_i - \mathbf{V}_n) \)
  \[
  \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \nabla \times \left( \frac{\mathbf{J} \times \mathbf{B}}{\rho_i \nu_{in} c} \times \mathbf{B} \right) \quad (12)
  \]

The rightmost term corresponds to ambipolar drift.
Ambipolar drift: simple geometry

Suppose \( \mathbf{B} = B_z(x, t) \hat{z} \) with \( \mathbf{V} = 0 \). Then

\[
\frac{\partial B_z}{\partial t} = \frac{\partial}{\partial x} \left( D_{AD} \frac{\partial B_z}{\partial x} \right)
\]

(13)

where

\[
D_{AD} \equiv \frac{V_A^2}{\nu_{ni}}
\]

(14)

Ambipolar drift acts like nonlinear diffusion in slab geometry.

The magnetic field decouples from the bulk flow when

\[
R_{AD} \equiv \frac{LV}{D_{AD}} \lesssim 1
\]

(15)

Ambipolar diffusion can facilitate current sheet thinning and singularity formation, and can result in plasma heating.
There are separate energy equations for different species (e.g., Meier & Shumla 2012)

- Heating due to ion/neutral friction
- Thermal conduction
  - Isotropic for neutrals
  - Anisotropic for ions & electrons
- Heat transfer between species
  - Collisions
  - Charge exchange
- Ionization energy
Partial ionization impacts chromospheric reconnection dynamics (including during the plasmoid instability)

- The magnetic field sweeps excess ions into the current sheet
- The overabundant ions recombine to become neutrals which reduces the bottleneck associated with ion mass conservation
Non-equilibrium ionization (NEI)

- Ionization equilibrium occurs when the ionization and recombination time scales are much shorter than thermodynamic evolution time scales
  - This assumption is not met in many diffuse, quickly evolving plasmas!
- Examples of NEI plasma:
  - Solar wind & CMEs (outside of a few $R_\odot$)
  - Supernova remnants
- Relatively simple to model hydrogen, but we also care about heavier elements
Suppose you rapidly **heat** plasma (e.g., shocks)
- Ionization takes time to catch up to temperature changes
- The charge state distribution will imply that the plasma is cooler than it actually is
- The plasma is **underionized**

Suppose you rapidly **cool** plasma (e.g., adiabatic expansion)
- Recombination takes time to catch up to temperature decrease
- The charge state distribution will imply that the plasma is hotter than it actually is
- The plasma is **overionized**
How do you model non-equilibrium ionization plasmas?

- Following a parcel of plasma, evolve the equation for every charge state of each element of interest

\[
\frac{dn_z}{dt} = n_e n_{z-1} q_i(Z, z - 1, T) - n_e n_z [q_i(Z, z, T) + \alpha_r(Z, z, T)] + n_e n_{z+1} \alpha_r(Z, z + 1, T)
\]  

(16)

where \(z\) is the charge, \(Z\) is the atomic number, \(q_i\) is the ionization rate, and \(\alpha_r\) is the recombination rate. Assumes collisionally dominated.

- Beware: atomic data have errors!
  - \(\sim\)10–20\% errors for best data
  - Higher errors for less well known data & theoretical calculations
  - Energetic particles can increase ionization rates

- The thermodynamic history of a plasma out of ionization equilibrium is encoded in the charge states
Examples of partially ionized plasmas include stellar chromospheres, molecular clouds, and Earth’s ionosphere.

Partially ionized plasmas are described using separate equations for the neutrals, ions, and electrons.

These equations include momentum transport and energy transfer between species.

Ambipolar diffusion arises when the induction equation is written using the bulk velocity instead of the ion velocity.

Partial ionization impacts dispersion relations for waves & instabilities.

Non-equilibrium ionization is important in diffuse plasmas when temperature changes occur more quickly than ionization and recombination can keep up.